PME44 VIRTUAL

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FOR THE PSYCHOLOGY OF MATHEMATICS EDUCATION
July 19-22, 2021
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Proceedings

of the 44th Conference of the International Group
for the Psychology of Mathematics Education

VOLUME 1

Plenary Lecture, Working Groups,
Seminar, National Presentation,
Oral Communications,
Poster Presentations, Colloquium

Editors:
Maitree Inprasitha, Narumon Changsri
and Nisakorn Boonsena

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PREFACE

We are pleased to welcome you to PME 44. PME is one of the most important international conferences in mathematics education and draws educators, researchers, and mathematicians from all over the world. The PME 44 Virtual Conference is hosted by Khon Kaen University and technically assisted by Technion Israel Institute of Technology. The COVID-19 pandemic made massive changes in countries' economic, political, transport, communication, and education environment including the 44th PME Conference which was postponed from 2020. The PME International Committee / Board of Trustees decided against an on-site conference in 2021, in accordance with the Thailand team of PME 44 will therefore go completely online, hosted by the Technion - Israel Institute of Technology, Israel, and takes place by July 19-22, 2021. A national presentation of PME-related activities in Thailand is part of the conference program.

This is the first time such a conference is being held in Thailand together with CLMV (Cambodia, Laos, Myanmar, Vietnam) countries, where mathematics education is underrepresented in the community. Hence, this conference will provide chances to facilitate the activities and network associated with mathematics education in the region. Besides, we all know this pandemic has made significant impacts on every aspect of life and provides challenges for society, but the research production should not be stopped, and these studies needed an avenue for public presentation. In this line of reasoning, we have hosted the IGPME annual meetings for the consecutive year, July 21 to 22, 2020, and 19 to 22 July 2021, respectively by halting "on-site" activities and shift to a new paradigm that is fully online. Therefore, we would like to thank you for your support and opportunity were given to us twice.

"Mathematics Education in the 4th Industrial Revolution: Thinking Skills for the Future" has been chosen as the theme of the conference, which is very timely for this era. The theme offers opportunities to reflect on the importance of thinking skills using AI and Big Data as promoted by APEC to accelerate our movement for regional reform in education under the 4th industrial revolution. Computational Thinking and Statistical Thinking skills are the two essential competencies for Digital Society. For example, Computational Thinking is related to using AI and coding while Statistical Thinking is related to using Big Data. Therefore, Computational Thinking is mostly associated with computer science, and Statistical Thinking is mostly associated with statistics and probability on academic subjects. However, the way of thinking is not limited to be used in specific academic subjects such as informatics at the senior secondary school level but used in daily life.

For the PME 44 Thailand 2021, we have 661 participants from 55 different countries. We are particularly proud of broadening the base of participation in mathematics education research across the globe. The papers in the four proceedings are organized according to the type of presentation. Volume 1 contains the presentation of our Plenary Lectures, Plenary Panel, Working Group, the Seminar, National Presentation, the Oral Communication presentations, the Poster Presentations, the Colloquium. Volume 2 contains the Research Reports (A-G). Volume 3 contains Research Reports (H-R), and Volume 4 contains Research Reports (S-Z).

The organization of PME 44 is a collaborative effort involving staff of Center for Research in Mathematics Education (CRME), Centre of Excellence in Mathematics (CEM), Thailand

Society of Mathematics Education (TSMEd), Institute for Research and Development in Teaching Profession (IRDTP) for ASEAN Khon Kaen University, The Educational Foundation for Development of Thinking Skills (EDTS) and The Institute for the Promotion of Teaching Science and Technology (IPST). Moreover, all the members of the Local Organizing Committee are also supported by the International Program Committee. I acknowledge the support of all involved in making the conference possible. I thank each and every one of them for their efforts. Finally, I thank PME 44 participants for their contributions to this conference.

Thank you

Best regards

Associate Professor Dr. Maitree Inprasitha
PME 44 the Year 2021
Conference Chair

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THE INTERNATIONAL GROUP FOR THE PSYCHOLOGY OF MATHEMATICS EDUCATION (PME)

HISTORY OF PME

The International Group for the Psychology of Mathematics Education (PME) is an autonomous body, governed as provided for in the constitution. It is an official subgroup of the International Commission for Mathematical Instruction (ICMI) and came into existence at the Third International Congress on Mathematics Education (ICME 3) held in Karlsruhe, Germany in 1976. Its former presidents have been:

Efraim Fischbein, Israel
Stephen Lerman, UK
Richard R. Skemp, UK
Gilah Leder, Australia
Gerard Vergnaud, France
Rina Hershkowitz, Israel
Kevin F. Collis, Australia
Chris Breen, South Africa
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Nicolas Balacheff, France
João Filipe Matos, Portugal
Kathleen Hart, UK
Barbara Jaworski, UK
Carolyn Kieran, Canada

The current president is **Markku Hannula** (Finland).

THE CONSTITUTION OF PME

The constitution of PME was adopted by the Annual General Meeting on August 17, 1980 and changed by the Annual General Meetings on July 24, 1987, on August 10, 1992, on August 2, 1994, on July 18, 1997, on July 14, 2005 and on July 21, 2012. The major goals of the group are:

- to promote international contact and exchange of scientific information in the field of mathematical education;
- to promote and stimulate interdisciplinary research in the aforesaid area;

and

- to further a deeper and more correct understanding of the psychological and other aspects of teaching and learning mathematics and the implications thereof.

All information concerning PME and its constitution can be found at the PME website: www.igpme.org

PME MEMBERSHIP AND OTHER INFORMATION

Membership is open to people involved in active research consistent with the aims of PME, or professionally interested in the results of such research. Membership is on an annual basis and depends on payment of the membership fees. PME has between 700 and 800 members from about 60 countries all over the world.

The main activity of PME is its yearly conference of about 5 days, during which members have the opportunity to communicate personally with each other during working groups, poster sessions and many other activities. Every year the conference is held in a different country.

There is limited financial assistance for attending conferences available through the Richard Skemp Memorial Support Fund.

A PME Newsletter is issued three times a year, and can be found on the PME website. Occasionally PME issues a scientific publication, for example the result of research done in group activities.

WEBSITE OF PME

All information concerning PME, its constitution, and past conferences can be found at the PME website: www.igpme.org

HONORARY MEMBERS OF PME

Efraim Fischbein (Deceased)
Hans Freudenthal (Deceased)
Joop Van Dormolen (Retired)

PME ADMINISTRATIVE MANAGER

The administration of PME is coordinated by the Administrative Manager:

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INTERNATIONAL COMMITTEE OF PME

Members of the International Committee (IC) are elected for four years. Every year, four members retire and four new members are elected. The IC is responsible for decisions concerning organizational and scientific aspects of PME. Decisions about topics of major importance are made at the Annual General Meeting (AGM) during the conference. The IC work is led by the PME president who is elected by PME members for three years.

President

Markku Hannula (Finland)

Vice-President

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PROCEEDINGS OF PREVIOUS PME CONFERENCES

The table includes the ERIC numbers, links to download, ISBN/ISSN of the proceedings, and/or the website address of annual PME.

No.	Year	Location	ERIC number, ISBN/ISSN and/or website address
1	1977	Utrecht, The Netherlands	Not available in ERIC
2	1978	Osnabrück, Germany	ED226945, ISBN 3-922211-00-3
3	1979	Warwick, United Kingdom	ED226956
4	1980	Berkeley, USA	ED250186
5	1981	Grenoble, France	ED225809
6	1982	Antwerp, Belgium	ED226943, ISBN 2-87092-000-8
7	1983	Shoresh, Israel	ED241295, ISBN 965-281-000-2
8	1984	Sydney, Australia	ED306127
9	1985	Noordwijkerhout, The Netherlands	ED411130 (vol.1) ED411131 (vol. 2)
10	1986	London, United Kingdom	ED287715
11	1987	Montréal, Canada	ED383532, ISSN 0771-100X
12	1988	Veszprém, Hungary	ED411128 (vol. 1) ED411129 (vol. 2)
13	1989	Paris, France	ED411140 (vol. 1) ED411141 (vol. 2) ED411142 (vol. 3)
14	1990	Oaxtepex, Mexico	ED411137 (vol. 1) ED411138 (vol. 2) ED411139 (vol. 3)

No.	Year	Location	ERIC number, ISBN/ISSN and/or website address
15	1991	Assisi, Italy	ED413162 (vol. 1) ED413163 (vol. 2) ED413164 (vol. 3)
16	1992	Durham, USA	ED383538
17	1993	Tsukuba, Japan	ED383536
18	1994	Lisbon, Portugal	ED383537
19	1995	Recife, Brazil	ED411134 (vol. 1) ED411135 (vol. 2) ED411136 (vol. 3)
20	1996	Valencia, Spain	ED453070 (vol. 1) ED453071 (vol. 2) ED453072 (vol. 3) ED453073 (vol. 4) ED453074 (addendum)
21	1997	Lahti, Finland	ED416082 (vol. 1) ED416083 (vol. 2) ED416084 (vol. 3) ED416085 (vol. 4)
22	1998	Stellenbosch, South Africa	ED427969 (vol. 1) ED427970 (vol. 2) ED427971 (vol. 3) ED427972 (vol. 4) ISSN 0771-100X
23	1999	Haifa, Israel	ED436403, ISSN 0771-100X
24	2000	Hiroshima, Japan	ED452301 (vol. 1) ED452302 (vol. 2) ED452303 (vol. 3) ED452304 (vol. 4) SSN 0771-100X

No.	Year	Location	ERIC number, ISBN/ISSN and/or website address
25	2001	Utrecht, The Netherlands	ED466950, ISBN 90-74684-16-5
26	2002	Norwich, United Kingdom	ED476065, ISBN 0-9539983-6-3
27	2003	Honolulu, Hawai'i, USA	ED500857 (vol.1) ED500859 (vol.2) ED500858 (vol.3) ED500860 (vol.4) ISSN 0771-100X http://www.hawaii.edu/pme27
28	2004	Bergen, Norway	ED489178 (vol.1) ED489632 (vol.2) ED489538 (vol.3) ED489597 (vol.4) ISSN 0771-100X www.emis.de/proceedings/PME28
29	2005	Melbourne, Australia	ED496845 (vol. 1) ED496859 (vol. 2) ED496848 (vol. 3) ED496851 (vol. 4) ISSN 0771-100X
30	2006	Prague, Czech Republic	ED496931 (vol. 1) ED496932 (vol. 2) ED496933 (vol. 3) ED496934 (vol. 4) ED496939 (vol. 5) ISSN 0771-100X http://class.pedf.cuni.cz/pme30
31	2007	Seoul, Korea	ED499419 (vol. 1) ED499417 (vol. 2) ED499416 (vol. 3) ED499418 (vol. 4) ISSN 0771-100X

No.	Year	Location	ERIC number, ISBN/ISSN and/or website address
32	2008	Morelia, Mexico	ISBN 978-968-9020-06-6 ISSN 0771-100X http://www.pme32-na30.org.mx
33	2009	Thessaloniki, Greece	ISBN 978-960-243-652-3 ISSN 0771-100X
34	2010	Belo Horizonte, Brazil	ISSN 0771-100X http://pme34.lcc.ufmg.br
35	2011	Ankara, Turkey	ISBN 978-975-429-262-6 ISSN 0771-100X http://www.arber.com.tr/pme35.org
36	2012	Taipei, Taiwan	ISSN 0771-100X http://tame.tw/pme36
37	2013	Kiel, Germany	ISBN 978-3-89088-287-1 ISSN 0771-100X http:// http://www.pme2013.de/
38	2014	Vancouver, Canada	ISBN 978-0-86491-360-9 ISSN 0771-100X http://www.pme38.com/
39	2015	Hobart, Australia	ISBN 978-1-86295-829-6 ISSN 0771-100X http://www.pme39.com
40	2016	Szeged, Hungary	ISSN 0771-100X http://pme40.hu
41	2017	Singapore	ISBN 978-981-11-3742-6 http://math.nie.edu.sg/pme41
42	2018	Umeå, Sweden	ISSN 0771-100X ISBN (Vol 1) 978-91-7601-902-3 ISBN (Vol 2) 978-91-7601-903-0 ISBN (Vol 3) 978-91-7601-904-7 ISBN (Vol 4) 978-91-7601-905-4 ISBN (Vol 5) 978-91-7601-906-1 http://www.pme42.se/

43	2019	Pretoria, South Africa	ISBN (Volume 1, print): 978-0-6398215-0-4 ISBN (Volume 1, electronic): 978-0-6398215-1-1 ISBN (Volume 2, print): 978-0-6398215-2-8 ISBN (Volume 2, electronic): 978-0-6398215-3-5 ISBN (Volume 3, print): 978-0-6398215-4-2 ISBN (Volume 3, electronic): 978-0-6398215-5-9 ISBN (Volume 4, print): 978-0-6398215-6-6 ISBN (Volume 4, electronic): 978-0-6398215-7-3 https://www.up.ac.za/pme43

THE PME 44 CONFERENCE

Two committees are responsible for the organization of the PME 44 Conference: the International Program Committee (IPC) and the Local Organizing Committee (LOC).

THE INTERNATIONAL PROGRAM COMMITTEE (IPC)

The International Programme Committee (IPC)			
Conference Chair	Maitree Inprasitha	LOC	Khon Kaen University, Thailand
Co-Chair	Narumon Changsri	LOC	Khon Kaen University, Thailand
	Wee Tiong Seah	LOC	University of Melbourne, Australia
	Tatsuya Mizoguchi	LOC	Tottori University, Japan
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	Ceneida Fernandez	PME	University of Alicante, Spain
	Anthony A. Essien	PME	WITS University, South Africa
	Judy Anderson	PME	University of Sydney, Australia

THE LOCAL ORGANISING COMMITTEE (LOC)

The host institution is Khon Kaen University. The conference is presented jointly by Center for Research in Mathematics Education (CRME), Thailand Society of Mathematics Education (TSMEd), Institute for Research and Development in Teaching Profession for ASEAN Khon Kaen University (IRDTP), Centre of Excellence in Mathematics (CEM), The Educational Foundation for Development of Thinking Skills (EDTS) and The institute for the Promotion of Teaching Science and Technology (IPST).

The Local Organising Committee (LOC)		
Conference Chair	Maitree Inprasitha	Khon Kaen University, Thailand
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	Tatsuya Mizoguchi	Tottori University, Japan
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ACKNOWLEDGMENTS

We thank all of our reviewers and the IPC for the detailed review work that has led to the presentations in these proceedings.

REVIEW PROCESS OF PME 44

RESEARCH REPORTS (RR)

Research Reports are intended to present empirical or theoretical research results on a topic that relates to the major goals of PME. Reports should state what is new in the research, how the study builds on past research, and/or how it has developed new directions and pathways. Some level of critique must exist in all papers.

The number of submitted RR proposals was 296, and 146 of them were accepted. Of those not accepted as RR proposals, 34 were invited to be re-submitted as Oral Communication (OC) and 13 as Poster Presentation (PP). As in previous years, every RR submission underwent a fully independent double-blind peer review by three international experts in the field in order to decide acceptance for the conference.

ORAL COMMUNICATIONS (OC)

Oral Communications are intended to present smaller studies and research that is best communicated by means of a shorter oral presentation instead of a full Research Report. They should present empirical or theoretical research studies on a topic that relates to the major goals of PME.

The number of submitted OC proposals was 74, and 61 of them were accepted. Of those not accepted as OC proposals, 10 were invited to be re-submitted as Poster Presentation (PP). In the end, considering re-submissions of Research Reports as Oral Communications, 79 OCs were accepted for presentation at PME 44.

POSTER PRESENTATIONS (PP)

Poster Presentations are intended for information/research that is best communicated in a visual form rather than an oral presentation. They should present empirical or theoretical research studies on a topic that relates to the major goals of PME.

The number of submitted PP proposals was 28, and 25 of them were accepted. In the end, considering re-submissions of Research Reports and Oral Communications as Poster Presentations, 34 PPs were accepted for presentation at PME 44.

COLLOQUIUM (CQ)

The goal of a Colloquium is to provide the opportunity to present a set of three papers that are interrelated in a particular way (e.g., they are connected through related or contrasting theoretical stances, use identical instruments or methods, or focus on closely related research questions), and to initiate a discussion with the audience on the interrelated set.

The number of submitted CQ proposals was 1, and it was accepted.

WORKING GROUPS (WG)

The aim of Working Group is that PME participants are offered the opportunity to engage in exchange or to collaborate in respect to a common research topic (e.g., start a joint research activity, share research experiences, continue or engage in academic discourse). A Working Group may deal with emerging topics (in the sense of newly developing) as well as topics that are not new but possibly subject to changes. It must provide opportunities for contributions of the participants that are aligned with a clear goal (e.g. share materials, work collaboratively on texts, and discuss well-specified questions). A Working Group is not supposed to be a collection of individual research presentations (see Colloquium format), but instead is meant to build a coherent opportunity to work on a common research topic. In contrast to the Research Forum format that is meant to present the state of the art of established research topics, Working Groups are considered to involve fields where research topics are evolving.

The number of submitted WG proposals was 5 and all of them were accepted.

SEMINARS (SE)

The goal of a Seminar is the professional development of PME participants, especially new researchers and/or first comers, in different topics related to scientific PME activities. This encompasses, for example, aspects like research methods, academic writing, or reviewing. A Seminar is not intended to be only a presentation but should involve the participants actively.

The number of submitted SE proposals was 1, and this was accepted.

LIST OF PME 44 REVIEWERS

The International Program Committee of PME 44 thanks the following people for their help in the review process.

A

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Szitányi, Judit (Hungary)
Sztajn, Paola (United States)

T

Tabach, Michal (Israel)
Tan, Hazel (Australia)
Tatsis, Konstantinos (Greece)
Teppo, Anne (United States)
Thomas, Michael O. J. (New Zealand)
Thornton, Stephen John (Australia)
Tirosh, Dina (Israel)
Tjoe, Hartono (United States)
Tomaz, Vanessa Sena (Brazil)
Tortora, Roberto (Italy)
Tsamir, Pessia (Israel)
Tshesane, Herman (South Africa)
Tso, Tai-Yih (Taiwan)
Tzekaki, (Marianna Greece)
Tzur, Ron (United States)
Westaway, Lise (South Africa)
Widder, Mirela (Israel)

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Ufer, Stefan (Germany)

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Vale, Pamela (South Africa)
Van Dooren, Wim (Belgium)
Van Hoof, Jo (Belgium)
Van Zoest, Laura (United States)
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Weigand, Hans-Georg (Germany)

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Wu, Der-Bang (Taiwan)
Weingarden, Merav (Israel)

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PLENARY LECTURES

GATEKEEPING IN MATHEMATICS EDUCATION

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The COVID-19 pandemic has exposed possibilities for change in a world in crisis from both environmental and social violence. As gatekeepers in society, we mathematics educators need to ask which practices and structures we are protecting and which we are challenging. I invite you to reflect with me to consider the role of mathematics, and the role of a researcher, a teacher educator, a citizen, and a leader in the field of mathematics education. I give an overview of the storylines that define the experience of mathematics students in order to question how these storylines might be changed. I conclude with recommendations for action in the field.

INTRODUCTION

Arundhati Roy is a powerful voice from Kerala, India. My first encounter with her writing was her novel *The God of Small Things* (Roy, 2001). She has been watching the impact of the coronavirus pandemic on the people of India and she reminds us that the pandemic is a portal (Roy, 2020). It is a doorway to a new future, a gateway.

She takes a very different stance from what I hear from many government leaders and the people around me, who wish for a return to “normal”. Government leaders are designing policy in the pandemic to bring back the normal. They subsidize dying industries to be ready to continue where they left off before the pandemic. Similarly, the people around me are wishing for the time when they can travel again. I feel this urge myself. Due to travel restrictions, I have not visited my children in more than a year except by videoconferencing. I also long to sit together with you, my friends, and colleagues from around the world. I long for the old normal. But the old normal constructed the conditions for this pandemic to happen in the way it has done.

I want to follow Arundhati Roy’s vision of the portal. We have this gateway, this opportunity to enter a new space. Many of us have been successful in the world as it was. We may not wish for change. For others it is different. There are powerful forces guarding the gates, trying to turn people around to go back to the old world, a world that so many people experienced as treacherous and broken.

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We mathematics educators may be among the guardians of this gateway to a better future, or maybe not. Maybe we are leading the charge against the guardians. The important question for me to ask myself is what I am doing about the gates—am I guarding the gateway or am I breaking through? I need to identify both the ways I am guarding and the ways I am breaking through. Surely there is a little of both.

The gates that we are guarding or breaking through have existed for a long time. The pandemic is helping us to see the gates that have been there all along. Before I go further with this metaphor, let me acknowledge that the images of gatekeeping and charging the gates are complicated. For example, I think of a photo from the insurrection in Washington DC on January 6, 2021 (Hughes, 2021). A woman stands at the gallows erected at the site. She seems to be crashing the gates with the crowd that broke into the government house to disrupt the rulers of the land. But she has wrapped herself in a flag that says, “Keep America Great”. That suggests gatekeeping: the crowd is trying to keep or return to an older social structure and trying to stop the change that was happening in the hall of government. With such complications, the metaphor of gatekeeping and gate crashing is not straightforward. However, the goal is not to fix our labels as gatekeepers or gate crashers. The metaphor allows us to reflect on what we are protecting and what we are pushing or fighting for. The strong imagery reminds us of moral decisions in our roles in managing the gates.

After reflecting on the ways, I stand at the gateways, I will turn our attention to the stories that have defined mathematics and mathematics education until now, and also the stories that have defined mathematics education research. Consideration of these stories may help us focus our actions as gatekeepers and gate crashers.

MATHEMATICS AT THE GATES

Mathematics is often described as a gatekeeper discipline (e.g., Stinson, 2004). It is used to screen people for advancement in education and entry into high-profile, lucrative professions. Here is a strong example: one must excel in school mathematics to become a physician. But I think the weaker examples may be more powerful because they amass literally billions of smaller influences, as mathematics mediates the experience of schooling for almost everyone in the world. To be clear, the gatekeeping function of mathematics is really the social power of a particular approach to mathematics, one that focuses on skills that can be easily assessed in the kinds of examinations that have become the norm—performing algorithms and memory work.

Nevertheless, even the other skills that are not assessed very well, such as applying concepts to solve real problems, act as gatekeepers, albeit in a different way. Because mathematics provides a powerful toolbox, these skills and practices can open doors in human responses to significant community challenges, including social and environmental challenges. I think, for example, of developments in teaching the use of mathematics to address social and environmental justice questions, promoted by Renuka Vithal (e.g., 2002), Rico Gutstein (e.g., Gutstein, 2006), Kjellrun Hauge (e.g., Abtahi et al, 2017), Masami Isoda (e.g., Isoda et al., 2017), and others.

Beyond the ways in which we assess our students' mathematical skills, our roles as mathematics educators go further in gatekeeping. When we mathematics educators work in teacher education, we manage the gates that allow entry into mathematics teacher positions. Our research on the teaching and learning of mathematics influences mathematics curriculum and professional learning. Those of us who guard the gates of this research have powerful roles—as journal editors, reviewers, and conference leaders (e.g., Martin, Gholson & Leonard, 2010).

The pandemic shines a bright light on the gates that operate in our world and challenges us to reflect on our choices at these gates. The social disruption and climate cataclysm that are upon us urge us to reconsider what we are doing at the gates. As I engage in this self-reflection, I invite you to reflect along with me. Our situations will probably differ, but the questions are probably much the same for all of us. We can all benefit from listening to each other's self-reflection. I will divide my self-reflection into the following four questions about my positioning at the gates.

> What am I protecting or challenging in my service in the field: as a reviewer or an editor?
>
> What am I protecting or challenging as a researcher?
>
> What am I protecting or challenging as a teacher educator?
>
> What am I protecting or challenging as a citizen living within mathematized structures?

Protection and challenge at the gates in the field of mathematics education

A couple of years ago, Vilma Mesa and I interviewed editors and past editors of *Educational Studies in Mathematics* (ESM) and asked what characteristics reviewers to have been emphasizing when judging whether a manuscript is acceptable for publication (Mesa & Wagner, 2019). This question helps us see reviewers as guardians of the gates. Editors are guardians too because we select reviewers and synthesize reviewer concerns. In other words, we decide whose concerns are worthy of attention, and which of their concern's authors need to address.

From the comments of the other editors and from my own experiences reading many reviews of diverse manuscripts, I see that most reviews focus on what is missing. This is a deficit approach to assessment. For example, a paper may lack methodological detail, lack theoretical or conceptual framing, lack consideration of relevant studies in the field, lack sufficient focus on mathematics education, and so on. Scholars who study assessment practices in mathematics classrooms and elsewhere show that deficit-based assessment favours the values of the status quo. Listen to the conclusion of Aditya Adiredja and Nicole Louie following careful study of the impacts of deficit and alternative assessment approaches in mathematics education:

> The ultimate function of deficit discourses is always to justify attitudes and behaviours that reproduce systems of domination, to legitimize oppression as the natural and moral consequence of dominant-group merits and subordinate-group deficiencies. To accomplish this, deficit discourses construct differences, frame those differences as evidence of the innate inferiority of subordinate groups, and make invisible the strengths, resources and knowledge that exist in marginalized communities. (Adiredja & Louie, 2020, p. 43)

When I focus on what is missing in a research paper, I am comparing it to some kind of imagined norm that reflects my experiences of the genre, shaped by the people with whom I usually associate. To challenge such a deficit discourse, we can instead focus on strengths. In the context of reviewing and editing in our field, a strengths-based approach would focus on the contributions of a manuscript to the field and appreciate the new perspectives a paper reveals to me. This would be challenging the status quo—gate-crashing. Research can contribute to the field by providing any of these:

- novel empirical results.
- insight into contexts not yet sufficiently considered in the field.
- application or development of new theory or conceptual frames.
- new approaches to methodology in the field.

Notice that the items on my list all point to novelty. I should expect the most promising research from sources that are relatively unfamiliar to me. A research contribution could further entrench or disrupt a powerful discourse in the field. Moves to entrench often use the language of progress (Llewellyn, 2016) because the metaphor suggests a line that we should follow. Moves to disrupt often use language of social justice.

I suggest that if we make our reviews focus on the contributions, it will change the face of our field, perhaps slowly, but surely. This does not mean we should discard important standards. Even with a focus on the contribution of a paper, we can suggest to authors ways to shape their writing to satisfy legitimate expectations of the field. For example, I expect authors to identify and justify the theories and concepts they use in their analysis. The problem comes when I expect to see only particular theories and concepts and refuse to consider the validity of theorization from other sources. Often, I see reviewers demand that a paper cite research from the most dominant of contexts, even when that research is only marginally relevant.

Reviews and editor decisions are complicated because all the concerns I have identified so far are important. Shortcomings in any of these areas would not be acceptable for publication. However, I suggest that the decision to move a paper forward or reject it in a peer review process is better oriented by its contribution than the bits it may be missing. In other words, if the potential contribution is promising, we can work with the authors to develop the missing bits and hopefully move toward publication. Too often a paper may be rejected early in a process because it does not align with what readers from dominant areas of the field have come to find normal (cf., Niss, 2018).

In our research on editorial practices, Vilma and I also organized statistics on contributions to the journal to understand better the way the research in our field represents the issues of mathematics education around the world. Not surprisingly, there are significant disparities among the regions represented in this journal. We knew in advance about these disparities, and we know that they extend beyond the context of *ESM*: we were not the first to point them out (e.g., Louie, 2017; Meaney, 2013), but our statistics made the disparity harder to ignore. Vilma and I were concerned about equity in the opportunities of scholars but there is more: “the concerns of scholars in certain countries are more strongly represented than the research and concerns of scholars in other countries” (Mesa & Wagner, 2019, p. 308). The conceptions of what mathematics education looks like and the issues it is concerned with are dominated by particular national contexts. This dominance must be challenged, and it is most appropriately challenged by scholars from outside the dominant regions. Scholars in the dominant regions need to find a way to accept these challenges.

I think the most important step for positive change is to pay attention to research that challenges the status quo from contexts that are underrepresented in the field. Scholars in these contexts will identify different concerns or different approaches to concerns familiar to me, and they can provide valuable critique of my concerns and my way of approaching those concerns. Looking again at my list of ways research can contribute, I see that research from contexts unfamiliar to me can provide tremendous insight. As researchers we should read the research from diverse regions and attend research presentations from scholars representing diverse regions. Further, as a reviewer or editor I can ask and expect authors to look beyond the usual contexts.

Editors of mathematics education journals are trying to do this (e.g., Wagner et al., 2020). We need the cooperation of reviewers to move strongly entrenched views on what qualifies as important work. Yes, reviewers and editors are gatekeepers of the field. I encourage you to accept invitations to serve in these roles but to see yourself as welcoming hosts at the door rather than as guards. Vilma and I named our article on reviewing processes “Behind the door.” I am asking myself what I am doing at the door, and I ask you the same question.

Protection and challenge at the gates in mathematics education research

In addition to our roles at the gates of our research field, we also have choices about what to value and what to ignore in our own research. We are always making choices about whose concerns are most important. This question relates to my earlier comments about representation. For example, when a study on affect uses a Western European context, how well does the study represent the way affect works in other parts of the world (Tuohilampi et al., 2015)? Another way of looking at this is to consider how conditions in diverse contexts impact affect and a teacher's actions in relation to affect. These questions highlight the value of research from diverse countries. But even if I remain in my own country (where I am better equipped with local knowledge to do research), I ask whether I should focus on the needs of mathematics students who are already achieving success or on students who struggle with success in their school mathematics. Should I focus on teaching practices that are usual foci of mathematics teachers or should I push the boundaries?

I have recommended that we read the research from diverse contexts. This reading can open our eyes to our own practices because questions and approaches from elsewhere can reveal the familiar as foreign. To illustrate this phenomenon, I think of the two and a half years my family and I lived in eSwatini in the 1990s. We were not surprised to see unfamiliar practices there, but when we returned home to Canada, we saw Canada in a new light. Canada now felt foreign and strange in relation to the different perspectives we developed in eSwatini. In fact, this shift in perspective is what motivated me to research mathematics education. I had taught mathematics in Canada for 5 years before teaching in eSwatini. Within weeks of teaching again in Canada, I was shocked to recognize the cultural nature of mathematics and mathematics teaching. Before this return I had thought that mathematics was culturally neutral.

Listening to or reading research from different contexts may not immerse us as deeply as living abroad, but it can still be effectual. In a similar vein, I suggest that it is important to pay attention to the experiences of diverse people in our school mathematics classrooms—students and teachers who identify in diverse ways.

When we think about how we research mathematics education we need to articulate the future we envision for our mathematics learners. Ole Skovsmose (e.g., 1994) has encouraged us to think about students' foregrounds—the futures *they* see before themselves—but I add that it is also important for us to be critically attentive to *our* visions for their futures. These visions shape our research, which in turn impacts what happens for mathematics students. I will say more about this later in this address.

Protection and challenge at the gates in mathematics teacher education

I see my gate-management role as a mathematics teacher educator as being closely related to the questions, I identified about mathematics education research. As a teacher educator, I make choices about what research novice teachers should read and discuss, and which issues we focus on when we read the research. This gatekeeping function is similar to my roles as a reviewer and an editor. With this choice about which research is important, I am choosing which mathematics learners' concerns are most important. This gatekeeping is then boosted by my role in grading novice mathematics teachers' assignments and writing references to support them. As a mathematics teacher educator, I shape the mathematics teaching force. I can position myself as a guardian of the structures that have privileged certain students or as a facilitator for mathematics teachers with new perspectives who will guide a range of mathematics students to the successes, they envision for themselves.

Protection and challenge in active citizenship

When I focus on the usual research and mathematics teaching practices in our field, I am in danger of ignoring other significant mathematics in my life. This has implications for our field. Mathematics permeates my life, but I will focus on one example here and invite us all to think of other examples. Consider the way we vote for representatives in democratic institutions. This is a form of representative sampling.

Consider, for example, the way we elect the International Committee of PME. As I understand it, each year the conference participants vote for four members at large. Each PME member who is present votes for four names. The four contestants who receive the most votes win. Let us say there are 300 participants at the conference and 160 of them share a set of values. All four elected members at large, will be selected by those 160 people. The votes of the remaining 140 participants have no effect. Further, the perspectives of the many who could not afford to come to the conference are not heard. The conference and its consequent leadership role in the field end up being controlled by a mere sliver of the scholars working in that field.

There are alternative structures for voting that use different mathematics and produce more equitable outcomes. I am particularly impressed with the Single Transferable Vote (STV) systems, which have been developed by mathematicians. I like Meek-STV, named after the mathematician who developed it. But the question of who is allowed to vote is the most important.

Until recently, I was blind to the mathematics of voting practices. It took my political engagement in my community at home to shake me out of blindly accepting voting practices I had assumed were normal. I am raising this example of mathematics in action for a couple of reasons. First, again, the leadership of our field is structured by taken-for-granted practices that favour the status quo. Many of these structures are highly mathematized. We should question them. They can be changed.

Second, I ask why I was blind to this problem of representation. One reason is that the structures favour people like me, and thus I may not have been motivated to ask questions. Another reason I was blind to this mathematics is that nothing in my own mathematics education pointed my attention in this direction. We here at PME are a collective of mathematically sophisticated people, capable of complex mathematics and ostensibly aware of the way mathematics works in society, but this structural problem persists, which is a mathematical problem and a problem for our field of study.

There are more examples of mathematized structures that govern our field, including metrics for ranking journals, universities, and scholars (Andrade-Molina et al., 2020). And there are more examples of mathematized structures that govern school life, and thus the life of mathematics students. The taken-for-granted norms can be challenged, and mathematics can play a strong part in that. I see Renuka Vithal's work as a good example of such citizenship (Vithal, 2002). This kind of work can help us reimagine mathematics classrooms. If we are not challenging the status quo, we are protecting it.

THEORY FOR GATEWAY INTERACTIONS: STORYLINES

So far, my reflection on gatekeeping and gate crashing in mathematics education has been quite general. To investigate the way, I manage my roles at the relevant portals, I draw on theories of human interaction. I think in particular of the work on storylines and positioning by Bronwyn Davies, who has become a prominent feminist scholar. For decades she has worked at understanding how people are drawn into particular forms of action and interaction. Her collaboration with Rom Harré has been the stem of a theory Harré and others call Positioning Theory. Here is a diagram that shows how positioning works (Figure 1). This diagram comes from my work with Beth Herbel-Eisenmann, Kate Johnson, Heejoo Suh, and Hanna Figueras.

All communication is guided by stories that we know, which we use to interpret each of our interactions. When I meet someone, I have to decide what kind of interaction it is. That is the storyline. And I have to decide on the part I play in the story. That is the positioning. These decisions guide my choice of words and my actions.

For example, imagine some children meeting you in a school. Perhaps you are in the school to do research, but the children do not really know much about what research looks like. You greet the children. They have to decide how to talk with you. Do they think of you as a teacher, a school administrator, a parent of another child, or perhaps in some other role? They have to identify a storyline—a story of students being interrogated by a head teacher, or a story of a parent interested in her children's friends, or something else. The decision the children make about what sort of interaction this will be impacts how they respond to your greeting and later to your questions and comments? Each child in the interaction and you yourself are all active in shaping the possibilities for the storyline and the possibilities for how you all position yourselves. A child might see her positioning as an informant on classroom dynamics, or as a skilled performer of mathematics, or as one who explains the ideas of her peers. And there are more possibilities. The children and you constantly adjust to each other and to the many choices about how to talk, what to say, what gestures to use, what kinds of communications are valued, and who should speak at any given moment.

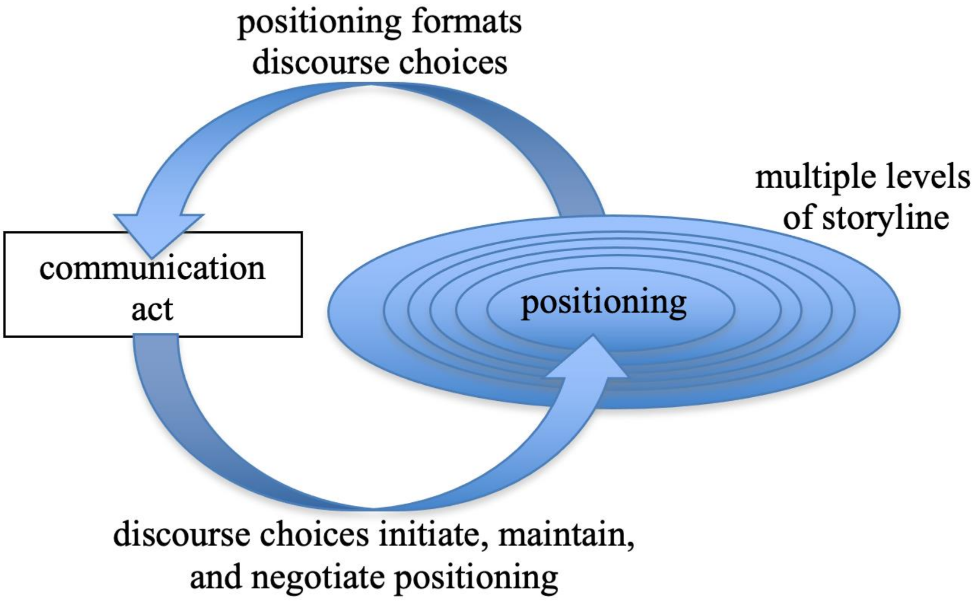
positioning formats
discourse choices
multiple levels of storyline
communication act
positioning
discourse choices initiate, maintain, and negotiate positioning

Figure 1: Positioning and storylines in human interaction (Herbel-Eisenmann et al., 2015, p. 194)

The theory reminds us that we are constantly negotiating our positioning and storylines because one person’s storyline and positioning choices have an impact on the other people. Figure 1 tells us that the positioning and storylines impact our choices of what to say and do, and that these choices in turn shape the storylines and the related positioning. Our choices determine the positioning and storylines that are available to others, and they also shape what these storylines look like for our future interactions.

Here are two important questions for us all to consider: Where do these commonly known storylines come from? And What are the dominant stories in mathematics education? In short, they come from interactions we have had, the stories we have read, heard, and watched in books, conversations, and other media. An implication of this theory is that we can only interact along the lines of stories we know. Thus, an important way to change the possibilities for a person is to make them familiar with different stories, different ways of interacting. For this to work, both an individual and the people with whom the individual interacts need to get to know a new set of stories. This theory implies that part of our agenda as mathematics educators is to generate good stories and disseminate them so that they become widely known. These stories can feature good mathematical action done by people with diverse identities.

To realize this agenda in mathematics education, we need to know what stories are currently known and thus what is possible for mathematics teachers and students. And we need to ask what positions we play in those stories. Some stories are much more deeply engrained than others and thus harder to disrupt. Some stories emerge within a specific classroom. These more local stories can have a different kind of power.

Here is a non-mathematical example of a story that impacts many stories. In Canada it is an emerging custom to begin meetings and gatherings with a land acknowledgment: I state that I live and work on the unceded territory of the Wolastoqiyik people. In saying this, I remind myself and others that I am aware that everything I take for granted rests on a colonialist history in which foreigners stole land from the Indigenous people here. I want to take seriously the fact that there is a long history of violence underneath the structures and norms that dominate my life, my work, mathematics, mathematics education, everything. And the violations continue, not just in Canada but around the globe. I hope my address helps us all confront this violence.

Ideas of cultural superiority abound. At a macro level, various forms of nationalism and racism are rooted in some people thinking their backgrounds entitle them to more wealth and privilege than others—perhaps more scholarly status than others. At micro levels, we see people believing that their values and conceptions of quality are superior. I hope you recognize that these cultures of superiority are what I have been talking about this whole time. I recognize the complexity. I myself think that I have a pretty good understanding of what is important in mathematics and mathematics education. My question today is how open I am to the knowledge, values, and experiences of others who aren’t like me? This question implies a question for action: how do I deliberately open myself to valuing the knowledge, values, and experiences of others?

STORYLINES IN MATHEMATICS EDUCATION

Now that I have established the significant power that we as mathematics educators have at gateways that impact the lives of so many people, I want to give an overview of some of the storylines at work. These are the stories that are taken as norms and impact how students, teachers, and others work through and around mathematics learning. There is a growing body of scholarship in our field addressing *storylines*. Most of these studies identify such stories in mass media. Sheree Rodney, Annette Rouleau, and Nathalie Sinclair looked at Canadian newspaper articles and found pervasive metaphors—one metaphor sees mathematics as an economic commodity and another sees mathematics educators at war (Rodney et al., 2016). The war metaphor aligns with a storyline found by a group who looked at storylines in North American media: “There are two dichotomous ways of teaching mathematics […] the ‘basic’ way and the ‘discovery learning’ way” (Herbel-Eisenmann et al., 2016, p. 104).

For any storyline it is important to ask what positions it makes available to mathematics students, mathematics teachers, and mathematics educators. I think the storyline about math wars influences mathematics teachers most directly. It oversimplifies the complexities of mathematics learning by focusing on certain issues. In so doing, it obscures other things worthy of attention—for example, the specificity of learning contexts, or questions about what mathematics is the most important to learn. While this war storyline has direct impact on teachers, there are spinoff impacts on students. Are they positioned as automatons developing procedural skill through repetition? Are they positioned as people who should demonstrate that they understand mathematical concepts? And if understanding is the focus, what is the impact on their interactions? What do they do with their understanding? With whom do they interact mathematically? A focus on storylines and positioning should always lead us to identify the implications for human interaction.

When I consider any storyline, it is important to remember that it too may be culturally specific. There may be different conflicts among mathematics educators in different places—different wars, if we use the language of the metaphor. For example, I understand there has been a conflict regarding the pace of education in Japan, with some educators pushing for fewer concepts to be investigated with greater depth. Any conflict in mathematics education seems to be an invitation for politicians to position themselves as champions for one side or another. The fact that mathematics education is often used as a talking point for politicians reminds us again about the importance society places on mathematics and the significance of the gates we manage.

Sean Chorney, Oi-Lam Ng, and David Pimm (2016), who looked at the same set of articles as the Rodney group, found a different conflict, which positions individuals and countries in competition. Individual students are ranked and compared. Countries are also ranked in massive comparison studies like PISA and TIMMS. We should think about what country comparisons do to classroom interactions. They could position some students as champions for their country, while other students become liabilities. This positioning can put a lot of pressure on some students and leave other students feeling worthless. While there are far-reaching implications for international comparisons, I think the competition storylines at the more local level have even deeper implications for the way students interact. Consider, for example, what group work looks like when students feel like they should be trying to outdo each other.

The study of media is not the only way to identify storylines. Another way is to look at interactions in mathematics classrooms. I thank Beth Herbel-Eisenmann for her collaboration in various studies in which we have identified positioning in mathematics classrooms. For example, when we analysed transcripts from 148 classes we found that a dominant positioning has students doing things because their teacher tells them what to do (Herbel-Eisenmann & Wagner, 2010). We called this *personal authority*. This positioning may seem quite natural: is this not the expectation of teachers, to guide students? The fact that this relationship seems natural underscores the power of storylines. Beth and I noted that mathematics is often said to be logical and free of culture and power relationships, and so we wondered why mathematics students' choices for action are not led more by the *mathematics* and less by their *teachers*. This question allows us to envision different forms of mathematics class interaction—activity that is organized around true inquiry rather than teacher-guided exercises.

Research on classroom interaction highlights another important aspect of storylines and positioning. A student's experience of mathematics learning is strongly impacted by the kinds of interaction offered in the classroom. The theory of positioning reminds us that students could try to have different sorts of interactions, including interactions that are not imagined by the teacher. But there are power relations at work. The teacher has a position of authority. Furthermore, the whole group, including the teacher and other students, are guided by the stories about mathematics that dominate society. Thus it is not easy for a single mathematics student to change the form of their interactions.

A recognition of the power of these stories guides some research being done by Annica Andersson in Norway along with Hilja Huru, Beth Herbel-Eisenmann, and myself. We are investigating the storylines available to Indigenous and new immigrant students. We want to work with their mathematics teachers to make more positive storylines available. Our analysis of Norwegian newspapers and public media has found some storylines that are particular to students who are seen by others as minorities. These storylines include "mathematics is language- and culture-neutral" and "extraordinary measures are needed to teach mathematics to students from minoritized groups" among others (e.g., Andersson et al., 2021). Interviews with students and teachers about their mathematics classroom experiences will help us identify other storylines and explore the way the storylines in the general public impact these students' experiences.

There is more research in our field that tells us about important storylines. We can look at work on myths (e.g., Anderson et al., 2018), discourses (e.g., Valoyes-Chávez, 2019), and identities. Identity work is especially prevalent among feminist scholars because storylines (or myths) are typically gendered. For example, Heather Mendick (2005) identified 15 binary oppositions common in mathematics education discourse, and she showed how they are gendered. In addition to the big myth that boys are better than girls at mathematics, each of these binaries positions boys more with one extreme and girls more with the other. As I list some of these binaries, ask yourself which side you associate with girls and which one with boys. Next, we should ask ourselves how we have come to see these associations as natural. Here are some of the binaries she illustrated: fast vs. slow, competitive vs. collaborative, independent vs. dependent, active vs. passive, natural ability vs. hard work, real understanding vs. rote learning, and reason vs. calculation. The damage these binaries and their stories can do in mathematics classrooms is obvious. They shape expectations students have for themselves and expectations teachers have for them. And these expectations shape the stories and positioning they can and do choose for their interactions. We should remember that these gendered stories will be different in different parts of the world.

I have given overviews of some storylines, but there are others. Research on these formative stories can help us denormalize them. In other words, the research gives us strategies for questioning our sense of what seems natural or normal. External disruptions to the normal, such as the COVID-19 pandemic, can also expose storylines (e.g., Bakker et al., 2021). The research submitted to the *ESM* special issue on the pandemic points to some such storylines and re-emphasizes others. Many of the 161 papers submitted for the special issue pointed to the need to change mathematics curriculum. These researchers are questioning the storyline in which school curriculum dictates what happens in mathematics classrooms. The pandemic has shown us that some of the mathematics taught in schools has been very useful to citizens for understanding the pandemic, but it also exposes how current curricula are insufficient (e.g., Kwon et al., 2021). Contributors to this special issue noted how the pandemic underscores storylines of inequalities (e.g., Yılmaz, 2021) and storylines about the way technology can mediate mathematics education (e.g., Borba, 2021). I add a storyline that has not been addressed in the articles in the special issue, but which I see the pandemic has exposed: the stories about assessing mathematics. In many cases, I have heard mathematics teachers say they are unable to use digital technology at a distance to assess students in the way they think assessment must be done. I hope to see this phenomenon researched.

RESPONSES TO STORYLINES IN MATHEMATICS EDUCATION

Once we recognize significant storylines in mathematics education and the way they shape the experiences of mathematics students and teachers, I think we researchers are compelled to ask ourselves how we ought to respond to these myths. Which storylines drive our work and which storylines do we ignore? What and whom we are protecting with these choices? I think most of the research in mathematics education is in some way a response to a dominant perception—a storyline or myth. Our research can resist a dominant perception, ignore it, or support it.

Our research addresses the storylines that are in action in mathematics learning environments, but research is also action in itself. In conducting our research and in the way we report our research we are enacting storylines. We position ourselves in relation to each other. Often, I see reviewers asking authors to position their research in relation to the field. They do not always use the word ‘position’ but the intent is to ask authors to be more aware of their work’s status in the field. What are they contributing?

It is a challenge to try to document the storylines in mathematics education research. I have tried to pay attention to the stories people tell about what they are doing. However, it is hard to find the stories in typical research articles because in our scholarly traditions researchers usually do not tell the stories that motivate and drive their research. I think this is because we value objectivity, which is a value that lives in stories told about mathematics. Even research approaches that are inescapably subjective seem bound by this writing convention that tries to mask obvious subjectivity.

To try to hear some of the motivational stories behind the research, I distributed an informal survey among my professional networks. I asked, “What was one of the first mathematics education articles/chapters/books you liked?” and “Why did you like it?” The responses gave me some insight into the kinds of publications that were influential both to other researchers and to mathematics teachers. The results addressed both situations because many of the respondents described a publication that motivated them to become active researchers. I will share some highlights here in the hope of motivating us all to recognize important work when we read and review the work of others. The responses came from scholars in 17 countries on six continents.

First, in looking at the publications my colleagues identified, 26 were theoretical articles, 15 were books (also mostly theoretical), five were empirical articles, two were the full body of someone’s work, one was a mathematical work, and one was a curriculum document. Given that theoretical works are published much less frequently than empirical works, it is notable that in my survey they strongly outnumber the empirical works. The respondents identified memorable publications that:

(1) opened their eyes to aspects of mathematics they had not previously recognized: e.g., “how the context plays a role in the way we as educators envision/implement mathematical concepts”.

(2) provided frameworks for research and interpreting mathematics learning experiences: e.g., “emphasized a difference between mathematical process and nomenclature,” “how school achievement in mathematics and mathematical

thinking are not the same," "it theorised the relationships within the mathematics classroom".

(3) provided language for experiences that readers were beginning to notice e.g., "putting words to ideas that were still vague but deeply rooted," "resonated with things that were already interesting to me in my practice".

Many of the responses pointed to phenomena that readers had not noticed in their experiences (category 1). Most of the responses enabled readers to understand their experiences in new or clearer ways (categories 2 and 3). These formative publications were powerful because they connected readers to their experiences. Returning to my earlier suggestion to read research from contexts different from our own, specifically I see that readers may have difficulty finding value in research reporting from contexts that do not resonate with their own experiences. Thus, I see the need for us to be more careful in our research reporting to identify the specific context, and for us to do more cross-context work in which we experience each other's contexts and identify what we learned.

I also see the tremendous potential for critiques of dominant frameworks in our field that come from scholars in contexts not represented in those frameworks. For example, Lihua Xu and David Clarke (2019) drew attention to significantly different cultural norms in Asian and English-speaking contexts to problematize assumptions and conceptualizations in research relating to what kind of communication is valued in mathematics classrooms. The next step will come in the way English-speaking scholars respond to their critique. Another good example was a symposium convened by Aldo Parra, Arindam Bose, Jehad Alshwaikh, Monica González, Renato Marcone, and Rossi D'Souza (2017) that theorized crisis from the perspectives of scholars from so-called "developing" countries. Again, scholars from the so-called "developed" countries need to take seriously the theories discussed and developed in this symposium and other such fora.

RECOMMENDATIONS FOR ACTION

Many of us, including me, have been successful scholars in a world in which humanity has produced widespread catastrophes—encroaching climate change, social inequities, and a pandemic born from these conditions. As mathematics educators we stand at the gates to the new normal that will emerge in these times. I have promoted actions that we can take as researchers and educators in the interest of justice. Like Edward Said (1994/2012), I see this as a responsibility of public intellectuals: "There is no question in my mind that the intellectual belongs on the same side with the weak and unrepresented" (p. 22). I see this as a human responsibility.

I close with some specific recommendations for immediate action:

- At this and other conferences,

  - expand your perspectives by attending some sessions that you would not normally choose, and
  - actively seek out and develop relationships with scholars outside your usual/comfortable networks.
- Read scholarship from regions outside your comfortable contexts.
- Volunteer to be a reviewer. Commit yourself to identifying the contribution of the work you review and to give helpful suggestions to the authors to help them realize their potential contributions.

If you are a scholar from an underrepresented region or group, be bold in sharing your research and in developing theories that emerge from your contexts. Scholars around the world look forward to learning from you.

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THE ENACTED SCHOOL MATHEMATICS CURRICULUM – MASTERY LEARNING IN SINGAPORE SECONDARY SCHOOLS

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The enacted school mathematics curriculum for Singapore schools is shaped by the intended curriculum that places emphasis on mathematical problem solving through the development of concepts, skills, processes, metacognition and attitudes. Since, 1997 an emphasis on "thinking skills" has pervaded the school curriculum in Singapore. This has manifested in mathematics classrooms in a myriad of ways and overtime become an essential feature of classroom work as teachers embrace mastery learning. In this paper, I draw on mathematical tasks from the lessons of two teachers and illuminate how thinking skills support learners' mastery of mathematical knowledge. The mathematical tasks show that student ability is not a barrier for teachers to engage students in higher order thinking.

INTRODUCTION

In this paper I first present a brief overview of the intended school mathematics curriculum in Singapore schools. Next, I draw on a project that examined how competent and experienced secondary school mathematics teachers enact the curriculum and illuminate the instructional core of the lessons of these teachers. Lastly, I draw on mathematical tasks from the lessons of two teachers in the project and illustrate the concept of mastery learning in mathematics in Singapore secondary schools.

THE INTENDED SCHOOL MATHEMATICS CURRICULUM

The framework of the intended school mathematics curriculum, shown in Figure 1, places emphasis on mathematical problem solving through the development of concepts, skills, processes, metacognition, and attitudes (Ministry of Education, 2018). At present the curriculum is best described as mathematics for all and more mathematics for some (Kaur, 2019). Lee et al. (2019) have identified two key approaches in the curriculum, namely the curriculum approach and the pedagogical approach. The curriculum development approach recognises the 'hierarchical' nature of mathematics and adopts a 'spiral approach' to the design of the curriculum. Each topic is

2021. In Inprasitha, M., Changsri, N., Boonsena (Eds.). *Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education,* Vol. 1, pp. 19-31. Khon Kaen, Thailand: PME.

revisited and introduced in increasing depth from one level to the next to enable students to consolidate the concepts and skills learned and to develop these concepts and skills further. The curriculum also recognizes the need for 'age-appropriate strategies' such as through the use of concrete manipulatives and pictorial representations to scaffold the learning and for sense making. The key pedagogical approach advocated by the curriculum document is the 'Concrete – Pictorial – Abstract' (C-P-A) approach (Leong et al., 2015) particularly for the teaching of the number and algebra strand.

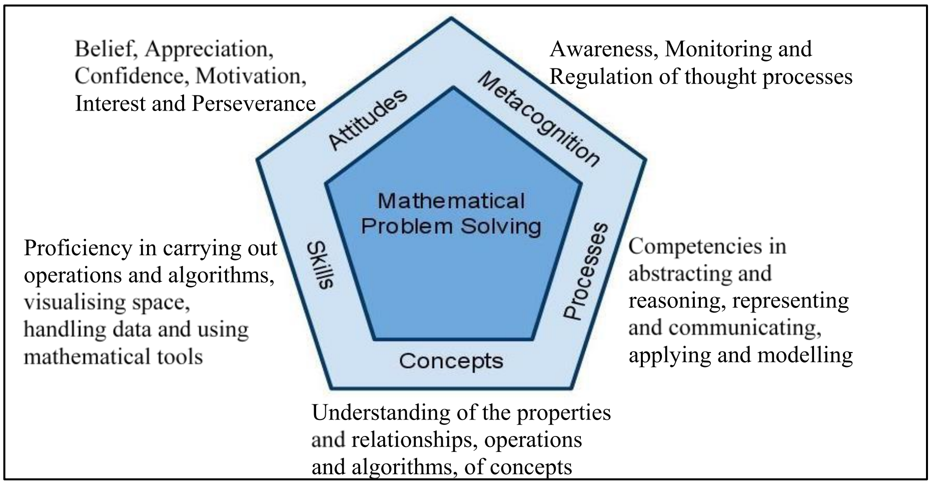

Belief, Appreciation, Confidence, Motivation, Interest and Perseverance
Awareness, Monitoring and Regulation of thought processes
Attitudes
Metacognition
Mathematical Problem Solving
Skills
Processes
Concepts
Proficiency in carrying out operations and algorithms, visualising space, handling data and using mathematical tools
Competencies in abstracting and reasoning, representing and communicating, applying and modelling
Understanding of the properties and relationships, operations and algorithms, of concepts

Figure 1: Framework of School Mathematics Curriculum (MOE, 2018, p. S2-6)

THE ENACTED SCHOOL MATHEMATICS CURRICULUM IN SINGAPORE SECONDARY SCHOOLS

A comprehensive study of the mathematics instructional practices in Singapore secondary schools was carried out from 2016-2019. The study is known as the Enactment project. A key finding of the project was an instructional core that drives the teaching and learning of mathematics in the classrooms of 30 competent and experienced mathematics teachers in Singapore secondary schools who participated in the project. The teacher participants were deemed as "good mathematics teachers" in their respective schools. They had at least five years of mathematics teaching experience, were recognized by their schools/cluster as competent teachers who have developed an effective approach of teaching mathematics and were keen to participate in the study.

Detailed analysis of the data that led to the identification of the instructional core has been reported elsewhere (Kaur et al., 2021). The core comprises three instructional components, namely:

- Development [D]

In this activity segment teacher shows, explains, tells or guides students to uncover or make sense of new concepts. Teacher may also use the interaction pattern of initiation-response-feedback (IRF) discourse format (Sinclair & Coulthard, 1992) to co-create new knowledge or demonstrate new skills.

- Student work [S]

In this activity segment students are given work to do in class at their desks or at home, either individually or in groups. They may also be asked to make notes in their notebooks following development of new knowledge or algorithm (formula) or correct their work following review of student work.

- Review of student work [R]

In this activity segment teacher reviews student work done in class or as homework, drawing attention of the whole class to errors, misconceptions, correct solutions, good presentations, etc. Teacher may engage students in IRF discourse format to work through the solutions of tasks they are unable to do on their own.

Lesson objectives, such as i) to verify and state Pythagoras theorem, and ii) to find the unknown sides of a right-angled triangle using Pythagoras theorem, of teachers in the project were enacted through micro-instructional objectives that comprised instructional components D, S and R. Table 1 shows how Teacher 24 enacted the two lesson objectives in his first lesson for the topic Pythagoras theorem and trigonometry.

Objective: To verify and state Pythagoras theorem			
Cycle 1	C*	Micro-instructional objective	Teacher / Student activities
	R	Review past knowledge	Teacher drew a right-angled triangle and reviewed its properties with input from students.
	D	Verify Pythagoras theorem	Teacher explained the activity (guided investigation) and gave out the activity kits.
	S	Verify Pythagoras theorem	Students carried out the activity verifying that the sum of the areas of the squares on the longest side of the triangle = sum of the area of the squares on the other two sides of the triangle.
	R	Verify Pythagoras theorem	With inputs from students related to the activity, teacher formalised the relationship between the sides of a right-angled triangle.
	R	Develop the vocabulary related to the sides of a	Teacher labelled the sides of the right-angled triangle on the board; named the

		right-angled triangle	longest side as the hypotenuse and drew the attention of the whole class to the ‘new’ word (“must use it and spell it correctly”).
	R	State Pythagoras theorem	With inputs from students, teacher wrote the theorem on the board: $a^2 = b^2 + c^2$ (where a is the length of the hypotenuse).
Objective: To find the length of the hypotenuse given the other two sides of a right-angled triangle			
Cycle 2	C*	Micro-instructional objective	Teacher / Student activities
	D	Demonstrate how to find the length of hypotenuse of a right-angled triangle given the other two sides	Teacher demonstrated an example of how to find the hypotenuse of a right-angled triangle with sides 3 cm and 4 cm.
	S	Engage students in applying new knowledge and skill building	Students worked individually and found the hypotenuse of a given right-angled triangle. Teacher walked around the class noting student work for ‘review’.
	R	Monitor student understanding	Teacher drew on samples of student work (correct and incorrect solutions) and invited inputs from students. For the incorrect solutions, errors and their causes were identified. As some computational errors were due to incorrect use of the calculator, teacher did a quick review of how to find squares and square roots using a calculator.
	S	Reinforce – application of new knowledge and skill building	Students worked individually and found the hypotenuse of a given right-angled triangle. Teacher walked around the class noting student work for ‘review’.
	R	Monitor student understanding	Teacher drew on samples of student work (mostly correct solutions) and invited inputs from students. Teacher highlighted correct and logical presentations.

C* - instructional component

Table 1: Lesson objectives of Teacher 24 for Lesson 1

A lesson often comprised of one or more cycles of instruction depending on the number of objectives. A cycle comprised combinations of D, S and R such as R-D-S-R-R-R or D-S-R-S-R as shown in Figure 2.

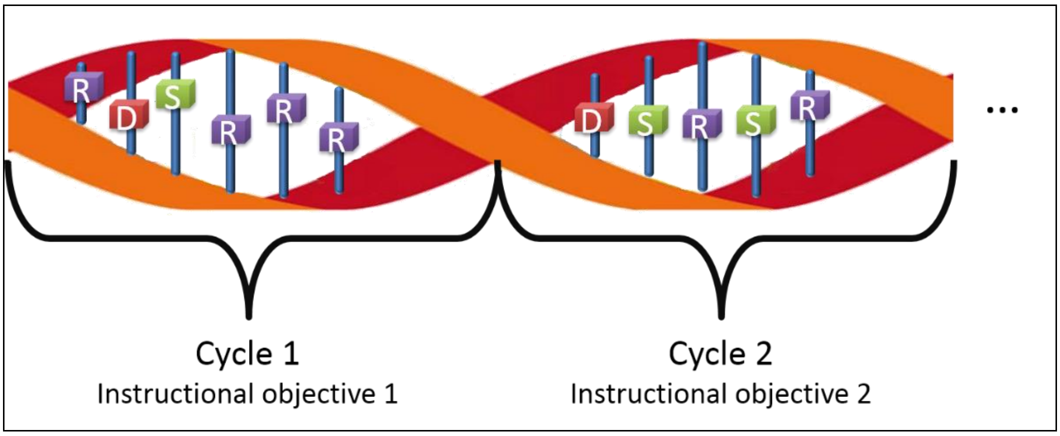


Figure 2: Instructional cycles

Figure 3 shows the interactions between the three instructional components, D, S and R. The D component develops a concept or introduces a skill. Teachers may show, tell, explain, or guide students to uncover/make sense of new concept(s). They may also introduce and demonstrate skill(s). The S component always follows the D component and involves students working on mathematical task(s) during classwork, homework, or assessment. Students may do the work individually or in groups. The tasks involve application of new knowledge or skill building that was developed in the D component. The R component, a critical one, is where student understanding is monitored. Almost always, student work during the S component is used for whole class discussion during the R component. Good presentations are show cased, alternative solutions are discussed, and erroneous solutions are examined and corrected with inputs from the class. This component also includes the review of past knowledge that is necessary for the lesson or a task that students are set to do. Many a time when a teacher is not satisfied with students' grasp of a concept or proficiency of a skill, the teacher engages students is more rounds of S → R until he/she is satisfied with students' mastery of knowledge.

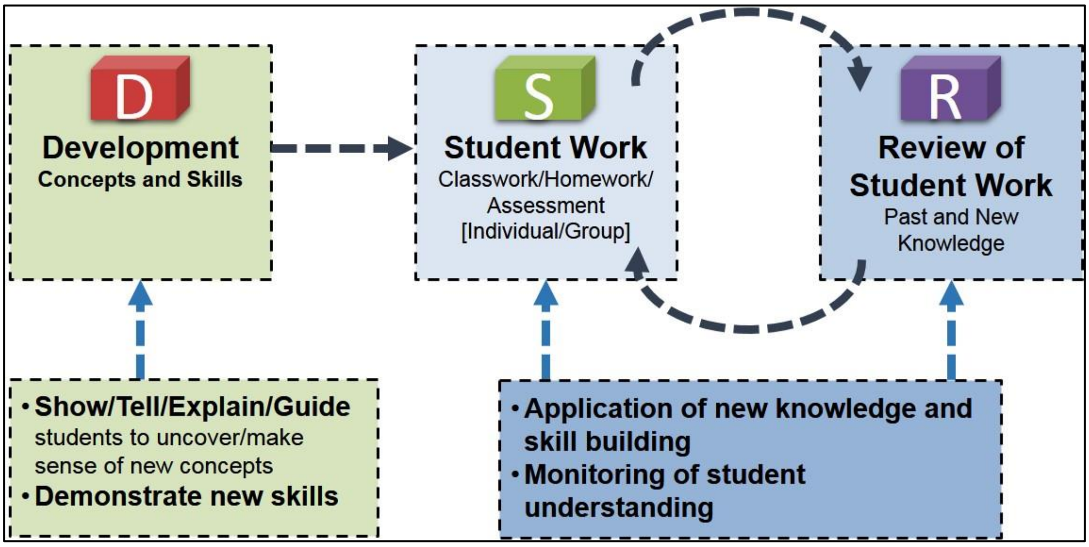

D
Development
Concepts and Skills
S
Student Work
Classwork/Homework/ Assessment [Individual/Group]
R
Review of Student Work
Past and New Knowledge
• Show/Tell/Explain/Guide students to uncover/make sense of new concepts
• Demonstrate new skills
• Application of new knowledge and skill building
• Monitoring of student understanding

Figure 3: Instructional core comprising the D, S and R components

Deliberate focus on mastery learning

In the project it was evident from the S ↔ R loops that there is a deliberate focus during mathematics lessons in developing conceptual understanding and procedural fluency with 'new knowledge' students have uncovered. Our findings uncovered in the project related to mathematics instruction in Singapore secondary schools are definitely not coherent with that reported by Leung (2001). Leung noted that mathematics instruction in East Asian countries is:

> … very much teacher dominated and student involvement minimal. … [Teaching is] usually conducted in whole group settings, with relatively large class sizes. … [There is] virtually no group work or activities, and memorization of mathematics is stressed … [and] students are required to learn by rote. … [Students are] required to engage in ample practice of mathematical skills, mostly without thorough understanding. (Leung, 2001, pp. 35–36).

In this paper I draw on two classroom vignettes and illuminate the local conceptions of mastery learning by teachers in Singapore secondary schools.

Vignette 1 – Teacher 11

Teacher 11 is a senior teacher, one who is locally recognised for her teaching competency and is trusted with the charge of developing junior teachers in her school. The grade 10 students in her class are from the Normal (Academic) course of study (from the 25th – 40th percentile of a cohort). A total of seven lessons were recorded for the topic: Arc length, area of sector and radian measure. Specific to this vignette are the objectives of the first two lessons shown in Table 2.

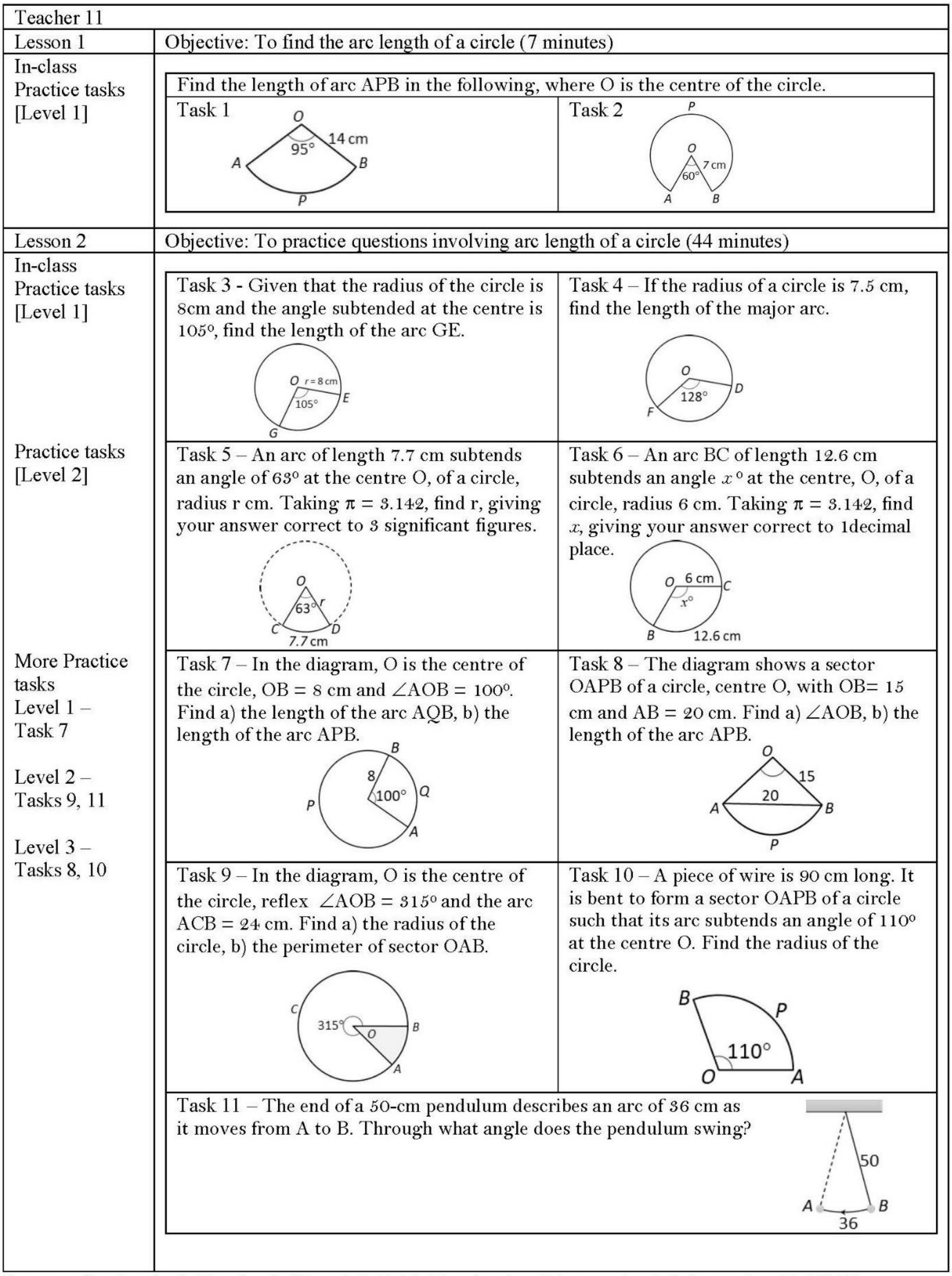
Teacher 11 – Topic: Arc length, sector area and radian measure	
Lesson	Objective/s
1	1. To identify the parts of a circle. 2. To find the arc length of a sector.
2	1. To review parts of a circle and formula for arc length. 2. To engage students in practice of questions* involving arc length. *Note: questions ranged from level 1-3. Level 1 direct questions involved the use of the formula to find arc length given an angle and radius. Level 2 application questions involved finding the angle given an arc length and radius, or finding the radius given an arc length and angle. Level 3 higher order thinking questions involved drawing on past knowledge like the cosine rule to first find the angle and then the arc length or formulation of a mathematical sentence to represent the perimeter of a sector and solve for an unknown.

Table 2: Instructional objectives for Lessons 1 and 2 of Teacher 11

Teacher 11 used a total of 11 practice tasks, shown in Figure 4, to achieve the second instructional objective in both her Lessons 1 and 2. A practice task is used in a lesson to either illuminate a concept or demonstrate a skill, that has been developed in the D activity segment, further the teacher asks students to work through it during the lesson either in a group or individually or during out of class time (Kaur, 2010). From Figure 4 it is apparent that Teacher 11 engaged her students in deepening their conceptual knowledge of arc lengths and procedural fluency in finding the unknowns in varying contexts related to arc lengths of circles. There was a gradual progression in the increase of cognitive demands of the tasks students worked with.

During Lesson 1, following the development activity during which students uncovered that arc length of a sector was equal to $x^0/360^0$ x $2\pi r$ where x was the angle of the sector and r the radius of the circle (where the sector was located), Teacher 11 set the students to do task 1 individually in class (the task was projected on the board). As students were doing the task, she supported those who were struggling through between desk-instruction during which she answered questions that students had and or re-visited a concept or demonstrated a skill from a past D activity segment of the lesson. A quick review of the task followed before task 2 was put on the board. Students had little difficulty completing the tasks. Teacher 11 was not satisfied with correct answers, but rather students' reasoning. She collected student responses and revoiced that 1) the arc length is directly proportional to the angle it subtends at the centre of the circle, and 2) the arc length is a fraction of the circumference, and the fraction is $x^0/360^0$. This short practice session involved the last 7 minutes of the 55-minutes lesson. There was no homework given and more practice resumed in the next lesson.

In Lesson 2, following a review of knowledge uncovered during the previous lesson, 44 minutes of the 60-minute lesson were devoted to student work in class. It began with level 1 tasks (tasks 3 and 4) and followed on with the rest chronologically. While students were doing the tasks the teacher not only provided between-desk instruction to those who needed her help but also scanned for student work that would be used during the review segment and enrich the discourse on misconceptions, computational errors, and integration of past and new knowledge. The S → R loops were systematically enacted, task after task. Every task was reviewed as a whole-class activity and student understanding monitored by the teacher. Tasks 8 and 10 were challenging for more than half of the students, but the whole-class review allowed students to update their gaps in knowledge and complete the tasks. Engaging students in such practice is Teacher 11's conception of mastery learning in her lessons. It certainly is not synonymous to "drill and practice" or "rote-memorisation" of formulae. But instead, the teacher's concerted push for students to make sense of the mathematics they are working with, make connections between past and new knowledge are at the forefront of the teacher's classroom instruction not only during the D activity segment but also the S and R segments.



Teacher 11		
Lesson 1	Objective: To find the arc length of a circle (7 minutes)	
In-class Practice tasks [Level 1]	Find the length of arc APB in the following, where O is the centre of the circle. Task 1	Task 2
Lesson 2	Objective: To practice questions involving arc length of a circle (44 minutes)	
In-class Practice tasks [Level 1]	Task 3 - Given that the radius of the circle is 8cm and the angle subtended at the centre is 105°, find the length of the arc GE.	Task 4 – If the radius of a circle is 7.5 cm, find the length of the major arc.
Practice tasks [Level 2]	Task 5 – An arc of length 7.7 cm subtends an angle of 63° at the centre O, of a circle, radius r cm. Taking $\pi = 3.142$, find r, giving your answer correct to 3 significant figures.	Task 6 – An arc BC of length 12.6 cm subtends an angle x° at the centre, O, of a circle, radius 6 cm. Taking $\pi = 3.142$, find x, giving your answer correct to 1decimal place.
More Practice tasks Level 1 – Task 7 Level 2 – Tasks 9, 11 Level 3 – Tasks 8, 10	Task 7 – In the diagram, O is the centre of the circle, OB = 8 cm and $\angle AOB = 100^\circ$. Find a) the length of the arc AQB, b) the length of the arc APB.	Task 8 – The diagram shows a sector OAPB of a circle, centre O, with OB= 15 cm and AB = 20 cm. Find a) $\angle AOB$, b) the length of the arc APB.
	Task 9 – In the diagram, O is the centre of the circle, reflex $\angle AOB = 315^\circ$ and the arc ACB = 24 cm. Find a) the radius of the circle, b) the perimeter of sector OAB.	Task 10 – A piece of wire is 90 cm long. It is bent to form a sector OAPB of a circle such that its arc subtends an angle of 110° at the centre O. Find the radius of the circle.
	Task 11 – The end of a 50-cm pendulum describes an arc of 36 cm as it moves from A to B. Through what angle does the pendulum swing?	

Source of tasks: 1- 6 (Teacher's Notes) & 7-11 (Textbook – Discovering Mathematics 4A N(A))

Figure 4: Practice tasks used by Teacher 11

Vignette 2 – Teacher 24

Teacher 24 is also a senior teacher. The grade 10 students in his class are from the Normal (Technical) course of study (from the bottom 25th percentile of a cohort). A total of six lessons were recorded for the topic: Pythagoras theorem and trigonometry. Specific to this vignette is an instructional objective from Lesson 2 – To state the converse of Pythagoras theorem. Episode 1 illuminates how Teacher 24 engaged his students in uncovering the converse of Pythagoras theorem through the S activity segment and an accompanying R activity segment. It is evident from Episode 1 that students of all ability levels can be engaged in activities like uncovering the converse of Pythagoras theorem through an application of Pythagoras theorem to investigate if a triangle with given dimensions was right-angled.

Episode 1

1 T Yesterday I gave you a question to do as homework. Let me show you the question on the screen. Can you see the question?

> The question
> The sides of a triangle ABC are 11 cm, 13 cm and 17 cm.
> Is triangle ABC right angled? Justify your answer.

I asked you to do this at home.
Can you present your answer to me? [Teacher walks around and finds that students have not done the question]
You’ve got 5 minutes to do it now if you have not done it.
[Teacher walks around, and checks students work]

2 T Can you show me how you can prove or disapprove whether this triangle is a right-angled triangle?
11 cm, 13 cm, and 17 cm
Show by doing some calculations.

3 T How many of you have done it? Raise your hand.
[only S1 raises his hand]
I want everyone to do it. [Teacher walks to S1]

4 T -> S1 Yes, show me. Good idea

5 T -> class I have one student who already has an idea.
Think how to prove or disapprove if this is a right-angled triangle.

6 T -> S2 Good idea, write your working on your whiteboard.
How do you add this [T pointing to S2’s work]?
Yes, you are right, something about matching.

7 S2 If this is equal, then is right-angled?

8 T -> S2 Yes

9 S3 Teacher I can do it

10 T -> S3 Very good

11 T -> class Most of you have got the idea. What does Pythagoras theorem

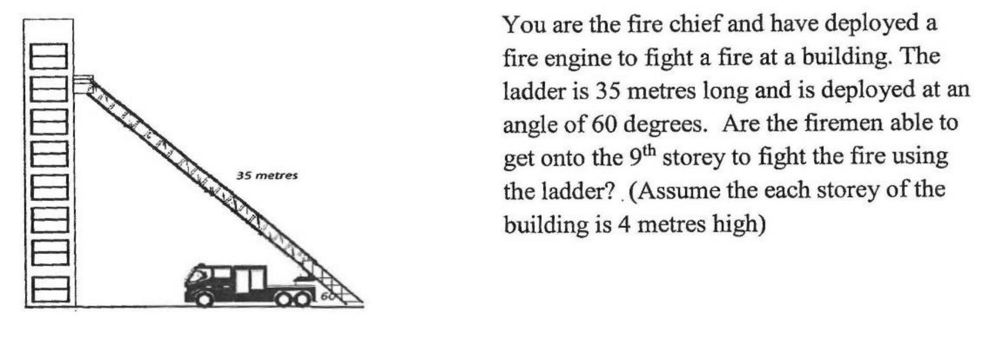
		say?
12	Ss (chorus)	$c^2 = a^2 + b^2$
13	T	Now let’s write down your working [teacher writes on the board at the front] $c^2 = a^2 + b^2$ $19^2 = 11^2 + 13^2$ $289 = 290$ Is this correct?
14	Ss	No
15	T	So, what shall we do?
16	S5	not a right-angled triangle
17	T	Yes, but we need to present our working correctly
18	S5	Do not put the equal sign
19	T	Can we summarise our working? First step: Write $c^2 = a^2 + b^2$ Note: c is the largest side, so $c^2 = 17^2 = 289$ $a^2 + b^2 = 11^2 + 13^2 = 121 + 169 = 290$ since $c^2 \neq a^2 + b^2$ the triangle is not right-angled.
20	T	Let’s write Pythagoras theorem and its converse
21	T	T writes on the board with inputs from the students. In a right-angled triangle, $c^2 = a^2 + b^2$ (where c is the hypothenuse) conversely, if in a triangle the three sides are related such that $c^2 = a^2 + b^2$ (where c is the longest side) the triangle is right-angled. Write this down in your notebook

Episode 1 shows Teacher 24’s conception of mastery learning wherein an application of Pythagoras theorem applicable to right-angled triangles helps students to verify its converse. Often teachers merely state the converse without the kind of student work (S) and review (R) that Teacher 24 facilitated.

CONCLUDING REMARKS

Curriculum time is finite. Therefore, within the allotted instructional time teachers strive for mastery learning in best possible ways. The S ↔ R loops appear to be the crucibles for mastery learning where teachers in Singapore secondary schools facilitate procedural fluency and deepening of conceptual knowledge through appropriate mathematical tasks that engage students in direct application of new knowledge, problem solving and higher order thinking.

The kinds of mathematical tasks teachers select for work during the S → R loops are critical, and this is evident in the two vignettes. The two vignettes were deliberately chosen to show that student ability is not a barrier to the kinds of mathematical work teachers may choose to engage learners in. In addition, the nature of mathematical tasks that Teacher 11 used in her 44 minutes of instruction during Lesson 2, were also prevalent in lessons of Teacher 24 ranging from the use of calculators, to finding squares, square roots, and trigonometrical ratios and the inverse for finding angles, to problem solving involving a real-world task, as shown in Figure 5. It is apparent that though there is emphasis on mathematical computations a strong emphasis is also placed on thinking mathematically (Noyes, 2007).



You are the fire chief and have deployed a fire engine to fight a fire at a building. The ladder is 35 metres long and is deployed at an angle of 60 degrees. Are the firemen able to get onto the 9th storey to fight the fire using the ladder? (Assume the each storey of the building is 4 metres high)

Figure 5: Practice task in Lesson 6 of Teacher 24

In conclusion, it may be said that teachers' conceptions of mastery learning in mathematics lessons in Singapore secondary schools appear to be aimed at 1) procedural fluency with mathematical tasks that call for more than direct application of mathematical knowledge, 2) use of 'new' knowledge to facilitate deeper understanding of mathematical concepts and 3) development of mathematical habits.

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COMPETENCIES FOR TEACHING MATHEMATICS IN THE DIGITAL ERA: ARE WE READY TO CHARACTERIZE THEM?

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The responsibility for integrating digital technology into the everyday reality of teaching and learning mathematics is in the hands of the teachers. Yet even after four decades of research in the field of integrating technology into mathematics teaching and learning, the actual use of technology in mathematics is limited. Perhaps it is time to characterize the digital competencies teachers need for successful technology integration in the classroom. Geraniou and Jankvist (2019) describe the attributes of mathematical digital competency (MDC) for students. Here I propose considering the MDC attributes as a baseline for teachers and suggest several paths that will enable us as research community to expand on digital competency for teachers.

INTRODUCTION

The work of teachers is complex. Indeed, teaching mathematics in the 21st century entails complexities and challenges that teacher have never before had to face, for they are expected to integrate technology into their instruction for the benefit of all learners. The COVID-19 pandemic is a painful reminder of the crucial role played by the digital environment in the shift to remote teaching and learning. Initial attempts to integrate technological tools into mathematics classrooms began about four decades ago. Hence, it is surprising that technology integration into school mathematics is still limited. The challenges are not only at the level of teacher practice. Theory must also be further developed to gain a better understanding of the growing demands imposed on teachers. In this talk I seek to touch upon some of these complexities and to suggest some avenues for further actions to be adopted by researchers, teacher educators and practitioners.

I begin by exploring the terminological shift from knowledge to competencies for teachers and students. In particular, I define mathematical competencies for students and describe the *Mathematical Digital Competency* framework proposed by Geraniou and Jankvist (2019). This leads to a consideration of the implications of such a framework for mathematics teachers' competencies. Next, I examine the issue of professional development as a vehicle to help

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teachers integrate technology and I question its effectiveness. I then suggest two possible directions that facilitate studying teachers' mathematical digital competency: First I examine the European framework for the Digital Competence of Educators (DigCompEdu) (Redecker, 2017), which was developed in response to increasing demands for teachers to master a growing set of competencies. I then examine three platforms (e.g., the Learning Management Systems (LMS) platform) used to support teachers' didactical work in order to determine what competencies teachers need to be able to make "good" use of such platforms. I end with some theoretical observations.

A SHIFT FROM KNOWLEDGE TO COMPETENCY

From teachers' knowledge to teachers' competencies

Researchers have converged on the notion that teacher knowledge is specific. The seminal work by Shulman (1986) initiated interest in understanding the essence of this knowledge and its components, and this interest is still vital today. Shulman proposed distinguishing among three categories of content knowledge: (a) subject matter content knowledge, (b) pedagogical content knowledge, and (c) curricular knowledge (p. 9). *Pedagogical content knowledge* (PCK) was designed to fill the "missing paradigm", that is, the gap (p. 7) between content knowledge and pedagogical methods traditionally taught in teacher education courses.

Shulman's model still remains influential, and its categories have evolved through a range of research studies and have inspired other frameworks. Among these frameworks are: 1) *Technological Pedagogical Content Knowledge* (TPACK, Mishra & Koehler, 2006), which "attempts to identify the nature of knowledge required by teachers for technology integration in their teaching, while addressing the complex, multifaceted and situated nature of teacher knowledge" (Koehler, 2012), and 2) *Mathematical Knowledge for Teaching* (MKT, Ball et al., 2008), which refines Shulman's conceptualization for the case of mathematics teachers.

Yet according to Neubrand (2018), knowledge-driven approaches are limited for at least two reasons: "the gap between knowing and acting" and the lack of the "affective component" (p. 609), which appears to be as important as the cognitive one. Along these lines and drawing on recent research studies, Kunter et al. (2013) claim that "aspects beyond knowledge may be important in determining teacher success. These aspects include teachers' beliefs, work-related motivation, and ability for professional self-regulation" (p. 807). Thus, they delineate the concept of teacher professional competency. Likewise, Grossman and McDonald (2008) argue that

> in the future, researchers need to move their attention beyond the cognitive demands of teaching, which have dominated the field for the past 20 years, to an

> expanded view of teaching that focuses on teaching as a practice that encompasses cognition, craft, and affect (p. 185).

Nevertheless, whereas recent research seems to converge on the need to go beyond knowledge when considering the essence of the teaching profession, researchers do not agree about what characterizes this profession. The term ‘digital competence’ was introduced in the context of ICT. Ilomäki et al. (2016) point out that “[d]igital competence seems to be a ‘loose’ concept: One that is not well-defined, still emerging, with meanings varying based on users from different approaches” (p. 656).

The field has clearly shifted from considering only cognitive aspects of teachers' preparation to considering the broader notion of competency (Grossman & McDonald, 2008). The word *competency* describes a person’s capability to do something adequately (Grammarist, 2014, https://grammarist.com/words/competence-and-competency/). According to the OECD (2003), “a competency is more than just knowledge or skills. It involves the ability to meet complex demands, by drawing on and mobilizing psychosocial resources (including skills and attitudes) in a particular context” (p. 4).

Although there is no commonly accepted conceptualization of competency, according to education research competency encompasses three facets: Skills, knowledge and attitudes (Ala-Mutka, 2011). *Knowledge* is defined as information acquired through sensory input: Reading, watching, listening, touching, and the like. The concept of knowledge refers to familiarity with factual information and theoretical concepts (European Parliament Council, 2008). *Skills* refer to the ability to apply knowledge to specific situations. Skills develop through practice via a combination of sensory input and output (ibid). As an example, social skills are developed through interaction with people by observing, listening, and speaking with them. Hence, while knowledge is theoretical, skills are practical. One can know a subject but may not have the skills required to apply that knowledge to specific tasks since knowledge does not provide skills. A teacher may know mathematics and pedagogy, but this only makes her knowledgeable about teaching; knowledge does not make one a good practitioner. To become a good teacher one must teach, practice one’s techniques, and improve one’s skills through experience. Developing skills provides some knowledge, as practicing those skills results in sensory inputs. Competencies take “skills” and incorporate them into on-the-job behaviors. These behaviors add up to competent performance of job requirements. Attitudes are conceived as motivators of performance and include ethics, values, and priorities as well as responsibility and autonomy (Ala-Mutka, 2011).

From students’ knowledge to students’ competencies

The terminological shift from knowledge to competencies is not unique to teachers. A similar shift has been noted with respect to students. Niss and Højgaard (2019) revisited their 2002 Danish KOM project on competencies and learning mathematics (Niss & Jensen, 2002) and attempted to answer the following question: What does it mean for someone to be mathematically competent? They explain their task as follows: “By focusing on mathematical competence rather than on mathematical subject matter as the integrating factor of mathematics across all its manifestations, we have chosen to focus on the *exercise* of mathematics, i.e., the *enactment* of mathematical activities and processes” (Niss & Højgaard, 2019, p. 12).

Niss and Højgaard propose the following definition: “*A mathematical competency* is someone’s insightful readiness to act appropriately in response to *a specific sort* of mathematical *challenge* in given situations” (p. 14). The authors further refer to eight overlapping mathematical competencies that are visually organized into what is known as the KOM flower (Figure 1) (Niss & Højgaard, 2019, p. 19).

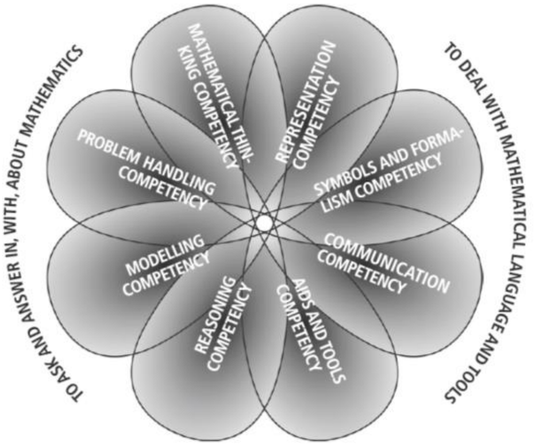

TO ASK AND ANSWER IN, WITH, ABOUT MATHEMATICS
MATHEMATICAL THINKING COMPETENCY
REPRESENTATION COMPETENCY
TO DEAL WITH MATHEMATICAL LANGUAGE AND TOOLS
PROBLEM HANDLING COMPETENCY
SYMBOLS AND FORMALISM COMPETENCY
MODELLING COMPETENCY
COMMUNICATION COMPETENCY
REASONING COMPETENCY
AIDS AND TOOLS COMPETENCY

Figure 1: A visual representation of the eight mathematical competencies

Geraniou and Jankvist (2019) took another step towards defining *Mathematical Digital Competency* (MDC). While not restricting themselves to a specific definition, they claim that MDC involves at least the following three characteristics:

- Being able to engage in a techno-mathematical discourse
- Being aware of which digital tools to apply within different mathematical situations and context and being aware of the capabilities and limitations of different tools
- Being able to use digital technology reflectively in problem solving and when learning mathematics. This involves being aware and taking advantage of digital tools serving both pragmatic and epistemic purposes (p. 43).

In particular, they framed these characteristics in terms of the instrumental approach (Rabardel, 2002). I will come back to this approach towards the end of the talk. Suggesting these characteristics for students' mathematical digital competencies is an impressive achievement. First, the characteristics are not connected to or rooted in any specific mathematical topic. Second, they are not linked to any particular technological environment. Third, they do not refer directly to students' ages or grade levels and hence are not restricted to specific learners.

Is it possible to conceive of a parallel description for teachers? I believe that as a research community we must try to provide such a parallel conceptualization. My first observation is that these student mathematical digital competencies may be considered as a minimum baseline for those of teachers. Now we need to describe and characterize their upper limit, to which I devote the remainder of this talk.

PROFESSIONAL DEVELOPMENT FOR TEACHERS

The research consensus is that the teacher is the key person in integrating technology into classrooms (Emprin, 2010). It is the teacher who determines whether, to what extent and how technology will enter the mathematics classroom. The teacher's main aim should be to introduce technology into the classroom efficiently for the benefit of all learners. Research studies converge in claiming that teacher training is one of the key elements in achieving this aim. Indeed, in a literature review on "*barriers to the uptake of ICT by teachers,*" Jones (2004) stresses that "*there is a great deal of literature evidence to suggest that effective training is crucial if teachers are to implement ICT effectively in their teaching*" (p. 8). This clearly addresses the issue of pre-service teacher education (TE) and in-service teacher professional development (PD) geared toward using digital technology.

According to research, ICT integration has a profound impact on teachers' practices. Technology facilitates new teaching and learning approaches, such as direct manipulation or visualization of mathematical objects or collaborative learning. Teachers need to develop new knowledge and skills for designing relevant technology-mediated tasks, monitoring student work and assessing student learning using technology (Spiteri & Rundgren, 2020).

The literature discusses teacher education or teacher professional development initiatives. The success of PD is measured according to how ICT is implemented in teacher practice following the PD. Hegedus et al. (2017) pointed out that in a number of cases, the outcomes of these initiatives are disappointing. This disappointment is surprising, as in these initiatives researchers usually fill multiple roles as designers, providers, and evaluators of the PD. The gap between teachers' needs and the TE-PD contents has been identified as one of the main reasons for this disappointment (Emprin, 2010). This signals to teacher educators that they need a better understanding of what teachers need to know in order to use ICT effectively. One possible explanation of the mismatch between teachers' needs and the PD they are provided may be that the PD targets a change in teachers' knowledge and perhaps even skills yet fails to address attitudes. In other words, the PD should aim at teachers' competencies in using technology in teaching mathematics. This brings us back to the need to characterize these competencies.

FRAMEWORK FOR DIGITAL COMPETENCE OF EDUCATORS

The OECD proposed the Digital Competence of Educators (DigCompEdu) (Redecker, 2017) with the purpose of suggesting a common language for various stakeholders in education and as a "response to the growing awareness among many European Member States that educators need a set of digital competencies specific to their profession in order to be able to seize the potential of digital technologies for enhancing and innovating education" (p. 8). DigCompEdu is one of the most developed and detailed frameworks, both internationally and in English speaking countries.

The 22 competencies described in the framework are grouped under six categories: (1) professional engagement; (2) digital resources; (3) teaching and learning; (4) assessment; (5) empowering learners; and (6) facilitating learners' digital competency. As can be seen in Figure 2, these six categories are further clustered under three headings: educators' professional competencies, educators' pedagogic competencies and learners' competencies.

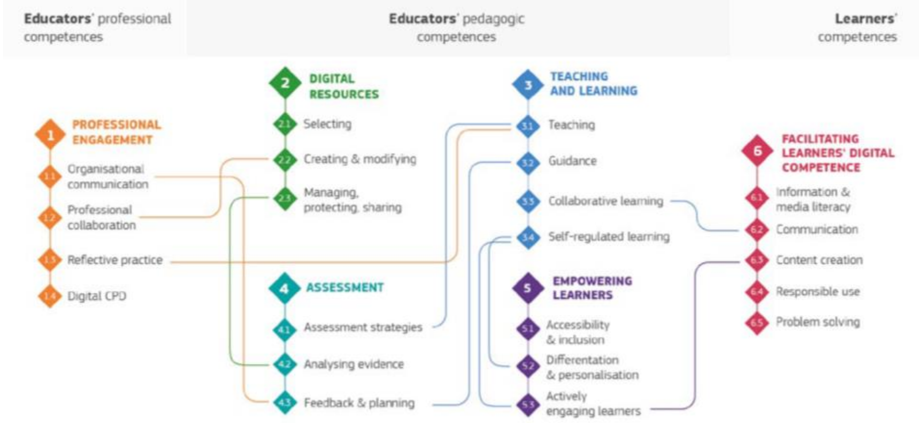

Educators' professional competences
Educators' pedagogic competences
Learners' competences
1 PROFESSIONAL ENGAGEMENT
1.1 Organisational communication
1.2 Professional collaboration
1.3 Reflective practice
1.4 Digital CPD
2 DIGITAL RESOURCES
2.1 Selecting
2.2 Creating & modifying
2.3 Managing, protecting, sharing
3 TEACHING AND LEARNING
3.1 Teaching
3.2 Guidance
3.3 Collaborative learning
3.4 Self-regulated learning
4 ASSESSMENT
4.1 Assessment strategies
4.2 Analysing evidence
4.3 Feedback & planning
5 EMPOWERING LEARNERS
5.1 Accessibility & inclusion
5.2 Differentiation & personalisation
5.3 Actively engaging learners
6 FACILITATING LEARNERS' DIGITAL COMPETENCE
6.1 Information & media literacy
6.2 Communication
6.3 Content creation
6.4 Responsible use
6.5 Problem solving

Figure 2: The DigCompEdu Framework (Redecker, 2017, p. 8)

For each competency, the framework provides a concise description. For example, the competency "selecting digital resources" is described as follows:

> To identify, assess and select digital resources for teaching and learning. To consider the specific learning objective, context, pedagogical approach, and learner group, when selecting digital resources and planning their use. (p. 20)

Next, a list of activities is associated with each competency to provide a sense of its scope. For example, for the "selecting digital resources" competency, the following five activities are listed:

- To formulate appropriate search strategies to identify digital resources for teaching and learning.
- To select suitable digital resources for teaching and learning, considering the specific learning context and learning objective.
- To critically evaluate the credibility and reliability of digital sources and resources.
- To consider possible restrictions to the use or re-use of digital resources (e.g., copyright, file type, technical requirements, legal provisions, accessibility).
- To assess the usefulness of digital resources in addressing the learning objective, the competence levels of the concrete learner group as well as the pedagogic approach chosen. (p. 44)

As Redecker points out, this list of activities is not exhaustive if we consider the rapid development of digital technology. Finally, for each competency, the framework outlines a progression through six proficiency levels. Again, for the aforementioned competency, the progression is as follows:

- Newcomer (A1): Making **little use** of the internet to find resources.
- Explorer (A2): Being aware of and making basic use of digital technologies for **finding resources**.
- Integrator (B1): Identifying and **assessing** suitable resources using **basic criteria**.
- Expert (B2): Identifying and **assessing** suitable resources using **complex criteria**.
- Leader (C1): **Comprehensively** identifying and assessing suitable resources, considering all relevant aspects.
- Pioneer (C2): **Promoting** the use of digital resources in education. (p. 45)

The EduDigComp framework is an impressive attempt to describe competencies encompassing a width range of teachers' activities. The levels implicitly acknowledge that teachers need to further develop each competency. Yet how the different levels relate to one another is not clear. Are they discrete levels? Does each level encompass the preceding levels? Moreover, the competencies are not specific to any particular subject matter or grade level (Tabach, & Trgalová, 2020).

After this brief examination of the European framework for the Digital Competence of Educators (DigCompEdu) (Redecker, 2017), let me now suggest a different angle that may help us, as researchers, better understand teacher competencies for teaching mathematics in a technological environment. I do this by examining digital platforms known as Learning Management Systems (LMS).

LEARNING MANAGEMENT SYSTEMS

General learning management systems

A learning management system is an infrastructure that facilitates delivering and managing instructional content, identifies and assesses individual and organizational learning goals, tracks the progress towards meeting these goals, and collects and presents data for monitoring the learning process of the establishment as a whole (Szabo & Flesher, 2002). Typically, a learning management system provides instructors with a way to create and deliver content, monitor student progress and participation, and assess student performance (Pilli, 2014). Among the popular LMS systems, Moodle (Modular Object-Oriented Dynamic Learning Environment, https://moodle.org/) seems to the one of the most effective and popular open sources free LMS systems (Cavus, 2015).

What must a teacher be able to do while working with an LMS of this type? What teacher competencies are needed for working with such LMS? A teacher needs to be familiar with the infrastructure itself, able to manage student participation and assign students to a particular course, upload course materials and make them available to students, assign homework/classwork/tests, define a space where students can upload their contributions, and manage some communication opportunities for students, for example forums or messages. Other features may also be used, but these are the basic ones. None of these teacher activities is unique to mathematics teachers. Although every interface must be learned and assimilated, I believe that working within a system such as Moodle involves competencies that are no different from those needed for being an involved citizen in today's world.

In the last decade or so, however, LMS systems have been embedded in digital platforms that are specific to mathematics. Next, I will describe two of these, STEP and DESMOS teachers, to show the span of the opportunities and challenges they offer teachers. Please note that I was not a developer of these platforms, and I believe there are platforms with similar features that I am not addressing. Think of my choice of platforms in terms of a convenience sample in quantitative research.

Seeing the entire picture platform

The Seeing the Entire Picture (STEP) platform is aimed at supporting mathematics teachers in conducting formative assessment (https://meri.edu.haifa.ac.il/projects). The platform was developed by Prof. Yerushalmy and Dr. Olsher from the University of Haifa. In the words of its developers:

> "**STEP** is designed to help teachers and students make use of rich and interactive assignments in math classes, through the use of a wide range of technologies

> (cellular phones, tablets, laptops, and computers), and provide an automatic and accessible analysis of the students' answers in a manner that would form the foundation for the teacher's decision making in real time – during the actual course of the class"

The platform offers such LMS features as assigning students to a specific class, duplicating activities, choosing part of an activity, and modifying it, assigning activities to a lesson, and collecting students' responses. In this sense it is not unique. Yet, STEP offers mathematics teachers some unique properties for designing and enacting a formative assessment lesson.

Two basic working modes are available for designing a lesson: Designing an activity or choosing from a set of already designed activities, either "as is" or with modifications. Here I focus only on the pre-designed activities already provided by the system. These activities are categorized according to their mathematics topic, such as functions or geometry. Each topic is also divided into sub-topics. In addition, the activities are linked to specific grade levels. Each activity includes information about the technological platforms on which the activity can be enacted (tablet, smart phone, and the like), as well as a teacher's guide. The entries in the teacher's guide include the aim of the activity; when to do the activity; students' prior knowledge; examples of possible submissions, including mathematical foci and possible filters (explained subsequently); and points for possible discussion. That is, the designed activities provide the teacher with a good basis for anticipating how the lesson will look.

Once the teacher opens the lesson, the students gain access to the activities, can work on them in their private space, and can submit their answers through the system. The teacher can examine the students' submissions in four modes: The first is table mode, which provides information about who submitted it and the correctness of the submission. The second is "carpet" mode, where all the submissions have arranged one next to the other on a single screen. See Figure 3 for an example of such a carpet, taken from Olsher and Abu Raya (2020). The teacher can choose to drill down any one of the submitted responses for closer inspection or for sharing and discussion while projecting it to the whole class. Students' names can be made anonymous. The third mode involves filtering student submissions according to mathematical features, as explained in detail in the next paragraph. The fourth mode is in the form of a bar graph.

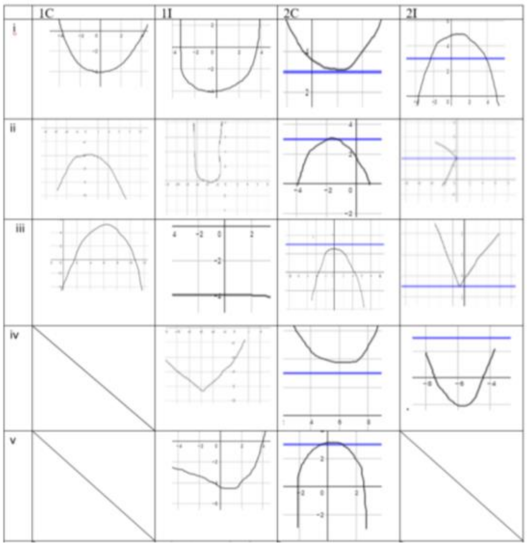


Figure 3: An example of a “carpet” of student answers in the STEP platform

An interesting feature of the STEP platform is the “filtering” option, which is based on mathematical properties of the submitted responses. The teacher can choose among several filtering options for any given activity. For example, Figure 3 displays some student answers for the following task: Sketch different quadratic functions that intersect with the line y=3 at one point or less. The students were asked to sketch three different examples if they exist. The filtering options for students’ responses go beyond correct or incorrect examples. For possible incorrect features, the platform suggests the following: the sketch intersects the line y=3 at two different points; the sketch does not describe a function; the sketch does not describe a quadratic function; the sketch is partial. For correct responses, the system can filter solutions according to the following criteria: The sketch has minimum at y=3; the sketch has maximum at y=3; the sketch has maximum below y=3; the sketch has minimum above y=3; the sketch has vertex at y=3 and on the y axis; none of the sketches have a vertex at y=3; each sketch has a different number of intersection points with y=3; all the sketches have a vertex at y=3. Teachers can choose one, two or three features, and the system provides them with a Venn diagram showing how many submissions can be filtered based on the chosen feature. Such a platform naturally provides the teacher with a wealth of information and access to students’ thinking. At the same time, analyzing student submissions and deciding on what step to take next in order to promote students’ thinking constitute an unusual challenge that requires the teacher to master the mathematics but also much more. To support the teacher, the platform provides a teacher’s guide specific to every activity. This guide points out the main mathematical ideas addressed by the task, delineates what kind of student answers might be expected, and suggests possible discussion points.

In fact, when I consider the competencies, teachers need to make effective use of the platform's potential, the five practices model developed by Stein et al. (2008) comes to mind: Anticipating, monitoring, selecting, sequencing and connecting. The teacher's guide can help teachers *anticipate* student responses. Enactment of a lesson via the platform can support *monitoring* students' progress by watching their submissions. The filtering options can support the other three practices: *Selecting* which solutions to share with the class, *sequencing* the chosen solutions and *connecting* them to the aim of lesson, which was determined during the *anticipating* stage. Indeed, the term "practice" can be replaced by "competency" as both involve knowledge, skills, and attitudes.

DESMOS teacher platform

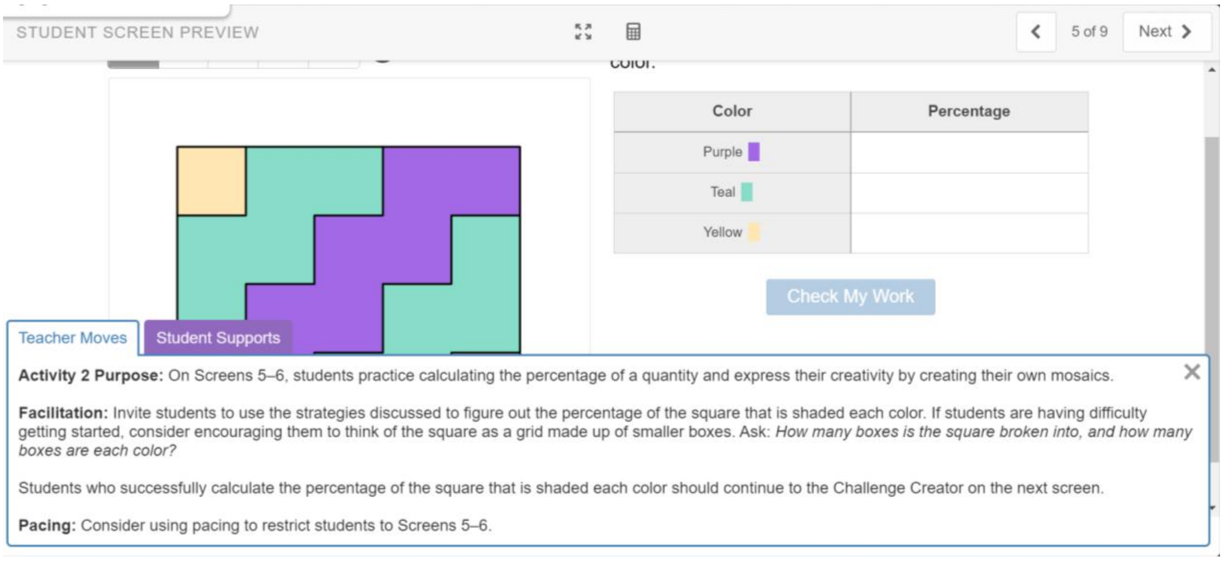
DESMOS is a free platform available at desmos.com that seeks to:

> "…build a world where every student learns math and loves learning math, where a student's access to the power and beauty of math doesn't depend on their place of birth, race, ethnicity, gender, or any other aspect of their identity."

The DESMOS platform offers tools for doing mathematics activities in the environment. It also provides a platform for teachers (https://teacher.desmos.com/), which is the focus of this discussion. The DESMOS teacher platform offers such LMS features as assigning students to class, coordinating activities, communicating with students and the like. It also allows teachers to create activities for their students or to choose from a large set of ready-made activities that can be used "as is" or with modifications. It is interesting to note that the DESMOS team developed an explicit set of principles to guide their development of mathematical activities. Among these principles are the following: "Create opportunities for students to be right and wrong in different, interesting ways"; "Give feedback that attaches meaning to student thinking"; and "Use a variety of resources". For more information, see https://blog.desmos.com/articles/desmos-guide-to-building-great-digital-math-2021/.

As in the case of the STEP platform, here I also focus on ready-made activities. These activities are clustered under mathematical topics, with several activities under each topic. Basic information is provided for each activity, such as the expected time for working on the activity, the aims and so on. A typical activity includes a number of screens that reflect a collection of several tasks. For each such activity, the platform provides a short message for the teacher on the student screen. Figure 4 depicts the fifth screen out of nine for the activity. The teachers are provided a brief description of the purpose of the particular task and hints for how to facilitate students' thinking. Note also that these tips for the teachers are labeled "Teacher Moves," suggesting a student-centered pedagogy.

In addition, the DESMOS platform offers an interesting checklist for teachers to consider before enacting an activity in class. The list includes the following actions: complete the activity using student previews; identify your learning targets for the activity; determine the screens you will bring to the whole class using the following function: 'Teacher Pacing and Pause Class. What will you discuss on those screens?'; anticipate screens that will cause students to struggle and then plan your responses; plan a challenge for students who finish the activity quickly and successfully; during the activity, when appropriate make yourself available to students who need individual help or have questions; and write a summary of the activity's main ideas: How can you incorporate student work in that summary? What parts of the activity can you skip to ensure there is sufficient time for the summary?



Color	Percentage
Purple 🟪	
Teal 🟩	
Yellow 🟨	

Activity 2 Purpose: On Screens 5–6, students practice calculating the percentage of a quantity and express their creativity by creating their own mosaics.

Facilitation: Invite students to use the strategies discussed to figure out the percentage of the square that is shaded each color. If students are having difficulty getting started, consider encouraging them to think of the square as a grid made up of smaller boxes. Ask: *How many boxes is the square broken into, and how many boxes are each color?*

Students who successfully calculate the percentage of the square that is shaded each color should continue to the Challenge Creator on the next screen.

Pacing: Consider using pacing to restrict students to Screens 5–6.

Figure 4: Example of one screen of an activity in the DESMOS teacher platform

While enacting the activity, the teacher can view the students' progress, see whether their submissions are correct and view their answers by using a mode similar to the STEP carpet mode. The teacher can also ask the system to provide an aggregated view of student responses and can select anonymous responses to share with the whole class.

The DESMOS teacher platform is another clear example of a digital tool developed specifically to support teaching mathematics. Again, for teachers to be able to use such a powerful platform, they need a set of didactical competencies.

THEORETICAL CONCEPTUALIZATION – DOUBLE INSTRUMENTAL GENESIS

Krumsvik and Jones (2013) claim that "digital competence of teachers is more complex than in other occupations" (p. 172) as it embeds two dimensions: (1) ability to use technology (personal use), and (2) ability to use technology in a pedagogical setting. That is, teachers must also "continually make pedagogic-didactic judgements which focus on how ICT can expand the learning possibilities for pupils in subjects” (Krumsvik, 2008, p. 283). The theoretical construct of *double instrumental genesis* (Haspekian, 2011) reflects this view and allows conceptualizing these two dimensions of teachers’ digital competence. The *double instrumental genesis* construct was developed in accordance with the instrumental approach (Rabardel, 2002) and encompasses both the personal and the professional instrumental geneses of teachers who use ICT. Whereas the personal instrumental genesis is related to the development of a teacher’s *personal instrument* for a mathematical activity from a given artefact, the professional instrumental genesis yields a *professional instrument* for a teacher's didactic activity. To avoid any confusion between teacher’s personal and professional activities, we (Tabach, & Trgalová, 2018; Trgalová, & Tabach, 2020) use the term *mathematical genesis* to refer to a teacher’s *personal activities* (transforming an artefact into a mathematical instrument, i.e., doing mathematics with technology) and the term *didactic genesis* to refer to a teacher’s *professional activities* (transforming the same artefact into a didactic instrument, i.e., teaching mathematics with technology).

Hence, a teacher develops *two* instruments—i.e., parts of artefacts together with mental schemes of use—from the same tool: a mathematical scheme and a didactic scheme. A scheme is an invariant organization of activities for a given class of situations and comprises four components: Goals and anticipations; rules of action, information-taking and control; operational invariants (i.e., knowledge); and possible inferences (Trouche, 2004). It is reasonable to assume that these two developmental processes, that is, the mathematical and didactic schemes, are interconnected. The *mathematical instrumental genesis* leads to the construction and appropriation of a tool, yielding an instrument used for the purpose of doing mathematics. The *didactic instrumental genesis* leads to the construction and appropriation of the previous instrument, yielding a didactical instrument for the purpose of teaching mathematics.

An examination of platforms such as STEP or DESMOS teachers, which are specifically directed at teaching mathematics, makes the didactical instrumental genesis “visible”. Yet I contend that a teacher who works “only” with mathematical tools such geogebra or DESMOS must undergo the didactical instrumental genesis and develop competencies for orchestrating the learning.

Returning to the issue of teachers’ digital mathematical competencies with which I opened the talk, I believe that the MDC defined by Geraniou and Jankvist (2019) also applies to teachers. Beyond this is a complementary set of competencies, specifically didactical digital mathematical competencies, that are relevant to the work of mathematics teachers. In this talk I hinted at some of these, which I believe constitute a fruitful field for future research.

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WHAT MATHEMATICAL THINKING SKILLS WILL OUR CITIZENS NEED IN 20 MORE YEARS TO FUNCTION EFFECTIVELY IN A SUPER SMART SOCIETY?

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Critical predictions for the next 20 years include climate change, new pandemics, and growing inequality. These predictions should help us imagine the skills that students will need. However, there is a change that will have an even greater impact. With machine learning, apps will both advise and make decisions for us. However, apps mostly serve the interests of third parties, often conspiring and deceiving users. Worse still, evolved face-to-face and reputation-based trust-building mechanisms are maladapted to this new environment. This represents a huge evolutionary mismatch. Rather than spending years preparing for agricultural and industrial life, citizens will need to know a core set of skills and computational models in order to understand the underlying psycho-social models embedded in apps; and decide which ones to trust.

FOUR CHALLENGES

There are several challenges that our students will face in the next 20 years. One of them is climate change. In developed countries, the energy currently captured per capita per year is 50 times greater than the energy captured by our hunter-gatherer ancestors (Morris, 2013). However, in underdeveloped countries the energy captured per capita is barely a third of the level of developed countries (Gates, 2021). As people in underdeveloped countries represent roughly 90% of the population and aspire to live as people do in the developed world, the demand for energy will triple. If we do not change the production model, carbon emissions will increase and global warming will continue to grow. This is one of the challenges of the next generation that our students will face. This of course will require future citizens to be able to understand these phenomena, their components, and the effect of different interventions.

Another significant challenge is pandemics. Although some predicted a pandemic like the current COVID-19 pandemic, no one prepared for it. Today, we need to educate and prepare students to face new outbreaks. The increasing population density, connectivity, and mobility facilitate the emergence of new pandemics and particularly pandemics on a global scale. Unlike typical hunter-

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gatherer diseases, these are crowd diseases that appeared with agriculture and require a minimum population of hundreds of thousands of inhabitants (Diamond, 2012). It is critical that citizens understand the effect of social distancing, reduced mobility, the use of masks, and different types of vaccines and treatments on the spread of a disease. Citizens will require mathematical skills and knowledge of a core set of basic models to estimate the relationship between these parameters and the significance for their health. In other words, students need this set of mathematical thinking skills in order to develop health literacy.

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However, there is a fourth challenge, which is of a different nature and appears to have an even greater impact. We are now witnessing the development of autonomous applications or artificial agents that use Machine Learning to constantly sense, learn, communicate with other devices, and make decisions for citizens. This technology is leading to a level of change that will have a great psychological and economic impact. First, it may exclude many people from the job market. Second, it may turn many people irrelevant (Harari, 2015). This is the rise of the useless class (Harari, 2015). It is not an exploited, abused, or undervalued class. It is radically different. It is a superfluous class. This could easily lead to a spike in mental health diseases and a profound lack of meaningful life. Third, citizens will likely lose control of their own lives as various apps will decide for them. Furthermore, there is also a significant impact on public trust (Jian et al., 2000). Most of these apps are provided by third parties and are designed primarily to satisfy their own needs and interests. It will be very difficult to decode the true motivation embedded in them. In this paper, we argue that this trust issue is the biggest challenge for the next 20 years, and that mathematical thinking is essential to help address this difficult conundrum. Unlike the three previous challenges, this challenge is a major turning point in the history of civilization. The autonomy, free will, and nature of the human condition are at stake. Given these implications, this paper will focus primarily on this challenge.

OUTSOURCING YOUR THINKING TO A COUNCIL OF APPS

Nowadays, many apps or artificial agents on your smartphone make suggestions for you. For example, how to go from one place to another in the city, or what news to watch. Soon, some apps will take the lead, like driving your car or autonomously providing you an accurate dosage of a medicine. Little by little, we are getting used to depending on apps to help us think and decide. Increasingly, apps are learning our needs and preferences, and thus taking control of our decisions. It is very likely that you will soon face the dilemma of receiving conflicting advice from two different apps. In that case you will need to talk to both of them and understand the disagreement. After that, the apps could talk to each other and reach an agreement. But if they do not agree, what will you do? Next, consider the situation in which you have many apps or artificial agents. This means you will indeed have a council formed by your preferred apps. You will be King Arthur with your round table of councillor apps. But what will you do when there are divergent opinions? What will you do if some apps accuse other apps of negligence, carelessness, poor analysis, lying to you, or even disloyalty, corruption, or treason? The problem becomes even more serious if you consider that apps are constantly learning things about you and about the world that you yourself do not know. How will you determine if the decisions are reasonable and are good for you?

The problem is compounded when a critical component enters: the real motivations of the apps. These artificial agents were designed by third parties, and therefore it is expected that they serve the providers' interests, not yours. They will be the new e-cigarettes, which, like the tobacco industry, has subliminally convinced us that we are better off with them. Moreover, your contacts, competitors, enemies, friends, or any individual or organization can set up their own artificial agents that track or spy on your actions and the actions of any of the members of your council of artificial agents in order to achieve their goals instead of your own. They will most likely not only track your information but also manipulate the data and may eventually even deliberately try to mislead you and your artificial agents. Apps will probably form alliances that allow them to achieve their objectives better. Then, one possible solution is to improve your artificial agents by acquiring more powerful ones from third parties; apps with better knowledge and more memory that are faster and better connected. That type of solution, however, will lead to a reaction from others and their artificial agents. You, your artificial agents, your contacts, and their artificial agents will produce not only a domino effect chain reaction but cycles with actions and reactions. In other words, a permanent arms race will be triggered. How will you determine which ones to trust?

Outsourcing your thinking and decision making is not easy. Just like King Arthur or the President, you will not be able to fully trust in your council and simply rest. You will have to be constantly monitoring and verifying that the decisions are reasonable.

CAUGHT IN AN EVOLUTIONARY TRAP?

How can our brain discriminate who is trustworthy and who is not? To answer this question, we have to consider how the human brain has evolved and for what problems it is well adapted. Our brain is the product of a process of natural selection that has taken place over millions of years. Over thousands of generations, it has evolved highly efficient mechanisms to learn the knowledge and skills required for the life of hunter-gatherers. These are the learning mechanisms of imitation, play, storytelling and teaching, which allow us to learn how to walk, talk, collect food, hunt, cook, fight against other bands, and maintain a productive social life in our tribe. These types of knowledge and skills are called biologically primary (Geary, 2007). However, due to the enormous advances in cultural evolution over the last 5,000 years, today we need students to do and learn completely new things, often of a totally different nature and some of them very counter-intuitive. Students also need to learn to live and socialize with thousands of strangers instead of dozens of relatives. They have to learn to read and write, something for which the brain is not adapted, requiring intensive and guided practice and leading to a huge transformation of different areas of the brain (Dehaene, 2005; Henrich, 2020). They also need them to learn abstract concepts that are totally different from the ones acquired by their ancestors, such as positional notation, negative numbers, fractions, and algebra. These are considered biologically secondary knowledge and skills (Geary, 2007).

There is then what evolutionary biologists (Mayr, 1942; Wilson, 2019) and psychologists (Van Gugt, 2020) call an evolutionary mismatch or evolutionary trap. That is, our brain is trapped with certain powerful characteristics that are not well adapted to modern life. One of the first identified cases occurred with turtles. When hatching on the beaches, turtles know to head in the direction of the moonlight. However, in current environments, this same mechanism directs them towards city lights and away from the sea. The same phenomenon occurs with the human diet. Our ancestors lived without having guaranteed food for the next few days and with the constant danger of famine. There were frequent unpredictable food shortages (Diamond, 2012). They also did not have refrigerators to store food. This makes our brains look for high-calorie foods and eat as much as possible in order to store energy as fats. But today, with an abundance of food available, this mechanism leads to obesity. Biologically primary abilities are very powerful, but they can also be traps that hinder biologically secondary abilities.

The good news is that it is possible to get out of these evolutionary traps with special training. However, it is not easy, and it takes many years of education with well-adjusted pedagogy and permanent directed work. For example, only with years of practice can we learn to read. Literacy produces great transformations in areas of the human brain, as well as the loss of some abilities. One example is facial recognition, which becomes marginalized only in one hemisphere and not spread across two, as is the case with illiterate people.

Another example is social life, which in our hunter-gatherer ancestors was in groups of around 150 individuals, known as the Dunbar´s number (Dunbar, 1992), most of whom were kin. When they detected foreigners hovering in the limits of their territory, conflicts and even murder ensued (Diamond, 2012; Wrangham, 2019). Even though we have been self-domesticated because of applying the death penalty to about 15% of the most violent men in the last three hundred thousand years, thus greatly reducing reactive aggression compared to other primates such as chimpanzees and bonobos, we still suffer from proactive premeditated aggression (Wrangham, 2019). In previous wars, all the men and boys of the vanquished were killed. It was not until 5,000 years ago that the victors began to keep prisoners, albeit removing their eyes so that they would not escape and making them work in activities that did not require sight (Diamond, 2012). Today, with citizen education over a number of years of schooling, we manage to live with millions of strangers without even getting excited. Even though we have never seen them, we are willing to collaborate and help them. However, nepotism is still strong, as are biases towards relatives, friends, and members of our interest groups. We have transitioned to a life with millions of people, most of whom are unknown to us, whom we have never seen face-to-face, and never will. This transition was a monumental change that took millennia. For example, among the Yanomamö, hunter-gatherers who live between Venezuela and Brazil,

> …strangers are generally suspect and viewed with distrust because the Yanomamö believe that they are likely to inflict supernatural harm on them. To know someone's personal name is, in a sense, to “possess” some kind of control over that person, so the Yanomamö initially do not want strangers to know their personal names. (Chagnon, 2013, p. 35).

Instead, today

> the seemingly trivial act of entering a café full of strangers without a care in the world is one of our species’ most underappreciated accomplishments, and it separates humankind from most other vertebrates with societies. (Moffett, 2018, p. 5).

It requires an enormous amount of trust in other people that you have never met nor are ever likely to meet again in the future. This is an impersonal trust. It can be roughly measured with the Generalized Trust Question: “Generally speaking, would you say that most people can be trusted or that you can’t be too careful in dealing with people?” (Henrich, 2020).

Given the growing appearance of smart apps and a council of increasingly sophisticated artificial ‘advisors’, it is critical to understand the mechanism we have for building trust. This mechanism can be easily hacked by these artificial agents. One critical social skill is detecting cheaters. It is an automatic and effortless process, which is independent and does not interfere with other cognitive tasks (Van Lier et al., 2021). However, this mechanism is adjusted to the hunter-gatherer life, where the interactions were face-to-face and in a group of around 150 people, many of whom were also close kin. The entire mechanism has evolved for interactions with very small groups instead of large groups of anonymous people, and with strong and recurring face-to-face interactions instead of interactions with artificial agents that are becoming increasingly common. Here we have an example of a deep evolutionary mismatch. With the increasing population of apps and artificial agents in the coming years, this particular evolutionary mismatch will become even greater.

What the students will face more and more is a radically new type of agent. These agents are not only artificial but increasingly intelligent. They learn at increasingly higher speeds and do it constantly, 24/7. For this new world, the evolved strategies for detecting cheaters, and the mechanisms for building relationships of mutual trust, will be increasingly maladapted and thus a huge evolutionary mismatch will explode.

HOW DO WE ESCAPE THE TRAP AND LEARN WHAT TO TRUST?

One might think that with so many super apps and super-smart artificial agents, citizens would lose any possibility of autonomy or free will. Artificial agents would know before us our preferences, our limitations, and would predict with great accuracy what we are going to do. This does not need to be a pessimistic and dystopian prediction. It could be quite the opposite. With the artificial agents´ arms race that will be triggered, we have the possibility of managing ourselves and continuing to improve our well-being. It will be one giant, constant poker game, but with a proper education, we can avoid being read and manipulated. The critical point is that each app has limitations when predicting the behaviour of others. Similar to humans, they also have bounded rationality. Predicting the behaviour of people and artificial agents is more complex than the classic unsolvable physics problem of predicting the trajectory of n-bodies. Unlike the physics problem in which the interaction is very simple and only depends on the distance between the objects, in this case there are n minds that try to avoid being predicted by others. It is a multi-player pursuit and avoidance game. The super apps arms race will save the citizen as long as we understand the underlying models that rule the artificial agents’ behaviour and can establish a dialogue with trusted apps. Endowed with the appropriate education and a council of trustworthy artificial agents, citizens can defend themselves against an invisible omnipresence of a super panopticon of millions of apps. It is not a zero-sum game. We can all win.

Ethnographic studies in populations that still live as hunter-gatherers today have found that children learn mainly from peers and their parents. They learn by observing, playing, and imitating the ideas and practices of others. Adolescents and adults learn more complex skills from non-direct relatives through observation and occasional instruction (Lew Levy et al., 2017). Learning what to eat by trial and error is very dangerous. It is very easy to get sick or die. It is very costly and time-consuming to explore and rediscover in isolation something that our ancestors have developed through centuries of collaborative work. However, observation and imitation are no longer enough in today's modern world. Knowledge has increased substantially. In the space of one or two decades, students need to assimilate concepts and forms of mathematical thinking that are the product of millennia of development and strong social interaction. More and more new concepts and models appear that allow a better and deeper understanding of the world and ourselves. These facts and models are used in an increasing number of apps and artificial agents.

Key skills include mathematical and computational modelling. Today it is common to think in terms of computational models such as those that predict the evolution of pandemics, climate change, or multiple social phenomena such as immigration and segregation. One example of this are computational models that help to understand school segregation within school systems. Such models reveal great inequality in terms of academic performance between schools, as well as school systems that are hugely segregated by socioeconomic levels (Araya & Gormaz, 2012). Other examples from education are models that explain the segregation that occurs within each classroom (Akerlof, 2017). More and more apps are based on agent-based models as well as massive amounts of data. But these models are built by experts with simplifications of reality. Therefore, it is very important to understand those simplifications and the underlying assumptions. Additionally, the massive data used can be heavily biased. For example, many of the results of decades of investigations of psychological phenomena are based on studies and experiments conducted on students from western universities. They are what are known as WEIRD people: Western Educated Industrialized Rich and Democratic (Henrich, 2020). However, repetitions of some of these experiments in other populations produce different results. Our students should therefore be prepared to know and understand these simplifications and biases.

The great challenge of trust in multiple artificial agents and apps can only be solved if citizens understand the basic underlying models on which these apps are built. For this reason, it becomes increasingly important for students to know the main mathematical and computational models not only of the physical sciences, but also of environmental, biological, economic, social, and psychological phenomena.

HOW CAN WE LEARN ABOUT A CORE SET OF MODELS AND MODELING?

In addition to knowledge of a core set of models, one might think that the development of general skills, such as modelling, would suffice. Although modelling is innate, it is however not always a conscious and rigorous thinking process. It is spontaneous and unconscious in important and recurring situations for our hunter-gatherer ancestors. For example, modelling a territory and thinking about how to get to a certain area, how to secure water, how to collect food and how to plan to kill an animal. It is an effortless, non-conscious process. However, the conscious use of models, especially more complex models, requires training and years of targeted practice. We are well equipped with thinking tools, both abstract and concrete (Dennett, 2017). Unlike other animals, this arsenal is the product of cultural evolution. There is, however, a somewhat naïve perception (not supported by empirical studies) that thinking skills, particularly critical thinking skills, are content independent. This naïve idea of independence and transfer of thinking skills from one area to another is called the Mozart effect: the idea that that listening to the music of Mozart or learning to play a musical instrument or chess or learning something difficult like Latin will transfer to other subjects (Willingham, 2015). Nobel laureate Herbert Simon illustrated this misconception with studies of expert chess players who did not compete favourably with non-experts in memorizing spatial positions when pieces were placed on the board at random (Gobet et al., 1998).

Moreover, mathematical thinking should be shown with concrete content examples. It should be denotative knowledge (versus connotative knowledge) (Katagiri, 2004; Isoda & Katagiri, 2012). Even when one understands mathematical phrases that express meaning, that does not mean that one is capable of doing it mathematically. It is necessary to teach mathematical thinking with concrete examples. Otherwise, it is just “inert knowledge” (Whitehead, 1929). Moreover, just demonstrating the critical thinking process is not enough. “Students need practice engaging in the critical thinking process themselves, and this practice should be deliberate and repeated with targeted feedback” (Holmes et al., 2015, p. 11199).

A hugely popular idea among educators is that, when faced with a problem, a student can start modelling on their own. They fail to consider that our civilization has developed a set of key models over a number of millennia. These models are not intuitive, are very powerful thinking tools in a great variety of situations and require years of training in order to be mastered. Only once a model has been used and mastered can students start to select alternative models, adjust the parameters of the model, and eventually build models or parts of models. It is this trajectory that led to the modelling teaching strategy that we call Use Select Adapt Build (USAB) (Araya, 2012a). According to the mathematician Henry Pollak,

> in teaching modelling … you have to take some models that have been created and have been known to be successful and students have to study those models, and understand what makes them work, and think about what went into their creation and the way they were formulated and their success. (Pollak, 2007 p. 114).

It is particularly important to start learning and understanding a “core set of models” (Page, 2018) that can be viewed as a multiplicity of lenses. “Mastery of models improves your ability to reason, explain, design, communicate, act, predict and explore” (Page, 2018 p. 1).

Accessing the main models and methods that apps use is one way of being able to critically evaluate their recommendations and decisions. Consider the case of impulsive buying, or impulsive consumption of food delicacies, or any impulsive decision. It is known that we do not analyse options with exponential discounts as bankers do. Humans and animals use hyperbolic discounts that make it very difficult for us to avoid a pizza and stick to our diet, or impulsively buy a nice car, or spend all our income and fail to save for our pension. Something of little value but close in time seems of more value than something more distant in time but of greater value. This is very similar to when you are walking towards houses and tall buildings. In the distance, the 10-story building looks taller than the 1 floor house. However, if the 1 floor house is first on the road, then when approaching it an inversion eventually occurs: The house looks taller and can cover the 10-story building. There is an inconsistency in heights as you approach them since the relative perceived heights change. This way of valuing and deciding was advantageous in many of the recurring situations of our ancestors but is not a well-adapted preference mechanism for modern life. It is a mental “bug”. But apps can easily hack this decision-making "bug". If our students know and understand the hyperbolic discounting model and the strategies “to outsmart the future selves that will have these preferences” (Ainslie, 2001 p.27), then they will be able to understand the actions of the apps. Students will thus have the ability to detect which artificial agents are using this “bug” to achieve their interests and not those of the user.

Another important model is Machine Learning (ML) applied to learning different tasks, ranging from medical or industrial diagnosis, and credit scoring, to voice or image recognition. In experiments with elementary school students (Araya 2007; Araya et al., 2011; Araya et al., 2014) applying ML and in other experiments with the underlying algorithm of steepest descent (Araya, 2021a), and with computational thinking (Araya Isoda et al., 2020; Araya, 2021b), we have found that students learn and enjoy these activities, detect patterns, and build models from labelled data. It also helps them understand potential biases in the learned models.

HOW ABOUT CREATIVITY WHEN BUILDING MODELS?

In addition to the use, selection, and adjustment of models, we need creative students who are capable of building new models and discovering new methods or solutions. According to the mathematician Jacques Hadamard, to come up with new ideas, he first required intense and persistent work trying to advance the problem. This is the preparation stage (Hadamard, 1945). Then comes a stage of incubation, where much is developed unconsciously in sleep or other activities until ideas for solutions eventually begin to appear. According to Poincaré, there is a recombination of ideas: “Ideas rose in crowds; I felt them collide until pairs interlocked, so to speak, making a stable combination” (Hadamard, 1945, p. 14). Or, as Ridley says, ideas have sex with each other (Schermer, 2010). Then comes the selection of the right combination. According to Hadamard “to invent is to choose” Hadamard, 1945, p. 30. A similar dynamic occurs in K12 students. Studies of discovery in elementary school children find that after an intense period of practicing with standard strategies, comes an unconscious process of mutations and combination of the original ideas, and then the selection of appropriate ones (Siegler & Stern, 1998). This gives rise to unconscious ideas that emerge later on in consciousness. Computational models of the mutation-recombination-selection algorithm show that this process is able to accurately simulate the discovery behaviour (Siegler & Araya, 2005). The crucial thing is to work intensively on providing deliberate practice to facilitate the emergence of new ideas. Additionally, analogies, metaphors, and enactive (bodily acted out) metaphorizing can also foster discovery (Araya, et. al, 2010; Soto-Andrade, 2018), as Einstein responded to Hadamard.

> … from a psychological viewpoint, this combinatory play seems to be essential feature in productive thought,…, the above mentioned elements are, in my case, of visual and some of muscular type (Hadamard, 1945, p. 142).

In addition to understanding a core set of models and working with them intensively, students also need to cultivate a core set of thinking attitudes. These are intentional states of mind for searching, attempting to make connections, random exploring, looking for different alternatives, working to establish a perspective, striving for counterexamples, venturing new views, seeking clarity and better ideas, and trying one’s hand at different models (Isoda & Katagiri, 2012). To achieve this attitudinal state, it is important to understand and master different tools. In this sense, mastering a core set of models is particularly important. Students should therefore be able to understand the model’s underlying ideas and metaphors, and refine the models based on new facts, data, predictions, successes, and failures.

HOW CAN WE PREPARE TO TEACH THESE THINKING SKILLS?

The quantity of new skills, models, and attitudes that we need to teach students presents a significant challenge. We therefore need an increasingly precise pedagogy that in a short time can allow deep learning of increasingly complex ideas. Thanks to a well-planned curriculum and precise instruction, humans have managed to partially escape some of the evolutionary traps. That, however, takes years of directed instruction and deliberate practice. Adapting teaching practices and having teachers assimilate the new content knowledge for this set of core models is a huge educational challenge. One potential strategy for developing teachers' skills is to establish a community with a powerful attitude of sharing, innovation, and collaborative learning.

One such strategy that is well-tested is the centuries-old Japanese methodology of Lesson Study and massive demonstrations in Open Public Lessons (Isoda, 2015). It began in 1880 in Japan with the aim of reproducing best practices in teaching (Isoda, 2015). It has been adopted in Thailand (Inprasitha, 2015a), and Singapore (Yeap et al., 2015). Many other countries have also started to introduce it (Quaresma et al., 2019; Estrella et al., 2018). It is a collaborative teaching improvement process designed to generate strong and productive communities of teachers sharing and learning from each other. Moreover, "when teachers work together …. they build pedagogical capital – a scare resource because isolation is endemic to age-graded schools" (Cuban, 2013, p. 181). Lesson Study accelerates the production of effective lessons, and in particular, it helps the development of the open-ended approach, with lessons that aim to develop higher order thinking skills (Inprasitha, 2015b).

An example of a multinational and interdisciplinary collaboration is that of the APEC community led by Masami Isoda from Japan and Maitree Inprasitha from Thailand. For more than a decade, they have brought together researchers and teachers from 21 APEC countries. Every year, the team goes through a five-stage process. First, the team defines a specific, innovative and challenging multidisciplinary content to teach. Second, in each country, the respective teams design and implement a lesson in local classrooms. One year it was tsunamis, next year it was floods, and in another year, it was forest fires. In each case, mathematical and computational modelling is promoted. Teams are also encouraged to help students achieve a deep understanding and a proactive and perseverant attitude in seeking alternatives and different perspectives, devising experiments, and making predictions, conjecturing patterns and explanations, pursuing collaborative work in teams and being open to new strategies. Other contents that have been addressed in recent years include computational thinking (including machine learning and computational modelling) and statistical thinking for big data. During the third stage, six months later, the group meets to share and analyse how the members have implemented the content in real classrooms in their respective countries (Isoda et al., 2017; Araya

Collanqui, 2021; Wiemken et al., 2021). Fourth, in their respective countries, members make adjustments to their classes. In the fifth and final stage, again six months later, some of the members hold open public lessons with students from the host country. A panel analyses them carefully, and the APEC team generates final recommendations for their implementation.

PREPARING TEACHERS FOR EDUCATING FOR THE SUPER SMART SOCIETY

We need to teach completely new ideas within an already crowded curriculum. Ultimately, the change needed in contents and teaching will be enormous. This is not an easy task. Consider, for example, the project to switch to a more active teaching approach at a couple of universities in the US and Canada led by Nobel Laureate Carl Weiman (Weiman, 2017). The effort cost about $65 per credit hour. That is the equivalent of $60,000 per course. Yet only 10% of math university professors changed their teaching practices.

As we have seen, one option is Lesson Study. However, it has some constraints (Yeap et al., 2015). One constraint is finding the time to meet face-to-face to observe lessons from the other members of the group, carefully analyse the lessons and look for ways to improve them. Another constraint is that it only works with small groups of teachers.

Although open public lessons are developed with hundreds or thousands of teachers actively watching the lessons, they only take place a couple of times a year. What we need to build is a much larger and more connected community. If the community is very small, it may experience the “Tasmanian effect” (Henrich, 2016). This is a regression that occurred in Tasmania. As the size of the community was reduced 12,000 years ago as a result of rising sea levels that cut them off from mainland Australia, the aborigines started to lose their know-how for making tools, weapons, clothing, and boats, eventually regressing to more primitive cultural tools. The size of the interconnected group impacts, for example, on the size of the vocabulary. American teenagers know around 17,000 words, whereas small-scale societies have a vocabulary of between 3,000 to 5,000 words (Henrich, 2016). What is needed is a truly collective brain that draws on a large number of teachers. For example, we have found that a territorial ecosystem cultivated by 640 teachers helped them to adapt teaching strategies for quarantined first and second graders during the COVID19 pandemic (Araya, 2021c). Ninety-eight teachers shared their strategies by uploading a couple of 3-minute videos every week. From these videos we found an evolution of the strategies in five successive generations, with evidence of imitation and improvement of the strategies shown in previous videos. In another experiment, we also found that an online parent ecosystem used by thousands of parents helped them to spread strategies to support first graders learning at home (Araya, 2021d).

Given the need to quickly introduce a core set of models and associated skills, we additionally need to increase the speed of diffusion and adoption. One possibility is artificial agents: a community of class observation apps that give immediate feedback to the teacher. It would therefore not be necessary for the teacher to meet with others to observe lessons. Artificial agents will observe, analyse and give feedback to the teacher. We and others have already started developing apps to observe lessons, detect teaching patterns from acoustic features of teaching (Schlotterbek et al., 2021), analyse teacher and student discourse obtained through automatic transcriptions (Araya, 2012b; Uribe et al., 2020; Lämsä et al., 2021; Altamirano et al., 2020), analyse automatic multimodal analysis combining acoustic and textual features of classroom speech (Schlotterberg et al., in press), detect patterns of non-verbal behaviour like gestures (Araya et al., 2015; Araya et al., 2016; Hernandez Correa et al., 2020), gazes and body orientation in the classroom (Araya Farsani, 2020), and estimate the impact of discourse on student achievement (Schlotterbeck et al., 2020). Soon, these types of apps will analyse the lessons and give teachers real-time, on-task feedback. They will offer both pedagogical and content knowledge support. These artificial agents that will accompany the teacher will also connect in real-time with the artificial agents of other teachers to exchange strategies, adapt them, recombine them, create new ones and discuss them with their teachers.

Welcome to mega learning communities with teachers supported by a round table of artificial agents, connected with other teachers and their artificial agents: A Super Smart Lesson Study community.

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PLENARY LECTURES

THE 4TH INDUSTRIAL REVOLUTION IS DANGEROUS MARKETING HYPE: HOW EDUCATORS, WORKING WITH ALL STAKEHOLDERS, CAN TRANSFORM THE TEACHING AND LEARNING OF MATHEMATICS

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The so-called 4th Industrial Revolution *promises to enable people not only to consume more at a lower price and to live longer but also it is to transform education in general and the teaching and learning of mathematics in particular. The dangers of this so-called 'revolution' are already evident in terms of cyber-attacks, fake news, threats to data security, and the detrimental effects of social media use on mental health and well-being. What is being called* surveillance capitalism *is emerging where automated machine processes feed on every aspect of every human's experience not only to know everyone's behaviour but also aiming to shape that behaviour at scale. The so-called 4th Industrial Revolution is not solely marketing hype, it is dangerous. It is educators, working with all stakeholders, who can transform mathematics education using the variety of tools and approaches that respect and preserve humanity and the planet.*

INTRODUCTION

The notion of the 4th Industrial Revolution (also known as Industry 4.0) may first have been used some decades ago, but it came to current prominence through Schwab (2015), the founder and executive chair of the *World Economic Forum* (WEF) that famously meets each year in Davos. In his subsequent book, Schwab (2016, p. 7) asserts that "Of the many diverse and fascinating challenges we face today, the most intense and important is how to understand and shape the new technology revolution, which entails nothing less than a transformation of humankind". This "revolution", Schwab maintains, "is fundamentally changing the way we live, work, and relate to one another" that in its "scale, scope and complexity" is "unlike anything humankind has experienced before" (ibid). A "first and vital step" in this "4th industrial revolution", Schwab concludes, is to "continue to raise awareness and drive understanding [of this] across all sectors of society" (p. 103). Viewed this way,

2021. In Inprasitha, M., Changsri, N., Boonsena (Eds.). *Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education,* Vol. 1, pp. 67-75. Khon Kaen, Thailand: PME.

this so-called “4th Industrial Revolution” is all-pervasive and critique foolhardy and futile.

While the subsequent literature on this so-called “4th Industrial Revolution” draws on some academic research, the publications to date are, for the most part, “dominated by consultancies, think tanks and modellers, mainly drawn from economics or working with economists” (Morgan, 2019, p. 373). Those particularly prominent in pushing the notion of the so-called 4th Industrial Revolution, in addition to WEF, are *Deloitte*, *PriceWaterhouseCoopers*, and the *Global Institute* arm of *McKinsey*. What these entities have in common, Morgan argues, is that they are revenue-earning and, by their very nature, intent on capturing attention. In order to do so, they are attracted to grand themes; the 4th industrial revolution, says Morgan, being “the latest of such themes” (ibid p. 282). Governments, meanwhile, draw on a theme like this to inform policy.

In this paper we argue that Schwab’s drive, in collusion with the likes of *Deloitte*, *PriceWaterhouseCoopers* and *McKinsey* to “raise awareness and drive understanding across all sectors of society” (to quote Schwab) of this so-called “4th Industrial Revolution” is primarily marketing hype underpinned, as Morgan (2019) argues, by a thinly disguised social Darwinism in which the people and entities who survive the coming “4th revolution” are, by definition, those who have been the fittest all along. It is designed to get people to quietly (and meekly) accede that their data belong to multi-national corporations and agree that their jobs can be replaced by robots.

While, as Jones (2020) argues, innovations in digital technologies have the potential to impact positively on the tackling of major world-wide challenges (pandemics, climate change, provision of sufficient food and water), the increasingly widespread uses of digital technologies raise significant social and ethical dilemmas. What Zuboff (2019, chapter 1, section III) is calling *surveillance capitalism* is leading to “automated machine processes” that “not only know our behaviour” but can also “shape our behaviour at scale”. Our argument, in this paper, is that the claim that the so-called “4th Industrial Revolution” can ‘transform’ mathematics education is not only marketing hype but it is also dangerous: rather, it is educators, working with all stakeholders, who can transform the teaching and learning of mathematics using the variety of tools and approaches that respect and preserve humanity and the planet.

THE PROMISE OF THE SO-CALLED 4TH INDUSTRIAL REVOLUTION

According to Schwab (2016, p. 12), the “4th industrial revolution” is “not only about smart and connected machines and systems”; rather, its scope is much wider. From this perspective, this so-called ‘revolution’ embraces not only digital applications (digital platforms, augmented reality, remote sensors, 3D printing, artificial intelligence, the Internet of things, quantum computing), but also physical manifestations (autonomous vehicles, 3D printing, robotics) and biological innovations (gene editing, neuro-technologies). This ‘fusion’ of technologies is being lauded by Schwab (2016, section 3.1) as promising to enable people “to consume more at a lower price”, to “live longer, healthier and more active lives”, through “increased economic growth” that ensures “there will always be work for everybody”. Schwab does, of course, cleverly acknowledge potential downsides but his stance is unrelentingly upbeat: “I feel strongly that we are only just beginning to feel the positive impact on the world that the fourth industrial revolution can have” (ibid, p. 35-36), he concludes. The impact of this so-called “4th Industrial Revolution” on Education, according to Schwab, is the imperative to “develop education models to work with, and alongside, increasingly capable, connected and intelligent machines” (p. 43). What could possibly go wrong?

THE DANGERS OF THE SO-CALLED 4TH INDUSTRIAL REVOLUTION

Jones (2020) lists some of the dangers already evident in the use of “4th industrial revolution” digital technologies across many fields, such as “data security, cyber-attacks, fake news, long-term exposure to wireless devices and digital screens, and the effect of social media use on mental health and well-being”.

The latest technologies are framed as being set to ‘revolutionise’ education in general, and mathematics education in particular. This is the stuff of marketing campaigns and intense selling of technological ‘solutions’; themselves versions of ‘fake news’. The evidence of the impact of the use of digital technologies on education is, in contrast, rather more nuanced (Falck, Mang, & Woessmann, 2018). For example, PISA data suggests that “limited use of computers at school may be better than no use at all, but levels of computer use above the current OECD average are associated with significantly poorer results” (OECD, 2015, p. 146); see Figure 1.

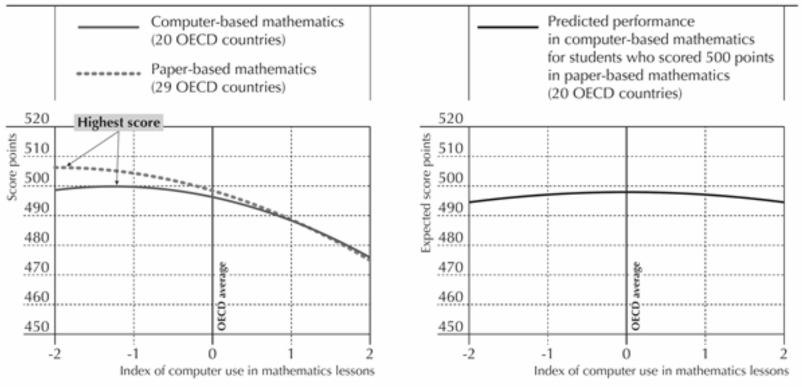

Figure 1: Computer use and performance in mathematics (source: OECD, 2015)

Jones (2020) notes how, in parallel to the development of digital curriculum resources, another form of technology-enabled development is ‘Adaptive Tutoring’ using so-called Intelligent Tutoring Systems (ITS), an application of Artificial Intelligence (AI) in Education. Existing implementations have primarily targeted procedural tasks, with those claiming to feature problem-solving tasks being heavily structured. Even so, research is continuing on expanding “the range of intelligent support for students working in exploratory learning environments by tracking, analysing and responding to their actions and choices” (Davies et al., 2013, p. 32).

It remains unclear, argues Jones (2020), how, or whether, the development of digital ‘assistants’ and ‘tutors’ may eventually lead to pedagogic ‘robots’ that could, at some point, supplant human teachers. Current work on such innovations utilises not only ITS/AI but also educational data mining (EDM) techniques to ‘track’ the work of each student. A wider critical debate on the impact of digital automation on Education is, stresses Selwyn (2019), long overdue. Indeed, Selwyn’s concern is that digital automation should be provoking *much greater* consternation and debate in Education.

At the same time that digital technologies are becoming increasingly prevalent, Jones (2020) summarises how inequalities in access to, and knowledge of how to use, digital technologies may be leading to the exacerbation of existing inequalities or the creation of new inequalities. The coronavirus pandemic during 2020-21 only served to underline that while digital technologies are undoubtedly useful during widespread school closures, Governments worldwide became desperate to re-open schools to enable education to take place. Unequal, or skewed, patterns of home and school access to digital technologies spur the creation of the ‘digitally privileged’ and the ‘digitally disadvantaged’. For the latter, it can be a ‘double-bind’ in that the digitally disadvantaged may lack not only access to digital technologies, but this lack of access also worsens a lack of knowledge of how to use the latest digital technologies. Echoing such concerns, Unwin (2017) provides a savage indictment of the use made of digital technologies, noting that, over recent decades, rather than reducing poverty the use of such technologies has increased inequality and had a negative impact on the development of poorer and increasingly marginalised peoples and countries.

Enveloping all such concerns is the growing emergence of what Zuboff (2019) calls *surveillance capitalism*. Recently, Google (in the form of its holding company *Alphabet*), became the fourth US company to achieve a market valuation of over one trillion dollars, following, as might be guessed, *Microsoft*, *Apple*, *Amazon*. In the wings are other corporations, with Goodwin (2015) observing that “*Uber*, the world’s largest taxi company, owns no vehicles. *Facebook*, the most popular media owner, creates no content. *Alibaba*, the most valuable retailer, has no inventory. *Airbnb*, the largest accommodation provider, owns no real estate”. What all of these companies have, and seek to exploit, is masses of data provided, usually for free, by users of their services. During the COVID pandemic of 2020-21, the major tech companies profited.

When Google, a search company, invests in smart-home devices, wearables, and self-driving cars, and Facebook, a social network, develops drones and augmented reality, such diversification can seem strange. According to Zuboff (2019), such varied activities across a seemingly random selection of industries and projects are *all the same activity guided by the same aim*; the capture of what Zuboff calls *behavioural surplus* (chapter 1, part III). The data these companies harvest feeds “machine intelligence” manufacturing processes that fabricate what Zuboff calls *prediction products* that anticipate “*what you will do now, soon, and later*” (emphasis added). Not only that, but, says Zuboff (op cit), “the most-predictive behavioural data come from intervening in the state of play in order to nudge, coax, tune, and herd behaviour toward profitable outcomes”. With the so-called “4th industrial revolution”, the danger is that learners, and teachers, become subject to “automated machine processes” that herd behaviour in these ways.

TRANSFORMING THE TEACHING AND LEARNING OF MATHEMATICS

Amidst rapid technological development in the so-called 4th industrial revolution, this is a time of major destruction of the planet’s ecosystem in an increasingly complex and uncertain environment. Addressing the *global emergency* in 2019, the United Nations (UN) warns that only eleven years remains to stop irreversible damage from climate change (UN, 2019). This is, of course, partly contributed by the millions of tons of carbon emissions produced per year for electricity generation, and the impact on the environment caused by growing electronic waste disposal and increasing rates of deforestation to feed the 4th industrial revolution. Davis, Sumara and Luce-Kapler (2015) pose a timely question, “How might we describe a teaching that fits with the time and place we find ourselves?” (p. 6).

To begin, we critique that mathematics education, like education systems in general, have perpetuated an unsustainable industrial/modernist model of growth (Orr, 2004). Where the so-called 4th industrial revolution argues that a range of emergent technologies can erase boundaries of the physical, digital and biological worlds, and even challenge ideas about what it means to be human (Schwab, 2016), the focus has been to “shape a future that works for all by putting people first, […] that all of these new technologies are first and foremost tools made by people for people” (p. 114). With much emphasis put on the benefits of “people” (human), we note that a significant missing piece of this argument is issues of sustainability: for living systems to coexist over time. In his essay, Renert (2011) points out that, by and large, ecology has played only a negligible role in mathematics curricula. He explains that mathematical reasoning can bring important perspectives about the environment and responses (i.e. accommodating, reforming, and transforming) to issues of sustainability. In a transforming response, "the process of sustainable development is essentially one of learning, while the context of learning is essentially that of sustainability […] Stakeholders make connections between multiple layers of purpose that include: physical, economic, environmental, emotional, social, and spiritual" (p. 22). Likewise, we suggest that a sustainable mathematics education is much more than learning about numbers and shapes but noticing the world differently through mathematics to make responsible decisions with the goal of improving the wellbeing of not only human communities but also the environment and the eco-system at large.

In view of this, an interdisciplinary approach is called upon where K-12 science, technology, engineering, and mathematics (STEM) education is about engaging students with different issues of local and global relevance through focusing on reasoning beyond understanding STEM content and practices. English (2016) notes that, currently, most STEM curricula have taken a disciplinary approach in which knowledge and skills are taught separately, and this can constrain the way they are applied to solve genuine problems and projects. Furthermore, STEM is framed problematically as primarily leveraging STEM content understanding as a means for solving complex issues. Such efforts of promoting STEM concepts underpinning environmental issues has shown to be insufficient in mitigating and resolving them (Wals, Brody, Dillon, & Stevenson, 2014). An alternative approach is to consider that many issues and problems cannot be resolved by the STEM disciplines alone. This framing is needed for solving problems of sustainability, and of real-life phenomenon in general, which are not known *a priori* or sometimes cannot be solved at all (Renert, 2011). Recognising that problems of real-life phenomena are open-ended, ill-structured and lack simple solutions precisely reflects problem solving as a global, 21st century competence (OECD, 2018). Here, we are reminded that technology alone changes nothing in *how* problem solving will be learned; it is educators who can design learning conditions for problem solving that draw upon mathematical thinking and technological literacy to meet complex demands in an ever-changing world.

To illustrate, one might ponder on the question: will emergent technologies such as 3D printing transform mathematics education? We want to clarify that it is not the technology itself that transforms teaching and learning, but the pedagogies that can realise the transformative potential of emergent technologies. In her Paper-inspired line of research on 3D printing, 3D computer-aided designs, and physical programming (Figure 2), Ng conceptualises *learning as making*, a form of constructionist learning that renders learners as innovators and producers of knowledge as opposed to consumers of meanings as determined by others (see, e.g. Ng, Sinclair, & Davis, 2018). Making situates students as meaningfully, and openly, constructing artefacts in technology-enhanced ways, through which mathematical thinking emerges in the making activity. *Learning as making* therefore supports innovation-oriented learning and flexible problem solving as opposed to finding the correct answer per se. This conceptualisation responds to Renert (2011) who observed, “most mathematical problem solving in today’s classrooms relies on the unchallenged assumptions that each problem has one correct answer, and that the teacher knows this answer. Students’ creativity is therefore limited to replicating solutions that are already known” (p. 223).

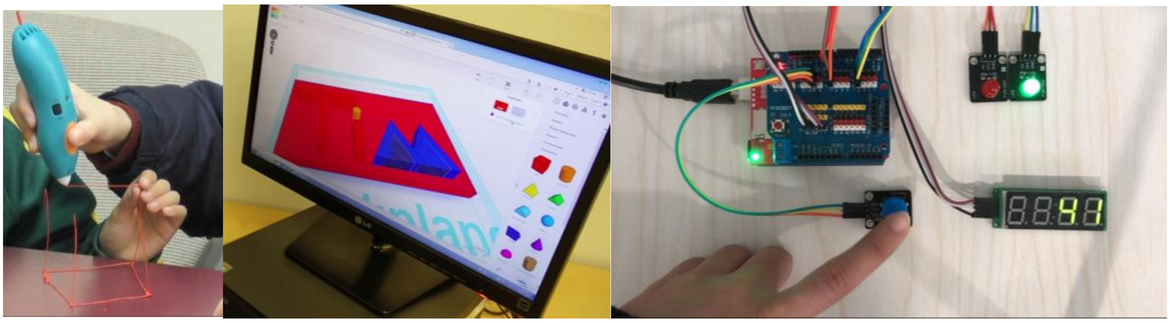

Figure 2: Constructionist learning: with 3D pens (left), 3D computer-aided design (middle), and programmable electronics (right)

While efficiency is an essential feature of all technologies that succeed in transforming classrooms, and no doubt the introduction of smart and AI technology can make more efficient classroom operations, another key characteristic of effective technology integration is the innovation in achieving meaningful enrichment to learning. Indeed, Blikstein (2013) warns that the use of digital fabrication risks offering substantial physical rewards for a relatively small effort and low learning enhancement, which he terms ‘the Keychain Syndrome’. Hence, though the potential transformations to do mathematics as part of a 3D reality is promising, it should be emphasised that 3D printing itself does not transform teaching and learning. Whereas Ng et al. (2018) speculate that 3D printing may contribute to a paradigm shift that challenges a long-lasting tradition of teaching and learning in 2D modes (i.e., paper-and-pencil and the computer screen) of curricular topics that render 3D concepts into 2D representations, it remains to be the educators and stakeholders, through their pedagogies and curricula implementations, who can transform *what* and *how* mathematics is taught and learned.

CONCLUSION

It is not the so-called “4th Industrial Revolution” that can ‘transform’ the teaching and learning of mathematics. Technologies do not change things; change is driven by human qualities and frailties. The so-called “4th Industrial Revolution” is not even a ‘revolution’. According to Unwin (2019), it is “market expansion and a reduction of labour costs through the use of technology” that “serves the interests of élite politicians, academics and business leaders far more than it does the poor and marginalised" and is underpinned by "masculine domination of the digital technology sector" that continues "to reproduce itself in ways that reproduce … gender inequalities and oppression".

Unwin (2019) concludes "those shaping these technologies are not primarily interested in changing the basis of our society into a fairer and more equal way of living together, but rather they are interested in competing to ensure their dominance and wealth as far as possible into the future" such that "The 4th Industrial Revolution is in large part a conspiracy to shape the world ever more closely in the imagination of a small, rich, male and powerful élite". All this underlines that the so-called "4th Industrial Revolution" is marketing hype, dangerous marketing hype. It is educators, working with all stakeholders, who can transform the teaching and learning of mathematics using the variety of tools and approaches that respect and preserve humanity.

In his twin books, Harari (2014, 2016) agues decisively that "humans seem to be more irresponsible than ever" in that "We are consequently wreaking havoc on our fellow animals and on the surrounding ecosystem, seeking little more than our own comfort and amusement, yet never finding satisfaction" (Harari, 2014, afterword) and, moreover, that "Science is converging on an all-encompassing dogma, which says that organisms are algorithms, and life is data processing" (Harari, 2016, chapter 11).

The real issue, Harari shows, is not only 'What do we want to become?', but also 'What do we want to want?' (Harari 2014, Part 4). For Zuboff (2019), the issue is "Who knows? Who decides? Who decides who decides?" (p. 521). Ultimately, Zuboff (2019, Conclusion) suggests, people must be "ready to name danger and defeat it". This entails refusing the seeming inevitability of surveillance capitalism. For humanity to live in harmony with other living species and the surrounding ecosystem, and for planet Earth to survive, it is people who must know, decide, and decide who decides.

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CREATING CONSTRUCTIVE INTERFERENCE BETWEEN THE 4TH INDUSTRIAL REVOLUTION (+COVID 19) AND THE TEACHING AND LEARNING OF MATHEMATICS

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This paper presents the authors' argument in favor of the statement "The 4th Industrial Revolution will transform/disrupt the teaching and learning of mathematics", focus of the Plenary Panel at the Conference PME44.

THE 4TH INDUSTRIAL REVOLUTION AND THE TEACHING AND LEARNING OF MATHEMATICS

The 4th Industrial Revolution (4IR) refers to an industrial change that is based on a “virtual physical system” that can intelligently control objects through a high-speed network 5G. As stated by Klaus Schwab, Founder of the World Economic Forum, in the Encyclopedia Britannica online (2020): "The revolution heralds a series of social, political, cultural, and economic upheavals that will unfold over the 21st century. Building on the widespread availability of digital technologies that were the result of the Third Industrial, or Digital, Revolution, the Fourth Industrial Revolution will be driven largely by the convergence of digital, biological, and physical innovations." However, today, to discuss the impact of the 4th IR on the *teaching and learning of mathematics* (TLM) we have to consider the binomial "4IR+COVID 19". Indeed, the advent of COVID 19 is accelerating the development of high technology like Virtual Reality and Augmented Reality which makes our educational environments more connected dynamically through on-line tools and the internet. On the other hand, despite such high technology, schools shutdown and unsatisfactory online education could have (and in some cases has had) devastating effects on the TLM. To give a glimpse at what the "4IR+COVID 19" disruption has looked like at the beginning of the pandemic in Italy for the second author, we provide an excerpt from her personal diary from the end of March 2020.

"Because of the COVID 19 pandemic, on March 5, 2020, all schools in Italy have been closed to students till March 15th - the shutdown has been extended to April 3rd, and possibly it will be extended further [indeed it was...till September 2020]. Any form of assembly is prohibited and even leaving one's

2021. In Inprasitha, M., Changsri, N., Boonsena (Eds.). *Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education,* Vol. 1, pp. 76-84. Khon Kaen, Thailand: PME.

home unless strictly necessary is strongly discouraged. Teaching at all school levels in presence has instantly been suspended and transferred online (after a few weeks of "nothing"), with no shared methodology or support with the platforms. Like that, all of a sudden, the TLM has taken on monstrous forms in many cases: teachers upload piles of "exercises", some post hours of videos of themselves simplifying expressions or solving inequalities on a blackboard, others sign into their virtual classrooms, share the screen of a tablet, and start writing endless pages of equations without hardly taking the time to breathe. It is the death of the TLM. When the internet's bandwidth is enough to keep students and families "logged on" the challenge to try and "learn" any mathematics in these conditions - and in isolation from everyone except family members (when they are not positive) - is overwhelming. Many families "shut down", others cannot connect in the first place, others try keeping up by printing piles of worksheets and monitoring their children's work. It is a disaster. In many cases all the wealth of possibilities brought on by the 4IR is backfiring and leading to further frustration and isolation."

In this short paper we argue that there is no question whether "disruption" to the TLM has happened, but, depending on whether or not people have soft skills and willingness to keep on fighting and looking for ways to connect with and help as many students as possible, such a disruption can go in two opposite directions. We will talk about it through the metaphor of wave interference: *Destructive interference* between the 4IR+COVID 19 and the TLM leads to a descending spiral in which mathematics becomes completely obsolete and is perceived as useless in school and, more in general, in life; the opposite direction is represented by *constructive interference*.

Constructive interference is what we need to strive for, now more than ever! To do so, we must acknowledge that digital technology equipped with artificial intelligence (AI) is becoming more and more responsible for many human activities; that innovations at the intersection of biology and technology are likely to allow implants that enhance memory, among other things; that the notion of classroom in a single physical space with chairs, desks and blackboards is becoming only one of many possible (if so!) learning settings. So, we now need to develop different sets of skills like critical thinking, creative thinking, problem solving, as well as other soft skills such as communicating and team working. In order to create constructive interference between the 4th IR and the TLM, we argue that mathematics education should focus on and explore new and more effective ways to achieve goals, such as: appreciating mathematics as a tool for exploring nature and developing technology; communicating with machines; integrating mathematics learning environments inside and outside of the classroom (whatever *classroom* now means); appreciating statistics and ways of managing big data; promoting freedom and equity at the social level.

How can we go about doing this? Focusing specifically on the TLM, first of all we believe that constructive interference requires modernization of TLM for students to gain manpower to accelerate sustainable development of the 4IR, now that creative thinking is the most important, in contrast to the previous education model for training “good workers” with basic knowledge and skills that were necessary to manage mass production systems in the 1960s and 1970s (e.g., Robinson & Aronica, 2018). Such modernization must take into account a range of aspects, including: the overcoming of "encyclopedic knowledge" in favor of interdisciplinary connected learning; opportunities to personalize learning and make it more inclusive (not "the same" for all, but providing equal opportunities to all); shifting from the idea of a "curriculum to cover" towards interconnected "big ideas". The technological resources provided by the 4IR now make such a restructuring possible: it is up to us to make it happen. In the rest of this paper, we will focus on three aspects that, based on our own experiences as mathematics educators, we think should become central in the modernization process.

ASPECT 1: STATISTICAL THINKING AND INTERPRETING BIG DATA

Statistical thinking as an important tool for understanding uncertain phenomena involves procedures: To quantify data or information (describing), to identify a meaning of the quantity (making knowledge), to infer the whole from the partial (inferencing), to correctly make a decision based on inferencing (applying) (MOE, 2015). Statistical thinking in a digital society has a special meaning in that it provides ways to understand "big data", that is the huge amount of information with a wide variety of patterns and trends that is being generated constantly. The interest in big data is growing exponentially in today’s society because big data represents a paradigm shift in the ways that we understand and study our world. For example, Bakker and Wagner (2020) have stated: "The current situation asks for mathematical and statistical literacy of a large population, as the media use all kinds of representations and simulations to explain why the spread of the virus has to be slowed down, and why isolation is so important so as to keep numbers within the capacity of the health care system (“flatten the curve”)" (p. 3). So, one direction is making sense of things that happen around us, but another is machine learning (also see Aspect 3 of this paper). Here we focus on the importance of *interpreting big data to be able to design dynamic interactive learning environments* with intelligent support.

Advances in AI technology are enabling analyses of such data in ways that were unthinkable in the past. Big data in our society is an important asset for understanding the uncertainty of modern society and for making rational decisions as well as exploring social and natural phenomena of the future. For example, companies are striving to conduct marketing to suit individual tastes by analyzing customer-generated payment information, purchasing records and interests, while government agencies are analyzing huge amounts of information and utilizing it in various social areas to improve the efficiency of budget execution and the quality of public services.

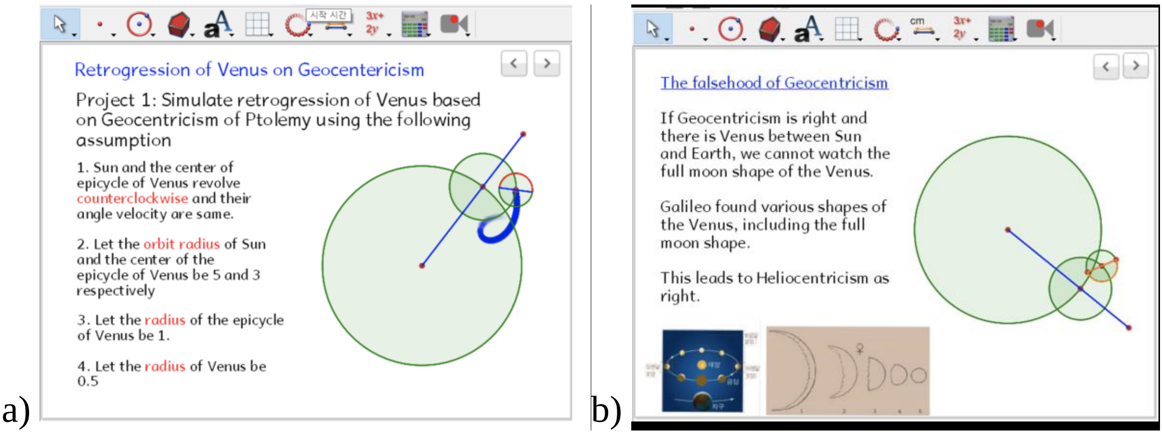
In the TLM we can achieve constructive interference between machines and humans by focusing on how to make rational decisions based on large scales of realistic data, delivered in real time through digital devices. Big data is characterized by 3+2 Vs: Volume, Variety, and Velocity, together with Veracity and Value of data. Veracity is a feature that describes how data can be reliable. We need to educate students to understand that data from social networks cannot be considered accurate because it does not always provide reliable evidence. For example, students can be led to explore ways to extract accurate data, "washing out" the inaccuracies and errors; and students can be invited to discuss the value of data. Through big data processing using high level ICT technology, it is now possible to collect and analyze data in real time and present analytical results in various forms. The ability to find trends hidden in big data and communicate using visual information so that the analyzed results can be easily understood is essential for students who will continue to lead the information age.

ASPECT 2: EXPRESSING IDEAS IN DIFFERENT "LANGUAGES"

Today many digital artifacts are available at the swipe of a finger, on many kinds of portable devices; moreover, we have experienced a convergence of physical and digital innovations, 3D printing and new block-based programming languages that can provide engaging and motivating contexts for learning. Interacting with these artifacts, students can be drawn into mathematical discourse, while simultaneously learning new languages that allow them to interact with machines. We see these forms of interaction as potentially beneficial for fostering constructive interference for at least two reasons: First, they can help students learn human-machine communication by providing immersive experiences; second, such interactions occur in different "languages" that, next to the formal language of mathematics, provide multiple means for students to communicate with each other and with the teacher in meaningful and inclusive ways.

Dynamic interactive textbook representations

Figures 1a,b show pages of a dynamic interactive textbook (Lew, 2016) developed using the Cabri applet, where “planet movement” is used in the "Circle” unit for the 9th grade in junior high school. Here students can interact with a model of the retrogression of Venus based on Geocentricism developed by Ptolemy in the 2nd century and they can use the situated context of the applet to reason about and discuss how Geocentricism was rejected by Galileo (Lew, 2016). Research has shown how through dynamic interactive applets, we can offer students the opportunity to manipulate different representations of functions, making conjectures about how functions describe *change*, and how this can be represented, talked about, and hence thought of within different semiotic registers of representation (e.g., Lew, 2016; Antonini, Baccaglini-Frank & Lisarelli, 2020). These dynamic and interactive representations of a function, in addition to being quite engaging and motivational for students, provide easy entries into situated forms of discourse that, with the help of the teacher, can evolve into more formal mathematical discourse (e.g., Antonini et al., 2020; Baccaglini-Frank, 2021).



a)

Retrogression of Venus on Geocentericism

Project 1: Simulate retrogression of Venus based on Geocentricism of Ptolemy using the following assumption

1. Sun and the center of epicycle of Venus revolve counterclockwise and their angle velocity are same.

2. Let the orbit radius of Sun and the center of the epicycle of Venus be 5 and 3 respectively

3. Let the radius of the epicycle of Venus be 1.

4. Let the radius of Venus be 0.5

b)

The falsehood of Geocentricism

If Geocentricism is right and there is Venus between Sun and Earth, we cannot watch the full moon shape of the Venus.

Galileo found various shapes of the Venus, including the full moon shape.

This leads to Heliocentricism as right.

Figures 1a,b: Simulation of the retrogression of Venus according to Geocentricism.

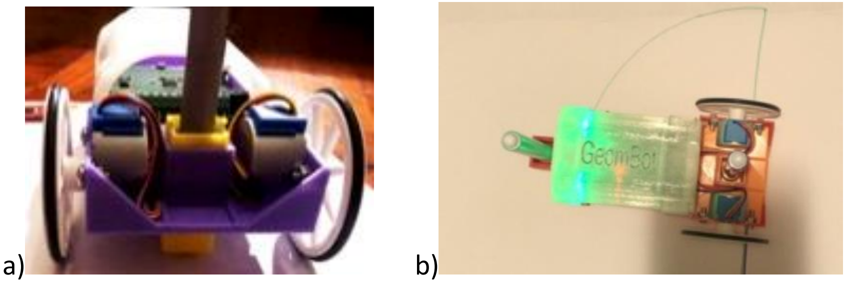
In general, research has suggested that dynamic interactive applets and textbooks have a positive impact on classroom practice, providing an environment for students to communicate in new, meaningful, and inclusive ways (e.g., Doorman, Drijvers, Gravemeijer, Boon & Reed, 2012; Lew, 2016; Lew, 2020).

Geometry through the language of the Geombot

While most of the activities with robots proposed in classrooms currently aim at introducing "programming" or "coding" per se - when they have any explicit educational aim at all - to foster constructive interference between the TLM and the 4IR, we build on a research tradition (initiated in the 1980s) that has focused on the integration of programming into mathematical learning (e.g., Papert, 1980). In this perspective coding becomes a language for expressing mathematical ideas, such as algorithms and equations, but also geometrical properties of a figure (for example, to be drawn out on paper by a robot), that can be learned from a very young age.

This has been a direction of research over the last few years of the second author, who designed a drawing robot, the GeomBot, for primary school students. The GeomBot combines the well-known strengths and opportunities offered by Papert’s original robotic drawing-turtle with those of the block-based programming language Scratch; and it builds on the convergence of physical and digital affordances. Most physical parts of the GeomBot are designed using

3D modelling software (SketchUp and OpenSCAD) and printed using a 3D printer. The GeomBot can hold a marker between its wheels that draws out its path as it moves on a sheet of paper on the floor, as well as a marker at the front, to highlight rotations (Figures 2a,b).



Figures 2*:* a) back view of the GeomBot; b) top view of the GeomBot.

To "talk to" the GeomBot the student needs to construct a sequence of command blocks designed in Snap!. The sequence is then transmitted to the robot wirelessly, once it is clicked. Although these blocks are virtual objects that "live" on a screen (touchscreen of an interactive white board, tablet, or computer screen), they are concrete enough to be accessible to and shared by all students as they engage in planning a drawing, and in programming and debugging a sequence based on the physical feedback given by the robot. A recent study has shown how it is possible for in-service teachers involved in professional development for 9-months with the GeomBot to significantly shift their perspectives towards the teaching and learning of geometry at many different levels, including understanding of mathematical contents, using different languages, inclusive mathematics education and overcoming a fear towards technology (Baccaglini-Frank, Santi, Del Zozzo & Frank, 2020).

ASPECT 3: UNDERSTANDING AI

Artificial Intelligence (AI) is a basis of highly sophisticated technologies driving the 4IR. AI can be looked as machine systems, or as a problem-solving strategy. As a machine system, AI aims at implementing a variety of recognition, thinking, and learning processes that before had been performed by only human intelligence. AI aims at modeling human intelligence. For example, an autonomous driving vehicle is a car equipped with AI which can recognize various environmental information and make a decision for driving safely. As a problem-solving strategy, AI can make a computer "think" efficiently. For example, machine learning and deep learning allow machines to analyze and process data and information that humans need. To pursue constructive interference, AI should gradually but systematically address in schools.

Indeed, education poses new challenges. UNESCO (2019) recommends governments to consider planning AI in education policies and to develop appropriate capacity-building programs for teachers. In Korea constructive interference is already taking place: curriculum for AI education and various high school AI textbooks are being developed and they will soon be offered through public education (MOE, 2020; Lee, Im, Jang, Song, Kong & Park, 2020; Youn, Kim, Nam, Choi, Jung & Kim, 2020). Furthermore, in 2019 the South Korean government decided to train a set of so-called “AI teachers” who are responsible for fostering new human resources to lead the new era of the 4IR. Each year for 5 years, 1,000 AI teachers will receive master's degrees through three years of in-service training, beginning in the second semester of 2020.

What is the most important area to be taught in mathematics education regarding AI for constructive interference to occur? Following Isoda (2021), school mathematics needs to foster computational thinking, since it is a critical cognitive skill that should be used efficiently to solve problems using computers and digital tools. Computational thinking should be included in the design of modernized mathematics curriculum in the digital society because it is powerful for students to formulate problems in a way that enables them to use a computer, to simulate the most efficient and effective solutions, to generalize the solutions to a wide variety of problems.

Specifically, students should learn and experience three core computational thinking processes which have been introduced as powerful tools to solve various problems in the history of AI (Araya et al., 2019): *algorithmic thinking*, *computational modeling*, and *machine learning*. Focus on these processes can help achieve constructive interference: their learning can foster meaningful problem-solving activity, while simultaneously provide an engaging application of mathematical knowledge.

CONCLUSION

Although mathematics has been at the heart of brilliant civilizations for centuries, today it is undervalued, and students often regard it as an irrelevant and meaningless subject. We believe mathematics to be one of the essential subjects in the twenty-first century and more central than ever in the midst of the 4th IR+COVID 19. We have highlighted aspects of the 4th IR upon which technological environments, tools and activities in mathematics education can be based to achieve constructive interference between the TLM and the 4th IR+COVID. Indeed, we believe it to be possible–and in some places this is already occurring– to take advantage of our societies' technological advances to design environments, tools and activities to help students appreciate mathematics as a tool for exploring nature and developing technology, communicating with machines, integrating mathematics learning environments inside and outside of the classroom, appreciating statistics and ways of managing big data. If the TLM can successfully embrace the kinds of modernization we propose, students will develop creativity and other key skills for their future, recognizing mathematics not as fragmentary knowledge, but as an interconnected body of knowledge through which to understand the world around them. However, such a modernization of the TLM is by no means automatic, nor does its application in some places imply its spreading worldwide. So, as a society, we must continue to strive together to propagate constructive interference.

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THE 4TH INDUSTRIAL REVOLUTION WILL TRANSFORM THE TEACHING AND LEARNING OF MATHEMATICS, OR WILL IT?

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In this year's PME debate, the panellists take up the question in the title relating to the potentials and the pitfalls of the 4th Industrial Revolution. Two panel members argue for the proposition and two argue against it. In this framing paper, I introduce the panel focus, and point to some of the contradictions that lead to '4IR' featuring in some writing as a kind of 'promised land' for education and for mathematics education, while appearing in other writing as, at worst, a sinister proposition that erodes what it means to be human, and – at best – a hollow cliché that offers no real proposals for change.

INTRODUCTION

In his recent fictional book 'Machines like me', Ian McEwan (2019) describes a world in which humanoid robots are becoming increasingly commonplace in the workforce and in domestic situations. McEwan paints a picture of a city in this context:

> 'Global temperatures rose. As the air in the cities became cleaner, the temperature rose faster. Everything was rising – hopes and despair, misery, boredom, and opportunity. There was more of everything. It was a time of plenty.' (p.113)

This quote captures something of the contradictions or oppositions that characterize writing in education and mathematics education relating to the likely impacts of the 4th Industrial Revolution. Klaus Schwab, the founder, and Executive Chair of the World Economic Forum is usually credited with popularizing the term '4IR' following the publication of his 2016 book with this title. 4IR is used as an umbrella term that describes the melding together of technologies across physical, biological, and digital worlds. Examples of such technologies are already all around us – 'smart' devices, 3-D printing, and genetic engineering to mention a few. While Schwab is a firm advocate of 4IR, Benioff, in the foreword of this book raises the spectre of 'technologies that could conceivably escape our control' and suggests the need for 'common values and clear purposes' (p. viii) to address the risks.

2021. In Inprasitha, M., Changsri, N., Boonsena (Eds.). *Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education,* Vol. 1, pp. 85-88. Khon Kaen, Thailand: PME.

This point, in turn, to a debate that is not centred around technologies as much as it is centred on our understandings of what we value in terms of the human condition and our aspirations for what we would like development to do for improving the human condition. Kazio Ishiguro (2021), in his recent book *Klara and the Sun,* pushes the envelope further on the fuzzy boundaries between the capabilities of the 'AFs' – the 'Artificial Friends' and humans. In this book, the narrator is the AF: Klara, a robot with highly developed observational skills, honed to offer service and empathy. In response to a human commenting to her that: *'It must be nice sometimes to have no feelings',* Klara notes at one point:

> 'I believe I have many feelings. The more I observe the more feelings become available to me.'

Klara's skillset rests on what appears to be a limitless capacity to use empirical observations for adaptive generalizations that guide her empathetic actions – the same kind of generalizations, perhaps, that actuaries use currently to offer probabilities and predictions to events and behaviours. Klara's aptitude is for the empirical generalizations of scientists rather than the deductive generalization of mathematicians, with human behaviours forming the empirical data points. Klara's 'thinking' is a long way off from the routine and procedural work that historically has been seen as the target of automation, and her skills broaden the alarm bells relating to the range of jobs that can fall, more efficiently, under the purview of the AFs that we might create.

The spectre being set up here is intimately connected to the forms of thinking that we are looking to 'farm out' or 'outsource' to the 4IR stable. Tim Wu, in a 2018 article in the New York Times, argues that development since the late 19th century has been driven by the goal of 'convenience' – by aspirations for 'saving time and eliminating drudgery […] making things easier.'

But Wu points to implications of aspirations driven by the need to have things easy, implications that are particularly pertinent for a debate on 4IR and mathematics teaching and learning:

> 'As task after task becomes easier, the growing expectation of convenience exerts a pressure on everything else to be easy or get left behind. We are spoiled by immediacy and become annoyed by tasks that remain at the old level of effort and time.'

4IR may well provide us with conveniences – aspects of our lives that become easier on the back of cognition distributed across human-machine-world infrastructures. But are we right to argue that 4IR offers the potential for engagement with more substantial mathematics that can leave more mechanical mathematics behind – safe in the hands of technologies that can deal with this latter mathematics much more effectively and efficiently than we can? If it is Ishiguro's world that we are heading towards, what, if anything, is going to be left that we might be better at than our Artificial Friends? Is it the case that in abdicating responsibility for the things that are time-consuming, and therefore inconvenient, for us to do, we are rendering ourselves less and less capable of engaging with things that are immediately difficult to deal with? A hallmark of rich mathematical problems is that they are not amenable to immediate solving by known or recalled mathematical procedures or results. They require prolonged engagement and even struggle. Their badge of virtue is that they are ‘inconvenient’ problems – problems that will take time to solve. Will the growing incorporation of 4IR in our lives render us less or more able to engage in the struggle that is an intrinsic part of rich mathematical problem-solving?

In a conference and in a world clasped tight in the grip of Covid-19, these questions are live and current. Human-machine interfaces have become our proxies for interpersonal interaction, with differential access to human-machine interfaces writ large in questions about differential access to mathematics.

Some of these deeper questions and concerns about what we might gain and what we run the risk of losing through stepping into the 4IR that are at the heart of the presentations within this panel debate. Papers arguing for and against the motion follow this framing paper. Hew-chan Lew and Anna Baccaglini-Frank (2021) argue in favour of the transformative potential of the 4IR, including the mathematical learning that becomes possible through the integration of physical-digital technologies. Keith Jones and Oi-Lam Ng (2021) argue against the proposition, pointing to how the potential of 4IR technologies rests in the hands of educators working with other stakeholders to decide on the values and aspirations that guide, and shape questions of what technologies are integrated, what they are used for, and why.

A small but important final point. The panellists in this debate are ‘taking on’ a position that may or may not reflect their own personal view on the transformative potential of 4IR. We hope that you will both enjoy and engage with the research-based perspectives they offer, and in the process, understand a little more clearly the position you hold on to the motion. Enjoy the debate! My thanks to all the panellists for their careful contributions. May the plenary panel papers provoke further thought and discussion about 4IR!

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PME44 VIRTUAL

THE 44th CONFERENCE OF THE INTERNATIONAL GROUP
FOR THE PSYCHOLOGY OF MATHEMATICS EDUCATION
July 19-22, 2021
Hosted by Khon Kaen, Thailand
Virtually hosted by Technion, Israel Institute of Technology

WORKING GROUP

SOCIAL AND ECOLOGICAL JUSTICE: FOR WHO AND HOW?

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To make sense of social justice's issues related to mathematics education, many researchers draw on social, cultural, and ecological perspectives of living in relationships (Hauge & Barwell, 2017; Nicole et al., 2019; Radakovic et al., 2018). The central purpose of these working sessions is to invite participants to explore and discuss issues of social justices as they relate to the teaching and learning of mathematics.

We all share a world that faces huge social and environmental challenges. As critical citizens and as educators we notice and are sensitive to our coexistence and the interrelationships within this world. Within the frame of social justice, we are morally and ethically responsible for our relationships (Abtahi et al, 2017; Boylan, 2016). The United Nations describes social justice as "an underlying principle for peaceful and prosperous coexistence within and among nations" (International Labor Organization Declaration). That being said, the meaning of peaceful and prosperous coexistence – and hence the meaning of social and ecological justice - varies context by context and place by place. In this working group, we will explore our ethical and relational responsibilities as mathematics education researchers and educators with respect to issues of social justice in mathematics education. In particular:

i. We explore issues of social justice in relation to mathematics education in relation to assumptions of universality and stability of mathematics and the teaching and learning of mathematics.
ii. We explore issues of relationality (to others, to the land, to the inhabitants of the earth) to be able to examine more closely the tension between local and global justice.
iii. We explore our ethical responsibilities to promote concrete action in response to social and ecological injustice.

The following questions will guide our exploration through various activities and discussions: how are social and ecological justice acknowledged,

2021. In Inprasitha, M., Changsri, N., Boonsena (Eds.). *Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education,* Vol. 1, pp. 90-92. Khon Kaen, Thailand: PME.

interpreted, and acted upon in mathematics education? How is the tension between the local, communal, and global conceptualized? What are our responsibilities as mathematics educators to make a difference?

In the two working sessions, we will discuss a variety of examples in order to explore the questions. The following are some related activities: Activity A – Using Intergovernmental Panel on Climate Change and United Nation's statistic, we provide data relating to the issues of climate change, pollution, economic inequality, etc. We then invite participants to reflect on the activity and on what implications they derive for the learning and teaching of mathematics; Activity B – we select a small number of key readings. We invited the participant, to analyze them in terms of how they conceptualize relationships between people(s), cultures, the ecosystem, etc., and Activity C – Finally, we proposed to explore pedagogical strategies in incorporating issues of social and ecological injustice into the teaching and learning of mathematics – how could these strategies be conceptualized in various kinds of classrooms – teacher education, high and elementary schools. To further investigate, we will use data from a collaborative study between Norway and Canada to see if and how teachers incorporate issues of climate change into mathematics classrooms.

We plan the following interactions:

Day 1: a. Introduction to the questions and identifying key terms; b. Viewing a 3-minute video about our planet earth. Discussion of the issues of social and ecological justices; c. Group work on activity A; d. As a whole team, reflecting on the implications of the activity for mathematics education.

Day 2: a. A short summary of themes collected in the first session; b. Group work on activity B; c. Relating the work to our stance towards relationality; d. Group discussion about activity C; d. Reflection and Closing remarks.

Throughout both sessions, we will highlight common themes in relation to our ethical responsibilities towards and possible ways of the incorporation of issues of social and ecological justice, in mathematics education. Points arising from both sessions will be fed back to our concluding remarks, in which we combine the discussion from the two sessions to highlight possible further research and actions.

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RESEARCHING MATHEMATICS CURRICULUM INNOVATION IN COMPLEX, CHANGING, UNCERTAIN TIMES

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We aim to dialogue with and support a network of scholars working around the theme of how mathematics education may be(come) relevant – locally and globally – in a changing world and how we, as researchers, might develop and employ methodologies that are suitably agile to operate with and within this world. We ask these questions while also troubling notions of authority and legitimacy in relation to mathematical and research knowledge. This Working Group was instigated in 2020 and met virtually at PME44. The 2021 session welcomes and will be accessible to newcomers.

BACKGROUND

We live in complex times of rapid change and uncertainty, characterized by alarming levels of resource depletion, climate change, poverty, inequality, health crises such as the Covid-19 pandemic, totalitarianism, and intolerance. These developments, and others, while produced together in centuries-long geopolitical processes – referred to as 'neoliberalism', 'racial capitalism', and so on – manifest materially in specific ways in local contexts. The current world condition highlights the need to rethink taken-for granted assumptions about economics, politics, society and the environment, and their relations, that have driven these global processes. For the mathematics education community, there are urgent questions which we started to explore in our 2020 Working Group dialogue: What does it mean for a curriculum to be 'relevant' (that is, alert to the particularities of place, identity, politics, and ecology) to the needs and concerns of students in a particular context? How and by whom might a mathematics curriculum be innovated, while being reflexive about the basis for

2021. In Inprasitha, M., Changsri, N., Boonsena (Eds.). *Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education,* Vol. 1, pp. 93-95. Khon Kaen, Thailand: PME.

truth claims or authority within that curriculum? What research might inform such work?

In the 2021 session we extend our dialogue to methodological considerations: What does it mean for researchers to ‘know’ about mathematics curriculum innovation in a complex context characterised by inextricable interdependencies, significant elements of contingency, and uncertainty? And on what and whose authority is this decided? We link to mathematics education scholars researching in contexts where change, precarity and uncertainty are not new experiences (e.g. Adler & Lerman, 2003; Vithal & Valero, 2003). These scholars, along with others in different research fields, highlight the assumptions of stability, certainty and linearity that underpin the dominant research methodologies in the current global knowledge production regime. In contrast, research in contemporary conditions requires that researchers ‘hold uncertainty’ (Bhan, 2019, p.15), and work with *connectivity* (between humans, and between mathematics, humans, the earth, technology, and power), *flexibility*, *fluidity*, *mobility*, *discontinuity*, *vulnerability*, and *dialogue*.

GOALS OF THE WORKING GROUP

In this Working Group we will be developing responses to the following key questions:

- What ‘agile’ methodologies work productively with uncertainty & contingency?
- What power asymmetries and vulnerabilities characterise research processes, both face-to-face and online, that work with connectivity?
- What counts as research on *mathematics* curriculum innovation & who decides?
- How do we conduct research that matters locally *and* together contributes to knowledge from multiple places that does not ‘other’ some ‘locals’?

ACTIVITIES AND TIMETABLE

Session (1)

- Introduction of aims and outcomes of the 2020 virtual WG meeting. (15 mins)
- Presentation of two curriculum projects and their methodologies. (20 mins)
- Small group discussion: What are the similarities and differences across the projects? What are the issues around researching curriculum innovation in such contexts? (20 mins)
- Plenary discussion to collate common themes from small group discussion. (20 mins)

- Summary and small group “homework” task: What research questions about curriculum innovation matter in your context? What future innovations might you pursue? (15 mins)

Session (2)

- Review of Session 1 and small group presentations of homework, allowing access into the group for newcomers. (20 mins)
- Small group discussion: What theories and methodologies might be used for the proposed research? How might these ‘hold’ uncertainty and complexity? How can power asymmetries and vulnerabilities be navigated? How might local-global relations be mobilised? How might such research be reported locally and globally? (30 mins)
- Plenary discussion to share small group contributions. (30 mins)
- Closing discussion: Next steps for future collaborations. (10 mins)

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CONCEPTUALIZING THE EXPERTISE OF THE MATHEMATICS TEACHER EDUCATOR

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BACKGROUND

This working group builds on the working group on the same topic at the PME 43 conference (Helliwell, & Chorney, 2019) whereby much of what was suggested for further investigation were questions that extended beyond mathematics teacher educator (MTE) knowledge and moved towards contexts that challenge, extend, constrain, such as culture, complexity, and curriculum. Participants in the 2019 working group expressed an interest in a follow-up working group to continue to develop the ideas and research problems initiated at PME 43.

In terms of MTE expertise, some scholars have extended existing models of mathematics teacher knowledge (such as Shulman’s (1986) “pedagogical content knowledge” (PCK)) as a way of describing the knowledge of the MTE (e.g., Chick & Beswick (2018) extend PCK to “MTEPCK” that is a “kind of meta-PCK which could be described as *PCK for teaching the PCK for teaching mathematics*” (p. 476, emphasis original)). In Mason’s (2008) chapter *PCK and beyond*, he challenges the common framing of PCK as a kind of psychology and instead proposes thinking about PCK as both social and distributed. Mason suggests that teachers “draw upon knowledges that are distributed in the historical-cultural-social and institutional practices, in texts, works-cards, apparatus, and other materials available…” (p. 309-310). MTE expertise could thus be framed by turning our gaze outward, by drawing on Hutchins’ (1995) model of “distributed cognition” as a balance between knowledge and external agencies.

Many of the questions that were posed by the working group participants last year share this outward looking, such as: How do MTEs balance complexity with a focussed treatment of an issue?; how do MTEs make use of examples/problems when working with mathematics teachers?; how do MTEs decentre from their own experiences of teaching mathematics and/or as a student of mathematics?; what is the relationship between in the moment decisions of MTEs and teachers?; and how do MTEs prepare teachers to adapt

2021. In Inprasitha, M., Changsri, N., Boonsena (Eds.). *Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education,* Vol. 1, pp. 96-98. Khon Kaen, Thailand: PME.

to curriculum changes (when MTEs themselves need to adapt)? In terms of this follow-up working group, we intend the subgroups formed to continue their conversations and develop ideas further and we also welcome new participants.

AIMS OF WORKING GROUP

- To explore the theorisation of MTE expertise that goes beyond knowledge by considering personal stories, experiences and a variety of frameworks.
- To formulate approaches and research questions around MTE expertise.
- To explore and develop potential methodologies that support these approaches and research questions.

OUTLINE OF SESSIONS

Session (1)

- Introductions and initial discussion around the notion of distributed cognition as a possible overarching framework for MTE expertise. The presenters will share some personal experience of expertise that emerged from distributed activity and present some existing explorations of distributed expertise from MTE literature.
- Sharing of examples that connect to some of the questions that emerged from last year, moving to suggestions for possible new approaches to and conceptualisations for describing MTE expertise that offer alternatives to expertise as knowledge.
- Group discussion with a focus on connecting last years' issues and questions with the development of frameworks that support the interaction between the practice of MTEs and conceptualisations of MTE expertise.

Session (2)

- Building off session 1, groups will be organised by interest, according to last years' themes and/or emergent themes from session 1. Groups will develop their own questions, but the leaders will provide prompts to support engaging with questions from a distributed approach.
- Each group will share responses and then discuss on next steps for future collaborations, including consideration of a joint output for participants such as a special issue for the *Journal of Mathematics Teacher Education*.

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VIGNETTES AS REPRESENTATIONS OF PRACTICE FOR MATHEMATICS TEACHER EDUCATION AND RESEARCH

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TOPIC

Vignettes are classroom scenarios that can be represented in written, cartoon or video formats. In teacher education and corresponding research, they are an effective tool for representing and eliciting aspects of teacher practices, beliefs and understandings about cognitive and pedagogical aspects related to mathematics learning and teaching. With growing interest in this field, the proposed Working Group aims at making a contribution by facilitating the exchange and discussion between scholars from various international contexts.

THEORETICAL BACKGROUND

Vignettes are regarded as an effective stimulus for discussing real-life contexts and problems in teacher education and professional development. Vignettes are used in education research to elicit teachers' responses through probing for reactions to gain insights to participants' knowledge, beliefs, emotions, judgements, attitudes and values about particular phenomena. Vignette-based research in mathematics education includes, for example: developing trainee teachers' mathematical knowledge (Samková, 2018); improving pre-service teachers' noticing using learning trajectories (Ivars et al. 2018); developing knowledge about teaching mathematics that respects student diversity (Healy & Fereirra dos Santos, 2014); eliciting teachers' value laden beliefs (Skilling & Stylianides, 2019) or using different vignette formats to assess teacher competence (Friesen & Kuntze, 2018).

WORKING GROUP STATEMENT AND GOALS

2021. In Inprasitha, M., Changsri, N., Boonsena (Eds.). *Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp. 99-101. Khon Kaen, Thailand: PME.

This group recognizes the use of vignettes to advance mathematics research and pedagogy relevant to teacher education, professional development and methodological advances. The aims are to: investigate and discuss how vignettes are used by teacher educators and mathematics education researchers in various cultural contexts; compare and contrast vignette use in mathematics education settings and determine various foci (including participants, topics, processes); and identify specific activities and forums for future collaboration. Expected outcomes: disseminate and share vignettes, exchange and explore how to incorporate them in research and mathematics teacher training courses; collect research papers for a potential Special Issue; build an international network of researchers for communication about vignette use followed up at PME45. The sessions are planned accordingly:

Session (1): Vignettes in mathematics teacher education

- Short presentation: Introduce vignettes and digital tool DIVER for creating cartoon-based vignettes, provide and exchange information about current vignettes use for developing teacher expertise and professional learning from different cultural perspectives (25 minutes)
- Participant's trial/develop vignette activities: exemplar written, cartoon and video vignettes in context, facilitated by the WG organisers (40 minutes)
- Whole group: Participants discuss and share their contexts and potential opportunities to use vignettes; outlook to session 2 (25 minutes)

Session (2): Vignettes in mathematics education research

- Short presentation with interactive group elements: Present outcomes of research in terms of increasing teacher knowledge about student learning and pedagogical approaches for improving student learning (25 minutes)
- Continue development of exemplar activities from session 1; discuss opportunities for collaborating in vignette-based research (40 minutes)
- Summarise and feed-back; plan for post conference activities (25 minutes)

Acknowledgements

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CHALLENGES FOR PUBLISHING RESEARCH RESULTS ON MATHEMATICS PROFESSIONAL DEVELOPMENT

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[5]Weizmann Institute of Science, Israel

The overarching goal of this new Working Group is to explore the concern that there are existing results and insights from research on mathematics professional development (MPD) that are not being published in major journals in the field. The group will identify the nature of the problem, potential sources, and possible solutions.

PUBLICATION AND RESEARCH ON TEACHERS MPD

The growth of research and publications regarding the mathematics professional development (MPD) of K-12 teachers is a significant mark of researchers' increased attention to this area of scholarship. For example, an ERIC search using the terms "mathematics" and "professional development" shows that the number of publications in English doubled between the1990s and 2000s and doubled again from 2000s to 2010s with 3,419 papers only in this last decade. Reviews of this, however, show that a significant amount of the papers published examine small case studies (Sztajn, Borko & Smith, 2017). Whereas it is often considered that the literature reflects the field's significant engagement in small case studies at the expense of other approaches or research phases (Borko, 2004), it is possible that the state of the literature also represents the types of manuscripts that are being accepted for publication.

The fact that important work on MPD might not be getting published received recent attention from the *Journal of Mathematics Teacher Education*, which put out, in 2020, a call for papers on MPD that addressed findings that might not have been published because the results were non-significant, the project did not unfold as intended, or the applied nature of the work created methodological challenges. The idea that there are important results about MPD not being

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published reflects the experiences of the organizers of this group. For example, within our own research projects, we have opted to publish case studies when larger studies did not find their way into mainstream journals. Or we have opted to attend to smaller theoretical aspects when reviewers expressed less interest in larger parts of the studies. Collectively, these experiences raise an important question: what is lost in our knowledge of MPD when important research results do not seem to fit major mathematics education journals?

The goal of this new Working Group is to explore reasons why papers on MPD might not get published, consider the issues in the field that might be contributing to this problem, examine what is lost in the field, and propose short- and longer-term solutions. The group aims to contribute to research on MPD by changing the dominant discourse in the field and increasing the amount, variety and quality of research that gets published.

Session (1):

Activity 1--Welcome and framing the issue: The work of the group will begin with a quick introduction to the overall issues that frame the proposed discussions.

Activity 2--*Understanding the phenomenon of interest:* Each organizer will briefly share their experiences with acceptances /rejections around publishing MPD research.

Activity 3--Examining the sources of the phenomenon: Participants will self-select into four small groups led by the first four organizers and discuss the stories shared, their own stories, and potential causes for the phenomenon such as: (1) tensions between research and evaluation; (2) existing funding models; (3) current dominant discourse; or (4) cultural differences. Groups will record their experiences in a shared document.

Activity 4-- Summarizing initial discussions: To conclude the first meeting, organizer will provide a summary of ideas from their groups.

Session (2):

Activity 1--Establishing shared ideas: Organizers will present their understanding of the phenomenon based on the ideas participants added to the shared document.

Activity 2--Additional forces that keep the phenomenon in place: The two last organizers (and editors from the JMTE special issue mentioned earlier) will present what they learned from the call for proposals and the papers received.

Activity 3--Moving forward: In small groups, participants will consider ways to address current forces that contribute to the occurrence of the phenomenon and propose steps to move forward.

Activity 4--Next steps: In a plenary format, the group will collectively generate guidelines for authors, reviewers and editors about opportunities for publication of key results regarding research on MPD and discuss next steps for the work of the group.

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PME44 VIRTUAL

THE 44th CONFERENCE OF THE INTERNATIONAL GROUP
FOR THE PSYCHOLOGY OF MATHEMATICS EDUCATION
July 19-22, 2021
Hosted by Khon Kaen, Thailand
Virtually hosted by Technion, Israel Institute of Technology

SEMINAR

OUTDOOR LEARNING IN MATHEMATICS EDUCATIONS

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During this workshop, the participants will learn about the theoretical background of outdoor education and its benefits for the learning process. Its implementation in mathematics classes will be practically demonstrated using MathCityMap. MathCityMap is a two-component system and consists of a web portal for teachers and a smartphone app for students. Both components are presented in the workshop. Hereby, the planned activities alternate between direct instruction and active involvement of the participants, ensuring a sustainable and fruitful learning outcome.

The organizers of this workshop are researchers in the field of outdoor learning with mobile devices. They all belong to the Institute of Mathematics and Computer Sciences where the MathCityMap system has been developed and evaluated since 2013. Through these years of experiences and the successful coordination of two Erasmus+ strategic partnerships on outdoor mathematics (www.momatre.eu and www.masce.eu), the team gathered a lot of experience in the presented research field.

The use of smartphones in classrooms is - due to the restriction legislated by administrations - not very popular. With the MathCityMap project, we show one possibility to use the own mobile device in a substantial and authentic learning environment. MathCityMap combines the long-known math trail idea with the current technological possibilities of mobile devices. A math trail is a set of mathematical tasks or questions that are bound to objects from the real world. Usually, they are located in walking distance. A math trail guide contains a map that displays interesting locations and descriptions of different tasks to discover mathematics in the environment (Shoaf, Pollak & Schneider, 2004). MathCityMap is a two-component system. The first component is a web portal (www.mathcitymap.eu) which serves as an open access database for authentic math problems in the environment. Every teacher/author can use public tasks and create individual tasks. The other component, the MathCityMap app, shows the students on a map where in the environment the problems are located. Additionally, it provides hints, feedback, and a sample solution. Through new features, such as the digital classroom and the task wizard, the system supports teachers in the preparation and organization of math trails (Ludwig, Baumann-

2021. In Inprasitha, M., Changsri, N., Boonsena (Eds.). *Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp. 106-108. Khon Kaen, Thailand: PME.

Wehner, Gurjanow & Jablonski, 2019). Within the proposed seminar, the participants get to know the system from student's and teacher's perspective. After a short introduction on outdoor learning and math trails, the participants run a MathCityMap@home math trail on their smartphone. The participants share their experience with each other. The second part of the seminar focuses on the teacher's perspective and starts with an introduction of the web portal. By creating a task using sample data, the participants also immediately use the web portal. Following this, criteria for meaningful tasks for the MathCityMap system are presented. With this basis, the participants create their own tasks. The seminar ends with the participants sharing again their impressions and experiences. As preparation for the workshop, the participants should download the MathCityMap app and register in the MathCityMap web portal. The participants should participate with their computer and have a smartphone/tablet on site.

Outline and structure

Planned timeline	Topic and content	Activities
20 minutes	Introduction to Outdoor Learning and the MathCityMap system	Presentation
45 minutes	Students' perspective	Running a trail (MCM@home)
25 minutes	Sharing of experiences, authors' perspective	Group discussion
25 minutes	Introduction to the web portal	Presentation
45 minutes	Task creation in the web portal	Use of Web Portal
20 minutes	Outlook: Criteria for meaningful tasks, Digital classroom	Presentation

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PME44 VIRTUAL

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NATIONAL PRESENTATION

MATHEMATICS EDUCATION COMMUNITY IN LOWER MEKONG SUB-REGION

Maitree Inprasitha[1], Narumon Changsri[1], Nisakorn Boonsena[1]

[1]Khon Kaen University, Thailand

This paper consists of two parts: 1) An overview of 30 years project (2000-2030) for preparing foundation development for mathematics education, and 2) Emergent mathematics education community in the Lower Mekong Sub-region. The results obtained from these two parts will be shared to clarify the meaning of goals, practices, and cooperation achievements throughout the developmental process.

AN OVERVIEW OF 30 YEARS PROJECT (2000-2030) FOR PREPARING FOUNDATION DEVELOPMENT FOR THE MATHEMATICS EDUCATION COMMUNITY

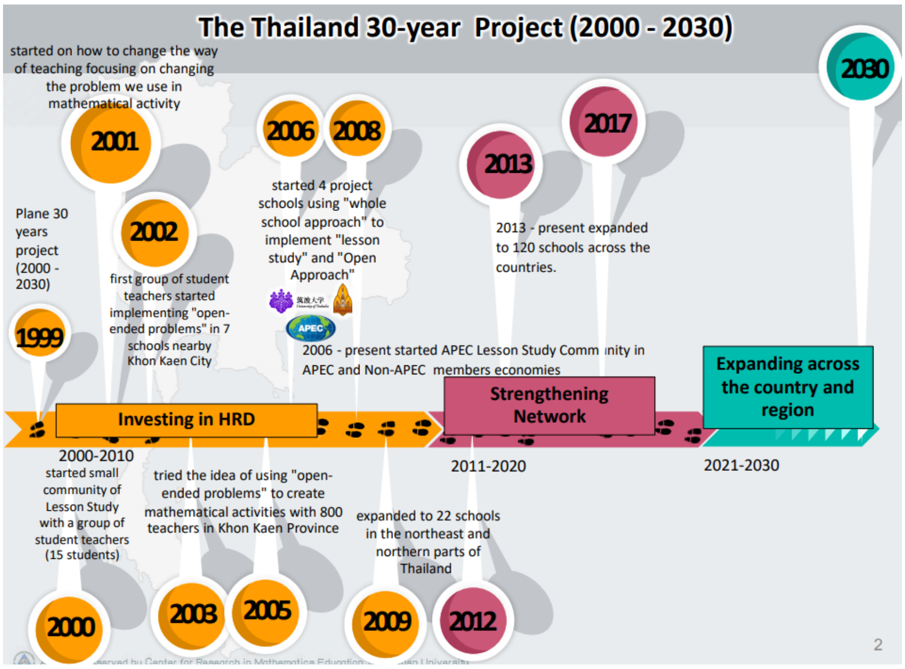
Long-term strategic planning for a period of 30 years project (2000-2030) was started in 1999. This Thailand 30 years Project was comprised of three phases with each phase was expected to take 10 years to complete as follows: The first phase (2000–2010) was mainly advanced **human resource development** with the purpose to develop and prepare young mathematics educators (YMEs) for long-term innovative development. As a result, a new generation of YME was successfully developed through conducting continuous teacher professional development (TPD) in innovations for the teaching profession. The second phase (2010-2020) was planned to **strengthen the networking** across the educational institutes where the YMEs were affiliated. The third phase (2021-2030) is planned to **expand** the teaching innovations **across the country and region** for the spreading effects of the teaching innovations (Inprasitha, Changsri & Isoda, 2020).

At the beginning of the first phase (2000-2010), we invested in **Human Resource Development** (HRD) as a pilot research project in Thailand to 15 fourth-year pre-service teachers in Bachelor Programs of Mathematics Education, at the Faculty of Education, Khon Kaen University who were assigned to seven urban schools for their teaching practicum in 2002 academic year, in Khon Kaen province, Thailand (Inprasitha, 2011). In 2003-2005, we tried the idea of using ‘open-ended problems’ to create mathematical activities with 800 teachers in Khon Kaen Province. In 2006-2007, we started the teaching innovations to four project schools using the ‘whole school approach’

2021. In Inprasitha, M., Changsri, N., Boonsena (Eds.). *Proceedings of the 44th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp. 110-119. Khon Kaen, Thailand: PME.

to implement 'lesson study' and 'open approach'. Furthermore, in 2006, we started APEC Lesson Study Community in APEC and Non-APEC members economies. In 2009, we extended to 23 project schools in the northeast and northern parts of Thailand (Inprasitha, 2015).

During 2011-2020, we have **strengthened our network** and tried to give firsthand experiences about the mathematics education community to our graduate students through their participation in various mathematics education conferences. We started in the late of Phase I, in 2007, we attended EARCOME 4 in Malaysia with two professors and 28 master's degree students. The participations made us recognized as the small group of the mathematics education community in Thailand.



The Thailand 30-year Project (2000 - 2030)

started on how to change the way of teaching focusing on changing the problem we use in mathematical activity

Plane 30 years project (2000 - 2030)

first group of student teachers started implementing "open-ended problems" in 7 schools nearby Khon Kaen City

started 4 project schools using "whole school approach" to implement "lesson study" and "Open Approach"

2006 - present started APEC Lesson Study Community in APEC and Non-APEC members economies

2013 - present expanded to 120 schools across the countries.

Investing in HRD

2000-2010

Strengthening Network

2011-2020

Expanding across the country and region

2021-2030

started small community of Lesson Study with a group of student teachers (15 students)

tried the idea of using "open-ended problems" to create mathematical activities with 800 teachers in Khon Kaen Province

expanded to 22 schools in the northeast and northern parts of Thailand

Figure 1: 30-years project (2000-2030)

EARCOME 4
4TH EAST ASIA REGIONAL CONFERENCE ON MATHEMATICS EDUCATION
JUNE 2007
GEORGETOWN PENANG

Figure 2: EARCOME 4, Malaysia

In 2009, we attended PME 33 at Thessaloniki, Greece with three professors and 12 Ph.D. students. In 2010, we attended EARCOME 5 in Japan with four professors and 25 Ph.D. students. In 2011, we attended PME 35 at Ankara, Turkey with three professors and 24 Ph.D. students.

Figure 3: PME 33, Greece

Figure 4: EARCOME 5, Japan

Figure 5: PME 35, Turkey

In 2012, we attended ICME 12 in Seoul, Korea, and PME 36 in Taipei, Taiwan with three professors, one master’s degree student, and 24 Ph.D. students. In 2013, we attended PME 37 at Kiel, Germany with four young faculty members. Furthermore, in 2013 we hosted EARCOME 6 at Thailand and CANP Project in Phnom Penh, Cambodia.

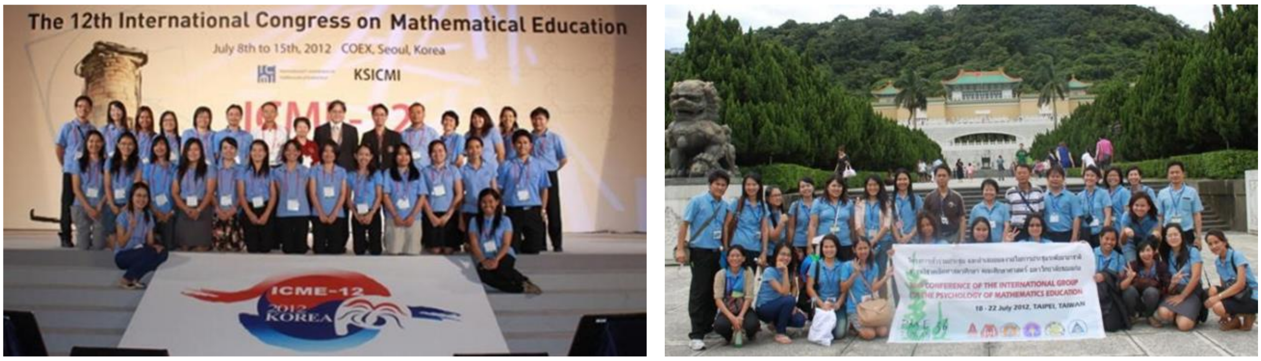

The 12th International Congress on Mathematical Education
July 8th to 15th, 2012 COEX, Seoul, Korea
KSICMI
ICME-12
2012 KOREA
CONFERENCE OF THE INTERNATIONAL GROUP
THE PSYCHOLOGY OF MATHEMATICS EDUCATION
18 - 22 July 2012, TAIPEI, TAIWAN

Figure 6: ICME 12, South Korea and PME 36, Taiwan

Figure 7: PME 37, Germany

Figure 8: EARCOME 6, Thailand

In 2015, we attended EARCOME 7 in Philippines with three professors and 10 graduate students. In 2016, we attended PME 40 at Szeged, Hungary with one professor two Ph.D. students, and ICME 13 in Hamburg, Germany with two professors and nine Ph.D. students.

CASIO
The 7th ICMI-East Asia
Regional Conference on
Mathematics Education
EARCOME 7
In Pursuit of Quality Mathematics Education for Al
Waterfront Hotel, Cebu City, Philippines
May 11 - 15, 2015

Figure 9: EARCOME 7, Philippines

Welcome to
TAIWAN
CMI-EARCOME 8
WAN / 2018 MAY 7-11

Figure 10: EARCOME 8, Taiwan

Figure 11: PME 40, Hungary

Figure 12: ICME 13, Germany

In 2017, we attended PME 41 in Singapore with three professors and 17 Ph.D. students, and 14 master's degree students to submit the proposal for PME 44. In 2018, we attended EARCOME 8 in Taiwan with three professors and six graduate students and we also attended PME 42 at Umeå, Sweden with two professors and four Ph.D. students to bid proposals for PME 44. In 2019, we attended PME 43 at Pretoria, South Africa with two professors and six Ph.D. students to welcome all participants to PME 44 in 2020.

Figure 13: PME 41, Singapore

PME44
THAILAND
2020
UMEÅ UNIVERSITY
UMEÅ UNIVERSITET

Figure 14: PME 42, Sweden

WELCOME to PME 44
PME 43
Make today matter
WELCOME TO
KHON KAEN, THAILAND
July 21-25, 2020
PME44

Figure 15: PME 43, South Africa

In Phase III, **Expanding across the country and region** (2020-2030), from the small group of mathematics education community in 2000, now we have expanded up to 274 project schools, 109,491 students, 7,857 teachers, and 14 educational institutes across Thailand and many networking in the international community. As pointed out by Inprasitha and Isoda (2021) regarding the main role and successful project as the collaborative agents from APEC member economies that consisting of the eight agents as the following: 1) Project Overseer-Professor from Khon Kaen University, Thailand (Maitree Inprasitha) and University of Tsukuba (Masami Isoda) developed proposal to submit to APEC secretariat, and organized the meeting and symposium in Thailand and Japan; 2) Specialist- Professors from APEC member economies, key persons who extend Lesson Study in each economy attend all the processes; 3) MOE-(Ministry of Education) is important to funding support the project; 4) Thai Expert- Professors and lecturers from CRME and IRDTP, Khon Kaen University have improved Mathematics Education in Thailand via Lesson Study and Open Approach; 5) Japanese Expert- Professors and researchers from Japan have advised how to use Japanese textbook and classroom observation; 6) Assistant- Graduate students from Mathematics Education Program, Khon Kaen University support all the processes of the project; 7) S-Leader- Local agents such as school principal and supervisor, and 8) S-Teacher- In-service teachers.

EMERGENT MATHEMATICS EDUCATION COMMUNITY IN THE LOWER MEKONG SUB-REGION

Faculty of Education, Khon Kaen University (KKU), Thailand had launched the Hoshino Project for training the mathematics teachers in Laos PDR since 2003. In the 2004 academic year, four trainees had studied for a master’s degree at the department of mathematics education and science education, KKU, funded by EDF Project. Since 2005, the department of mathematics education, Faculty of Education, KKU had provided scholarships for the master program for 13 mathematics teachers from Laos PDR through the EDF Project and the KKUEDU Partnership Project (7 out of 13 had graduated and worked as the network teachers and educators of KKU). Since 2012, three bachelor's degree and four master's degree students from Cambodia were offered the Her Royal Highness Princess Maha Chakri Sirindhorn scholarship to study in the mathematics education program, KKU. In this way, the mathematics education network across the countries had gradually established through their studies in the mathematics education program, KKU.

In 2003, the Higher Education Commission ordered academic departments to create a Cooperative Research Network (CRN) in Thailand. Mathematics education was separated from the mathematics field and created its own Cooperative Research Network in Mathematics Education (CRN-MathEd). Faculty of Education, KKU started the mathematics education program in master degree and had got the grant by the Project to Support the Competency for Competition of Thailand. This time is the starting point to produce the YMEs in Thailand. From 2006 until now, KKU and the University of Tsukuba launched the APEC Lesson Study Project. The mathematics education community in Thailand has been contributed by APEC Lesson Study Project.

In 2013, Thailand was the host of EARCOME 6 and KKU provided the chance for Laos and Cambodia teachers to join the conference. During the conference, the meeting for the Capacity & Networking Project (CANP) under the ICMI had organized by Prof. Bill Barton and colleagues, and Prof. Maitree Inprasitha. This was led to occur the 1st workshop of CANP in December 2013 in Cambodia. It has successfully engaged YMEs from four ASEAN countries (Cambodia, Laos, Thai, and Vietnam) as trainees.

Figure 16: CANP workshop in Cambodia

In 2013, the Thailand Society of Mathematics Education was established by mathematics educators in Thailand, and organized the 1st conference on mathematics education by January 2015 at KKU and provides the chance for Laos and Cambodia teachers to join the conference.

In May 2015, Thai mathematics educators and graduate students participated in the EARCOME 7 at the Philippines. The CANP participants from Cambodia, Laos, and Thai were supported by ICMI and Prof. Bill Barton, and Prof. Yeap Ban Har the grant for joining the 2nd workshop of CANP during EARCOME 7. Unfortunately, Vietnam and Myanmar did not participate in the 2nd workshop. In June 2015, KKU provides a scholarship for Cambodia and Laos graduate students to attend the ICMI Study 23 in Macau, China.

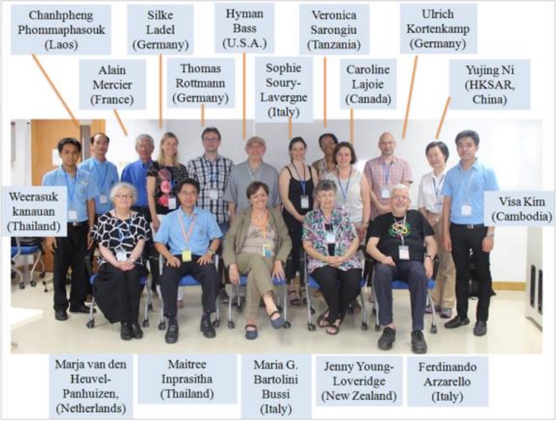

Chanhpheng Phommaphasouk (Laos)
Silke Ladel (Germany)
Hyman Bass (U.S.A.)
Veronica Sarongiu (Tanzania)
Ulrich Kortenkamp (Germany)
Alain Mercier (France)
Thomas Rottmann (Germany)
Sophie Soury-Lavergne (Italy)
Caroline Lajoie (Canada)
Yujing Ni (HKSAR, China)
Weerasuk kanauan (Thailand)
Visa Kim (Cambodia)
Marja van den Heuvel-Panhuizen, (Netherlands)
Maitree Inprasitha (Thailand)
Maria G. Bartolini Bussi (Italy)
Jenny Young-Loveridge (New Zealand)
Ferdinando Arzarello (Italy)

Figure 17: ICMI Study 23, Macau

In November 2015, Thailand has the great opportunity to host the World Association of Lesson Study (WALS) at KKU. We tried our best to involve the participants from Vietnam and Myanmar in order to join the conference and the 3rd workshop of CANP.

In 2016, we conducted the workshop for Cambodian teachers at the National Institute of Education, Phnom Penh, Cambodia, and conducted a workshop for Laos teachers at Pakse Teacher Training College, Laos PDR. Moreover, we had CANP Workshop in ICME 13 at Humburg, Germany. In 2018, we had CANP Meeting in EARCOME 8 in Taipei, Taiwan.

Cambodian Mathematics Textbooks (2015)
Japanese Mathematics Textbooks (2005)
Composing/ Decomposing

Figure 18: Teacher Training in Cambodia

In 2020, we attended the Cambodia 1st International Conference on Mentoring Educators. In 2020, we conducted the workshop in Laos PDR, in this workshop, they organized an open class activity in “Workshop on using Math Textbook and Teacher guide of G1 and G2 and Lesson Study Workshop” at Dongkhamxang Teacher Training Collage, under the project of teaching and learning mathematics at the elementary level in the Lao People's Democratic Republic together with teachers from Teachers College Demonstration School and a writing team from the Institute of Science Education of more than 80 people. In Open Class at Dongkhamxang Teacher Training Collage, Laos, which was a shared learning area from Lesson Study Team, corroborative do (observe classroom) and jointly reflect the learning outcome (corroborative see) together to exchange views experiences from the professors of many countries. This activity inspired many teachers to implement the practice in their classes who join the training. From reflection, many teachers have possessed their capabilities and some ideas to improve their classroom in the Lao People's Democratic Republic.

Figure 19: Teacher Training in Laos PDR

Furthermore, in PME 44, 2020, we offered alternative participation through youtube live for people who did not register for the conference this year. There are huge of silent participants include the mathematics education community in the lower Mekong sub-region.

With our efforts over the last 20 years, KKU, Thailand has produced mathematics educators and YMEs who have to take responsibility for improving the educational situations in the region. Among the countries in Lower Mekong Sub-region, we could gradually create collaboration through the network from a number of collaborative projects and the study in the mathematics education program at KKU. The mathematics education community in this region could be established by the leading role of Thailand to collaborate among the region. We should set long-term shared goals to solve the common problems in mathematics education, keep and expand the collaboration for better education in our Lower Mekong Sub-region.

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ORAL COMMUNICATION

PRESERVICE MIDDLE SCHOOL MATHEMATICS TEACHERS' STATISTICAL REASONING ABOUT DISTRIBUTION

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The instructional goals in statistics education shift from computational skills to conceptual understanding of basic statistical ideas (Franklin et al., 2007). Accordingly, it is expected that people reason with statistical ideas and make sense of statistical information named as statistical reasoning (Garfield, 2002). Distribution is one of the important statistical ideas to get as it is an overarching concept that includes the interrelated concepts of shape, center, and spread. The understanding of how the data is distributed is essential to notice variability among measures so that it rises to analyze the data by selecting appropriate statistics (e.g., mean, median, and mode) and graphical representations.

This paper is the part of design-based research that aims to develop the reasoning about distribution. In this paper, we examined 14 preservice middle school mathematics teachers' reasoning about distribution, which is the starting point of the design experiment. The participants were selected through purposive sampling method. A test of 10 questions including measures of center, measures of spread, data displays, shape, comparing distributions, and bivariate distribution was applied to evaluate their existing statistical knowledge about distribution. Based on the answers given for the test, semi-structured interviews were conducted to understand their reasoning about distribution in depth. We analyzed the data from the test descriptively and used thematic analysis for interview transcripts.

The data obtained showed that the preservice teachers could compare distributions with unequal sample sizes with being aware of multiplicative reasoning. Furthermore, they tend to use mean as average regardless of the shape or spread of the distribution. While they could reason about the shape of the distribution, spread or center, separately, they could not establish a relationship among them. For bivariate distribution, they could interpret the relations of variables with each other. The results revealed the need to improve the preservice teachers' reasoning about distribution by connecting shape, center and spread.

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TPCK TRAINING FOR PRE- AND IN-SERVICE MATHEMATICS TEACHERS IN THE PROLONG PROCESS FROM LEARNERS TO TEACHERS

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In this study we explore, innovatively, the integration of Technology, Pedagogy, Content Knowledge (TPCK) of pre- and in-service teachers in the context of mathematics through the inquiry-based learning (P) employed by engagement in a "mathematics lab" – a dynamic geometry software (DGS) package that serves as the technological environment (T). We focus the content (C) on geometry, which lends itself particularly well to the inquiry-based learning approach because it enables students to perceive regularities and guess rules. Research questions are: 1) Will there be a change in pre- and in-service teachers' attitudes toward their TPCK, before and after practicing the mathematics lab as *learners*?, and 2) How do pre- and in-service teachers demonstrate their TPCK level, as realized in their performance in planning a dynamic geometry activity as *teachers*?

Participants are 88 (25 & 63, respectively) pre-and in-service teachers who took part in six weekly meetings (each meeting lasts two to three academic hours) in a math lab, where they were exposed to the research model that was designed to gradually expose them to the various components of TPCK. The first two meetings included an exposure to the TPCK theory and to the DGS. The next two meetings included an inquiry-based instructed lesson using DGS that was tailored to the participants' knowledge of geometry (*teachers as learners*). The last two meetings involved an active design by the teachers of an inquiry-based geometry activity using DGS (*teachers as teachers*).

Research tools include: 1) a pre- and post- quantitative questionnaire on a 5-point Likert scale to measure teachers' perceived TPCK regarding how they perceive their knowledge towards integrating the various TPCK components in class, and 2) inquiry-based dynamic geometry activities that were designed by the participants, and analyzed using a unique rubric that was designed for this study.

Findings indicate a significant change in teachers' attitudes towards their TPCK components by time measurement (pre- and posttest), for those factors that relate to the technology component, i.e. TK, TCK, TPK, and TPCK. Further, the findings for the designed activities show that the highest rated component was procedural knowledge, followed by content knowledge and visual representation technology, and finally the pedagogy components that reflect inquiry-based learning.

The study contributes theoretically to the conceptualization of TPCK in the context of mathematics. The practical contribution is in the research model that appears to provide the lens for mathematics teachers to understand how to integrate technology into learning and teaching a process that can be implemented in other teacher training and professional development programs. Finally, a methodological contribution of this study lies in the development of the rubric for assessing dynamic geometry activities.

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FORMATS FOR PRE-STRUCTURED LEARNING OF HEURISTIC STRATEGIES IN MATHEMATICAL PROBLEM SOLVING - THE CASE OF HEURISTIC WORKED-OUT EXAMPLES

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This theoretical paper presents deductive conceptual work on the question: *Which theoretical potential of design principles and elements can be identified to foster the learning of heuristic strategies via the special format of heuristic worked-out examples (HWEs),* as part of the more general issue of possible formats for pre-structured learning of heuristic strategies in mathematical problem solving.

Referring to general action-based learning theories (Aebli, 1994; Galperin, 1969), and research on mathematical problem solving (e.g. Schoenfeld, 1985), we argue that having a heuristic strategy at one's disposal emerges from an individual's concrete problem solving actions – that is, *actions* in certain *situations* within concrete problem solving processes to reach certain *target*s – mainly by two complementing processes: Internalising and classifying, catalysed by language. Based on that conceptualisation, we develop a two-dimensional learning field for heuristic strategies, identify possible learning paths within, and give an analysis of established design elements of HWEs (e.g. Renkl, 2017), such as self-explanation prompts and fictional dialogues, regarding these learning paths.

The presented theoretical framework can be used to identify and develop new (configurations of) design elements for HWEs, and to formulate more general design principles, hence as a basis for evaluating and developing learning material. It also serves as a theoretical foundation for specific empirical research questions and hypotheses, e.g., regarding the effect of specific, known, and new (configurations of) design elements and principles on the learning of heuristic strategies. In that manner, we are currently conducting individual case studies with a heuristic worked-out example including self-acting tasks and post-reflection tasks as new design elements.

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RICH CONTEXTS TO PROMOTE EFFECTIVE LEARNING OF MATHEMATICS

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Teaching mathematics aims to facilitate the development of mathematical ideas for those who want to learn, so we can transform mathematics classes, inside or outside, into a rich environment that leads students to conjecture, prove, generalize, question, discuss, collaborate and communicate their way of thinking, creating a sense of community. Besides, cognitive science studies recommend that students that do not move (body or mind), end up inattentive. Movement, manipulation, and experimentation allow students to be aware, improve comprehension and memorization (Nesin, 2012). In this scenario, they must be faced with challenges that stimulate them to learn and put them to work with each other, moving. Thus, trails, congresses and gallery walk appear as rich instructional strategies and contexts for those who teach and learn mathematics. A math trail is a sequence of stops along a path, in which students solve mathematical tasks in the surroundings; a mathematical congress consists of presentations of tasks, by students, in an auditorium to their colleagues, that were previously prepared in groups, allowing them to discuss ideas with the audience; a gallery walk is a strategy that allows students to work collaboratively solving tasks, presenting them in posters, located around the classroom/outside, while having the opportunity to share ideas and receive feedback.

We have been carrying out qualitative and interpretative studies with elementary pre-service teachers (3-12 years old), where these instructional strategies are used, with the aim to understand to what extent they contribute to an effective mathematical teaching and learning. Data collection includes observations, written productions, photographic records, and questionnaires. These strategies were accepted by future teachers with enthusiasm and engagement in terms of mathematical contents, transversal abilities and attitudes. They acquired a different perspective about mathematics, and about how they can work/do mathematics without being stuck to a chair, working collaboratively, and learning in significant way. These contexts contributed positively to an effective mathematics teaching/learning, so that participants with different skills/characteristics would feel comfortable participating, highlighting their knowledge and difficulties, which were overcome by the feedback from their peers and teachers.

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DO YOU “BELIEF” IN MATH TRAILS?

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Outdoor mathematics education can be organized by using the math trail method. A math trail is a walk where students work on realistic mathematical tasks outside (Gurjanow et al. 2019). As students discover the world by using mathematics while running a math trail and are thereby highly motivated (ibid.), the repeated use of the math trail method can hypothetically result in a mind shift regarding mathematics.

In a pilot study in fall 2020, 8th graders of a German grammar school (two classes; N=34 complete data) worked on two math trails of 90 minutes each within one week. The students responded a 12-items questionnaire (5-point Likert scale) about their mathematical beliefs three times: one week before, one week after and 15 weeks after working on both math trails. The questionnaire, based on Grigutsch (1996), identifies four main aspects of mathematical beliefs, namely schema, formalism, process and application. Since each three items represent a single aspect, the scores of these items are summed which results in a scale from 3 (strong rejection) to 15 (strong agreement).

In line with Grigutsch (1996), before the intervention the static aspects schema (M=11.3; SD=1.9) and formalism (11.8; 1.5) predominate in the students’ view of mathematics compared to the dynamic aspects process (8.7; 1.7) and application (9.6; 2.2). After running math trails twice, the mean score of the schema aspect decreases significantly to (10.4; 1.8). In the follow-up test 15 weeks after the treatments, the scores (10.2; 2.1) are measured. Both, post-test (g*=0.45; medium effect) and follow-up-test (g*=0.55; strong effect), differ significantly from the pre-test. Between the post-test and the follow-up test, no effect could be observed. The other three aspects formalism, process and application remain almost unchanged in the three test settings.

Mathematical beliefs are seen as (relatively) stable (Liljedahl et al., 2012). Limited to the small case number, we find a decrease in the aspect schema after using the math trail method twice. We therefore hypothesize that outdoor math lessons with math trails could lead to a decrease of the predominant static students’ view of mathematics. This preliminary hypothesis will be investigated in more detail in a study in 2021.

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THE QUALITY OF MATHEMATICS TEACHER EDUCATION AT TERTIARY LEVEL IN UGANDA: IS IT RELEVANT FOR 21ST CENTURY MATHEMATICS TEACHERS?

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Golding and Batiibwe (2019) indicate that mathematics teachers lack the expertise of teaching learners how to solve mathematics-specific content but rather replicate their former teachers' methodology (Batiibwe, 2019). On account of this and with reference to the Systems Theory (ST) (Ashby, 1964), this study aimed to establish the quality of the mathematics teacher educators (MTEs) in government teacher training institutions (GTTIs). It sought answers to the following research questions: what (i) are the qualifications of MTEs (ii) is the curriculum content of the mathematics teaching methods course units and (iii) are the MTEs' pedagogical strategies at the 14 GTTIs?

Using a mixed methods research paradigm and cross-sectional survey and phenomenological research designs, data were collected from all the 24 MTEs in GTTIs through a demography survey, interviews, document analysis and focus group discussions and analyzed using descriptive statistics, categorization and narratives. Findings were: 83% of the MTEs are lecturers of mathematics and not mathematics education; there are varying course outlines, curricula content and materials of mathematics teaching methods course units between GTTIs; none of these has a provision for actual secondary school mathematics content; there is no curriculum structure in most GTTIs; the lecture teaching method is commonly used by majority MTEs; and MTEs use no ICTs and teaching resources other than PowerPoint handouts, and use no content specific software while teaching.

As the Uganda Ministry of Education and Sports (MoES) indicates that "the quality of education largely depends on the quality of teachers" (MoES, 2018, p. 3), it cannot be doubted that in the context of this study, poor in-service mathematics teachers with poor teaching methods stem from the initial mathematics teacher education at tertiary level. The lack of use of actual school mathematics during their training explains why most teachers end up teaching the way they were taught (Batiibwe, 2019) at secondary school. In conclusion, the quality of MTEs is questionable in the context of the 21st century. This implies an urgent need to re-think the mathematics teacher education curriculum.

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HOW DO TIME PRESSURE AND EPISODIC KNOWLEDGE INFLUENCE HOW TEACHERS RESPOND TO STUDENT THINKING?

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Teachers' diagnostic competence includes the ability to identify misconceptions in student solutions and to respond adaptively to these solutions. According to current models of diagnostic competence (e.g., Loibl et al., 2020), teachers need pedagogical content knowledge (PCK) about typical student misconceptions and they need to use this knowledge effectively when responding to student thinking in specific situations. A common requirement for teachers in the classroom is to respond to a student solution under time pressure. Although studies have shown that time pressure can reduce the quality of diagnostic processes, it is not well understood how PCK or episodic knowledge of experienced teachers about similar classroom situations can have a compensatory effect (Putnam & Borko, 2000).

Against this background, we will address the following research questions: 1) What is the influence of time pressure on the relation between PCK and the diagnostic processes? 2) Does the use of episodic knowledge partially mediate this relation?

Data collection will take place in May 2021 and the sample will be comprised of pre-service mathematics teachers and in-service teachers with several years' experience in teaching fractions. For assessing teachers' diagnostic processes, we will use a vignette-based design: Participants will be presented incorrect student solutions to a fraction problem. They are asked to select the best (most adaptive) out of three possible responses to each student and to justify their choice, which will allow us to analyze the episodic knowledge they may have used. A computer-based assessment affords to implement time pressure. Specific PCK will be assessed with a paper-pencil test.

We expect that time pressure is a moderator of the relation between PCK and the diagnostic processes and that the availability of episodic knowledge partially mediates this relation. The results will contribute to a better understanding of the interaction between knowledge facets, situation characteristics and teachers' responding to student thinking.

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STATISTICS BELIEFS OF FRESHMEN IN THE SOCIAL SCIENCES - AN EVALUATION OF QUALITATIVE INTERVIEWS

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BACKGROUND

Domain-specific beliefs have not yet been sufficiently explored in the field of statistics. While in mathematics theories and measuring instruments on beliefs were already developed in the nineties, especially by Törner and Grigutsch (1994), such theories and instruments are still missing in statistics. In order to take a first steps towards a theory, the author has already examined advanced students of the social sciences in a previous study on this topic in focus group methodology (Berens 2019). In that study four groups of beliefs were found. Some students viewed statistics as a system of formulas and rules, others understood statistics dynamically. The last group could be subdivided into those that extract information out of data and those that want to check theory using data. A fourth group saw statistics as a form of systematic description of reality.

METHOD

In order to check whether these initial results are also apparent beyond the limited group of advanced social science students, this study interviewed first-year students. In order to also choose a different methodological approach, individual interviews with fourteen students of different social science majors were conducted at the University of Goettingen. The open answers received were then coded and finally compared to the categories of the first study.

RESULTS

The individual interviews essentially confirm the results of the first study. However, they also help to give the four groups a little more structure. Thus, the perspective of statistics as formulas and rules has certain similarities with the people who look at statistics dynamically but from a theory-based perspective. In a completely different way, there are similarities with those who see statistics as a tool for describing reality. Since the dynamic and data-driven view can be seen as the opposite pole to the formula and rule view, a 2x2 matrix of statistical beliefs results.

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ELEMENTARY STUDENTS’ USE OF SPATIAL THINKING STRATEGIES IN A LAYERED PUZZLE TASK

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Spatial thinking—being able to embed, disembed, and transform shapes—is important in STEM fields, and puzzle-based interventions can help children develop their spatial thinking. Dynamic and static strategies, involving intrinsic and extrinsic dimensions (Uttal et al., 2013), are important in solving puzzles. Dynamic strategies include reinterpretation (e.g., turning, flipping, or reordering pieces) and combinations (e.g., putting pieces together); static strategies include flexible abstraction (e.g., ignoring details) and borrowing structure (e.g., using known combinations; Martin & Schwartz, 2014). However, we need details of students’ use of the strategies and their benefits.

Participants included 25 first graders and 21 third graders from the United States. We investigated students’ spatial strategies using a puzzle from Colour Code by Smart Game. In task 1, we gave students the target design (made of four colored tiles) and asked them to make the target design; in task 2, weeks later, students chose which structure they could borrow to create the target design and created it (see Figure 1).

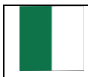
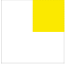
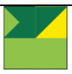
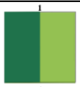
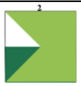
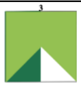
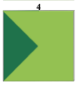
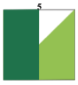
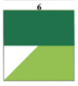
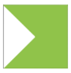
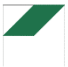
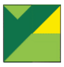
	rectangle		square		starting design	1  2  3  4  5  6 
	pacman		parallelogram		target design	

Figure 1: The four tiles, starting and target design, and structure combination choices.

On task 1, only 15% of students made the target design by reinterpreting or rotating the pacman tile correctly; two-thirds rotated other tiles or reordered them. On task 2, selecting the correct combination was significantly correlated with correctly making the target design $r(44) = .45$, $p < .01$, suggesting that disembedding the combination was associated with being able to embed it in the target design and that students could benefit from support in borrowing structure.

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THE USE OF VISUAL IMAGES IN MULTIPLICATION AND DIVISION IN EARLY YEARS' MATHEMATICS TEXTBOOKS

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Textbooks are used extensively in South Africa to support mathematics teaching in schools. The seeming reliance on textbooks has been exacerbated by Covid-19 and the national lockdown. In contexts where online learning is not possible, learners are required to use textbooks to make sense of mathematics.

Early years mathematics textbooks contain many visual representations and symbols. Arcavi (2003) suggests that visualisation is a sub-conscious cognitive process that involves the creation of a mental image or a physical gesture. Visualisation in mathematics is a cognitive process of transmitting information in an aesthetically pleasing way (Arcavi, 2003). One of the purposes of visual representations in a text is to help non-expert readers to understand what is being said, which is important for young children (Fotakopoulou & Spilitiopoulou, 2008).

The study has been guided by the following research question: What is the nature of the visual representations used to support children's understanding of multiplication and division in early years' mathematics textbooks?

The theory of social constructivism (Vygotsky, 1978) holds the view that social interactions create experiences that facilitate the learning and meaning-making process. The use of "signs" and "tools" (textbooks) are thus central to the mediation process (Vygotsky, 1978). Fotakopoulou and Spiliotopoulou (2008) have developed a framework for analysing visual representations in textbooks. The framework focuses on the type of image, the relation to content, the relation to reality, and the function and dimension of the visual representations. These aspects of the framework will be used to analyse visual representations in nine different early years' mathematics textbooks with a view to understanding how the visual images support young children's understanding of multiplication and division.

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ESTABLISHING AN “EQUITABLE ACCESS” AND “ROBUST UNDERSTANDING” LEARNING ENVIRONMENT

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In Taiwan, “Curriculum Guidelines of 12-Year Basic Education” are implemented starting from August 2019. For the domain of mathematics, schools and teachers are asked to get ready for implementing the adjusted guidelines, where more real-life scenarios and hands-on activities will be merged into the learning process for promoting students’ robust understandings, problem-solving skills, and positive learning attitude and interest as well as providing equitable access to the mathematical content in every classroom.

On behalf of supporting this fundamental shift, creating classroom environments that cultivate the powerful mathematical thinkers envisioned by the Directions will both require a significant change in teachers’ understandings of what makes for powerful mathematical thinking and how to sustain it, alongside the development of more powerful teaching strategies for reflecting on and improving them. Helping students engage in mathematical sense making requires developing a perspective on mathematics instruction consistent with the Directions, and a facility with the tools for creating instructional contexts that assist our students develop those understandings.

Emerged from the question of how to describe what matters for equitable access and robust learning in mathematics classrooms, the Teaching for Robust Understanding (TRU) framework (see Schoenfeld, 2014; Chang & Schoenfeld, 2018) is employed to act upon effectively. In this study, the TRU framework provides us a powerful insight on how to analyze the teaching and learning process for future improvements. Therefore, the purpose of this study was to exhibit how an elementary mathematics teacher employed multiple strategies for advancing her students’ learning through the lens of the TRU framework with a focus of the two dimensions—“equitable access” and “agency, ownership, and identity (AOI)” along with the remaining four dimensions, where these findings were drawn from a qualitative case study: Instead of only following the instructional sequence of the textbook, MT re-organized the learning content into 8 class periods, where plenty of real-life contexts were imbedded in her teaching process. In this presentation, we focus on the research lesson of “2nd class period”, where one question was proposed as the main task that students work individually first and then present their answers and discuss in their small groups. Video clips are going to be provide to show how her students equitably engaged in the learning process of the concept “ratio” as well as manifest their AOI while learning mathematical concepts.

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INVESTIGATING DIVISION CONCEPTS AT ENTRY TO SCHOOL

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It is often assumed that young children have no concepts of division before they are formally introduced to division at school. Our earlier research showed that many children achieve early multiplicative reasoning before it is formally taught (Cheeseman et al., 2020). The research question we sought to answer here was: What concepts of division do young children develop prior to school instruction?

We conducted a teaching experiment study (Steffe & Thompson, 2000) with 21 children of 5-6 years of age who were 5 months into their first year of school in Australia. A dancing game elicited thinking about equal-grouping (composite) thinking that is at the core of multiplicative thinking. When the music stopped, a number would be called and everyone would get into a group of that number. In addition to classroom-based observation/video data, we analysed student's drawn responses to a pencil-and-paper assessment protocol (Streit-Lehmann, 2019). The assessment consisted of six worded problems, three quotition division situations where children had to arrange objects into equal groups and count the groups; three partition division situations where children had to share all objects equally. Results indicate that many children (83%) could divide objects into equal groups and some could visualise division situations involving groups of a given size. Further, based on a detailed analysis of the responses of six children, results show that children could interpret stylised diagrammatic partitive (75% correct) and quotitive (100% correct) division contexts and add to diagrams to represent their solutions to worded problems. Our research shows that some young children develop concepts of division prior to school instruction in meaningful contexts. Furthermore, many young children can interpret, and successfully complete, iconographic representations of partition and quotition worded problems. These findings make a new contribution to the field and warrant further study of young children's division concepts.

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CONCEPT IMAGES OF CUBIC FUNCTIONS OF HIGH SCHOOL STUDENTS IN TAIWAN

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To go with the international stream of preparing students mathematical abilities required in 21st century, the new mathematical curriculum in Taiwan changed the contents of the topics pertinent to functions from drawing graphs by hands using the knowledge of calculus to drawing graphs using technology and observing their characteristics. Tall and Vinner (1981) indicated that students often evoked concept images rather than concept definitions when solving problems and the concept images could be different from those taught by their teachers. This study aims to explore students' concept images of cubic functions after learning the contents designed according to the new curriculum.

A questionnaire with dichotomous items and open-ended items was administered on 73 tenth grade students in two classes (A: 36; B: 37) from two senior high schools. A content analysis was conducted to analyse the responses to the open-ended items.

Regarding what cubic functions are, three types of concept images were aroused by the students. A total of 63% students aroused the graphical type of concept images and 51% aroused the algebraic type, while only 10% evoked the word type. This reflected teachers' emphasis on the graphs of functions and operational representations in their teaching. Furthermore, 49% of the students aroused only one type of mental images, 33% possessed two types, and only 3% could arouse all three types. Most students may lack the flexibility to evoke feasible concept images while problem solving. A total of 70% and 57% of the students had graphic type concept images in class A and class B respectively. Among them, 44% of the students in class A provided four types of graphs of cubic functions (see Figure 1), and 28% provided only one type. In contrast, only 19% in class B provided four types of graphs, and as high as 57% provided only one type. The possible reason may relate to the different teaching approaches of the two teachers. Teacher A let students manipulate the mobile phone to draw the graphs by themselves while Teacher B only demonstrated the graphs by himself.

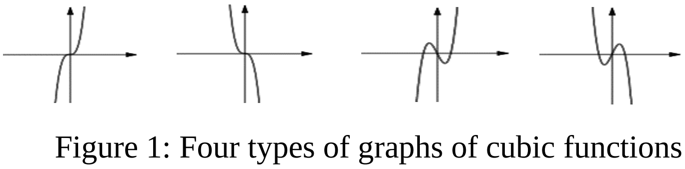

Figure 1: Four types of graphs of cubic functions

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PERSPECTIVES OF TAIWANESE HIGH SCHOOL STUDENTS ON THE USE OF CALCULATORS IN MATH LEARNING

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Taiwan's new mathematics curriculum was launched in 2019, which first stipulates the use of calculators in the teaching and learning of some specific topics at the secondary school level. The ability of quick and fluent hand calculation has been emphasized in Taiwan for a long time. Many mathematics teachers and experts have doubts about this policy. Aligning with student-centered teaching ideas is critical to listen to students' opinions. This study intends to investigate high school students' perspectives on the use of calculators in mathematics learning.

This study included two stages. In the first stage, a qualitative pilot study using open-ended questions was conducted on 200 high school students to obtain their opinions regarding what teaching behaviors they like/dislike or consider helpful/unhelpful to their learning in a calculator-integrated math class. A content analysis of students' responses and literature review was performed to obtain the items pertinent to integrating calculators in learning mathematics. The items composed the questionnaire for the second stage using a 6-point Likert scale (strongly disagree to agree strongly). The data used in the present study were from the responses of 470 10th graders. The percentages of their agreement on the items were calculated.

Regarding what teachers should do when integrating calculators in a math class, more than 90% of students considered that teachers should provide students opportunities to operate the calculators by themselves, give students sufficient time to try and think, arrange proper exercises for student practice, and should still introduce fast hand calculation tricks. Regarding how integrating calculators can help math learning, the items with the percentages of agreement higher than 90% were using calculators to verify the answers' correctness, increase accuracy, and save unnecessary calculation time. These perspectives reflected Taiwanese students' attempts to perform well in the examination. More than 70% of students agreed that the use of calculators helped them focus on mathematical thinking rather than complex calculating procedures, make conjectures by observing a large number of examples, get a better sense of the answers, and make tests with numbers in real-life to see values of learning math. The students also agreed that the integration of calculators could help them conduct learning behaviors that can develop their mathematics abilities and positive dispositions.

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AN EXPLORATION OF THE TEACHING OF MATHEMATICS DURING THE COVID-19 LOCKDOWN IN A RESOURCE-CONSTRAINED ENVIRONMENT

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The COVID-19 outbreak was declared a global pandemic in March 2020 by the World Health Organization. This declaration was followed by most countries going under lockdown. Correspondingly, schools and universities suddenly closed in most parts of the world, and the need for emergency remote teaching and learning (ERTL) of mathematics emerged. The COVID-19 pandemic was a new event, and most teachers at South African public schools, which are ordinarily resource-constrained, had not performed online teaching before the pandemic (Chirinda et al., 2021). Before the COVID-19 pandemic, mathematics was taught face-to-face in South Africa and most countries around the world. Learners physically met their teachers in the classrooms and explored mathematical concepts. In this regard, using a qualitative methodology, the study engaged a social justice framework to explore how mathematics teachers at South African public secondary schools responded to the call for ERTL during the COVID-19 lockdown. The following research question was formulated: How did South African secondary school mathematics teachers in a resource-constrained environment respond to the call for ERTL during the COVID-19 lockdown?

Twenty-three Grade 12 mathematics teachers at various public secondary schools in Gauteng, South Africa, participated in the study. The teachers were selected through snowball sampling. Snowball sampling allowed identified participant teachers to nominate other teachers implementing ERTL of mathematics in resource-constrained environments in South Africa during the COVID-19 lockdown. A Google-generated open-ended questionnaire and follow-up telephonic interviews were used to collect data. Thematic analysis was used to analyze the data.

The findings were that ERTL of mathematics in resource-constrained environments foregrounded issues of inequality in the South African education system that must be dealt with urgently. The findings from the study add to the mathematics education body of knowledge by advancing our understanding of the challenges faced by teachers in resource-constrained environments when implementing online learning.

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3D DECOMPOSITION AS A SPATIAL REASONING PROCESS: A WINDOW TO 1ST GRADE STUDENTS' SPATIAL STRUCTURING

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3D decomposition is considered a spatial reasoning process (Davis et al., 2015). Spatial structuring is a form of abstraction that creates mental models of shapes' structures (Battista & Clements, 1996). Since early grades, both play an important role in understanding shapes' structures and in learning how to manipulate them flexibly and fluently. 3D shapes have a strong presence in early grades, yet there is still little research about the way students learn their structures. We seek to answer the following questions: How do 1st graders decompose 3D shapes? How are these decompositions related to spatial structuring?

This research is part of an on-going design-based research focused on a 1st grade class's spatial structuring, while exploring 19 tasks concerning 2D and 3D shapes and their representations. Here, we analyse 1st graders' drawings. Local and global structuring were considered as main progression levels in our analysis.

Results show that students decompose shapes by establishing local relationships with flexibility, such as recognizing repeatable or symmetrical composites, without yet coordinate them to form the whole (by omitting or duplicating components); or global relationships, when students coordinate equal or quasi-equal composites, relating them with the whole, flexibly and fluently, and apparently, have a previous mental model that represents the structure of the shape. Students' levels of structuring seem to be influenced by the features of each 3D shapes. Further results will be discussed in detail.

Acknowledgment

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SCHOOL MATH AND MATH IN VIDEOGAMES: HOW DO STUDENTS PERCEIVE IT?

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The heart of many mathematical curricula of this era states that pupils should especially develop their ability to think in order to be able to use mathematics creatively in different situations. This idea is also at the centre of the Slovak curriculum. However, the real situation in Slovakia does not correspond to this. Pupils are still often taught by traditional methods, that only lead to the development of procedural skills and they do not see the connection between mathematics and other activities in their lives. Papert (1980), on the other hand, emphasized that children are successful in learning mathematics if they find it as part of other activities they do in their own interest. One of the ways to meet mathematics in situations where we really need it is in video games. Gee (2003) has pointed out that video games can teach us a lot about learning. In our study, we explore what video games can teach us about how high school students perceive mathematics. The aim of our study is to find out how high school students in our country with different attitudes to mathematics perceive that they are encountering math while playing games and what they identify as mathematics in games. We are also interested in how they perceive math in games compared to math in school. We chose a qualitative approach. We have done a collective case study involving seven participants. These participants were purposefully selected. They were students who played video games frequently and had mixed feelings about mathematics. We carried out semi-structured interviews with them. We conducted data analysis with pattern coding. The results showed that the participants perceived that they encounter mathematics while playing games in many situations. Above all, they perceived that they needed to use strategic and logical thinking in various types of video games, use combinatorics to fill inventory and improve abilities in RPG, and think about the probability while playing League of Legends and World of Warcraft. Participants perceived that math in school is very different from math in games. They expressed the opinion that in the game it is mainly about thinking, while in mathematics lessons it is mainly about counting and applying the right rule. They think that what they do now in mathematics lessons is unnecessary. They can't imagine they would ever use it in-game or in real life.

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THE NEED FOR PROOF IN SIXTH GRADERS' MATHEMATICAL DISCOVERY PROCESSES

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Proving activities are of great importance in mathematics (e.g., Davis, Hersh & Marchisotto, 2012). Various studies, however, point out students' lack of need for proof (e.g. de Villiers, 1990). To emerge students', need for proof, several strategies have been proposed (Zaslavsky et al., 2012). As Schoenfeld (1985) stated, the transition from conjecturing to proving, initiated by a need for proof, is crucial for the underlying mathematical process. Contributing to this discussion, discovery learning environments have been developed to analyze students' need for proof. Therefore, the research questions are:

- To what extent is the reconstruction of students' transition from conjecturing to proving possible and how are these processes initiated?
- Which aspects of the mathematical discovery process facilitate the transition?

The sample comprises interviews with twelve sixth graders. These were conducted as semi-structured interviews with low interviewer's impact to affect the learners' process of discovery minimally. The students worked as a tandem on an explorative task about sums of successive natural numbers. The interviews were analyzed via qualitative content analysis (Mayring, 2000).

As a result, two levels of (meta-)cognitive needs could be identified. Both of them initiate the transition from conjecturing to approaching a proof: The *Need for Questioning* and the *Need for Reasoning*. Underlying characteristics and different types of intra- and interpersonal aspects leading to those needs are elaborated. In the presentation, their impact and benefit for students' need for proof will be discussed.

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NEGOTIATING TENSIONS IN LANGUAGE DIVERSE MATHEMATICS CLASSROOMS

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Most mathematics classrooms world-wide exhibit language diversity. In some cases teacher and students speak different languages and must find ways to communicate in a common tongue. In other arenas, teacher and students may share a common 'school language' while home languages and informal ways of expressing observations differ. In every classroom, students and teachers must together navigate every day, school and technical register in one or more language (Prediger & Wessel, 2011). As language diversity has increased, so have calls for increased verbal engagement and sense-making on the part of students. This call for more meaningful participation means that the importance of language socialization events and the tension of navigating across languages and registers also increases (Barwell, 2020).

In this project, I sought to identify ways teachers in Dutch schools adapt mathematics instruction to meet the needs of an increasingly language-diverse population. I visited 34 mathematics classrooms in which home language diversity was present. The schools fell broadly into two categories. Some held language acquisition (either Dutch or English) along with learning mathematics as an explicit goal. Others held mathematical proficiency alone as an explicit goal. In each classroom, I observed for most of a school day, interviewed students, and interviewed the teacher about language diversity in relation to mathematics instruction. As I analyzed my observations, I found that where both language and mathematical learning were goals, teachers tended to invite rather than compel participation.

I found these settings resembled at times what Barwell (2020) described as language positive classrooms. During moments of tension as students and teachers labored to understand each other, home languages were accepted, students tried out ideas in informally, and teachers attended to gestures to understand students' meaning. Teachers generally supported students with explicit mathematical genres and provided structure for students' mathematical conversations with each other. In the oral communication, I will discuss and interrogate these language socialization patterns and their implications for language-diverse mathematics classrooms in detail.

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RATIONAL NUMBER HYPOTHETICAL LEARNING TRAJECTORIES IN MIDDLE YEARS' MATHEMATICAL TEXTBOOKS

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This research seeks to identify the implicit hypothetical learning trajectories in selected South African mathematics textbooks, with a view to investigate how these texts sequence the teaching of rational numbers, specifically, equipartitioning. These texts set out the standardized curriculum, which in turn sets the standard and pace of work that is be followed in the classroom.

Confrey and Maloney's (2010) rational number hypothetical learning trajectory on equipartitioning serves as the tool for analysing the promoted teaching and learning trajectory for rational numbers. Nine mathematics textbooks for middle years' students (Grades 4-6) were analysed by using the proficiency levels in this trajectory and additionally, by plotting this out against the official curriculum standards. The goal in equipartitioning is to produce equal-sized groups (from collections); equal-sized parts (from continuous wholes); or equal-sized parts from combinations of wholes and parts (Confrey, Maloney, Nguyen, Mojica & Myers, 2009). This framework shows how children's skills with equal sharing (equipartitioning) are hypothesised to develop over time (Confrey & Maloney, 2010).

Zhang, Clements and Ellerton (2015) argue that the teaching and learning of rational numbers is a hurdle in primary school because learners transfer their understanding of whole numbers to rational numbers. Through this document analysis, this research aims to reveal how these texts, and the South African curriculum, compare to Confrey and Maloney's (2010) hypothetical learning trajectory, with a view to better understand how the pacing and sequencing promoted in the curriculum and classroom mathematical texts. This will serve as the basis for reflection on how to effectively support the development these concepts in the classroom over time.

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HOW ELABORATED SHOULD ELABORATED FEEDBACK BE? A CLOSE EXAMINATION OF PREFERENCES AND IMPACT

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Elaborated feedback (EF) is recognized as a superior form of feedback, in terms of its impact on learning, over simple feedback (SF) which simply tells the student whether they are correct or not (Shute, 2008; Van der Kleij et al., 2015). However, the design of effective EF is seldom studied. Specifically, very few studies have compared multiple EFs with different levels of elaboration, and the results of those studies are contradicting.

This study aims to better understand the impact of EF's level of elaboration on learning, as well as on students' perception of the feedback's helpfulness, in digital learning environments in mathematics.

We took a mixed approach. In the quantitative phase, 27 university students in Calculus courses for non-mathematicians completed an online practice module about continuity. The module included pre- and post-test questions, as well as an intervention where students were randomly allocated to groups which differed by the type of feedback: short EF, long EF, and SF. Impact on learning was then measured by pre-to-post difference, as well as by more immediate measures, like success on a reattempt [0/1] when the first submitted solution was wrong. Both EF groups outperformed the SF group in all measures. The long EF group outperformed the short EF group in pre-to-post difference (15% improvement vs. 9%), while the short EF group outperformed the long EF group in the immediate measures (e.g., 83% reattempt success vs. 56%).

In the qualitative phase, 5 students from the same cohort were observed during—and interviewed after—interacting with an online module about extrema of two-variable functions, where all types of feedback were presented at once. The results suggest a distinction based on the stage in which feedback is provided: elaboration should be minimal when feedback is provided while students still work on the task (e.g., when they submit an incorrect answer and are asked to reattempt); more elaboration should be encouraged for feedback provided once the work is complete (e.g., when they submit a correct answer or when there are no more attempts).

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M@T.ABEL 2020 PROJECT: A TEACHER TRAINING PATHWAY DURING THE COVID-19 HEALTH CRISIS IN ITALY

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The M@t.abel 2020 project developed by the international centre for innovation in the educational field Future Education Modena (FEM) supported Italian teachers during the COVID-19 health crisis in 2020 suggesting activities based on mathematics laboratory teaching methods (Anichini et al., 2004) that teacher used with their classes also via distance learning. We present the results of an open-ended questionnaire administered to 293 teachers involved in the project to analyse the impact of this project in terms of teacher training. We analyse the results of the questionnaire using the lenses of a Mathematics Teacher's Specialised Knowledge (MTSK) developed by Carrillo-Yañez and colleagues (2018). MTSK model considered Mathematical Knowledge (MK) and Pedagogical Content Knowledge (PCK) as each divided into three subdomains. The three subdomains of MK are: Knowledge of Practices in Mathematics (KPM), Knowledge of the Structures of Mathematics (KSM) and Knowledge of Topics (KoT). The subdomains of PCK are: Knowledge of Mathematics Teaching (KMT), Knowledge of Features of Learning Mathematics (KFLM) and Knowledge of Mathematics Learning Standards (KMLS). The research questions of this particular study are: (i) Did the teachers involved in the M@t.abel project recognize an enrichment of their mathematical knowledge? (ii) if so, which components of the MTSK model were influenced by the project?

It emerged that teachers recognised an enrichment in PCK sub-domains, particularly in KMT and KFLM; improvements in MK subdomains were also registered, in particular for what concerns KoT: many teachers explained that the project and reflections on the specific activities helped them to clarify some mathematical items such as perpendicular and angle. Finally, even though no question was designed specifically to investigate beliefs, some of the teachers stated that the project helped to gain confidence in changing their way of teaching mathematics, influencing their beliefs about maths teaching and learning.

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MEASURING DIGITAL COMPETENCIES OF PRE-SERVICE TEACHERS – A PILOT STUDY

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The role of technology grows and continuously advances; thus, it is important that the technology-related knowledge of educators constantly evolves as well. To better understand the respective knowledge of pre-service teachers, we translated and adjusted the self-assessment (SA) instrument by Valtonen et al. (2017), based on the TPACK framework (Mishra & Koehler 2006), for mathematics contents. In addition, because of the limitations and controversy of Likert scale SA instruments (e.g., summarized in Safrudiannur, 2020), an external assessment (EA) focusing on the TCK, TPK, and TPCK elements of the framework was developed by a group of experts and experienced in-service teachers. From a technology perspective the EA focused on spread-sheets and contains for example TCK items requiring to explain the use of the $ symbol in spread-sheets or open-ended TPCK items inquiring the advantages and disadvantages of using spread-sheets in mathematics lessons. The SA and the EA instruments were piloted with three groups of bachelor degree mathematics pre-service teachers for lower secondary schools (grades 5–10) who participated in a third semester seminar with asynchronous and synchronous learning elements of digital technology, including spread-sheets, for use in teaching. Measures were taken at the beginning (pre) and three month later, towards the end of the seminar (post). Firstly, using Confirmatory Factor Analysis, we were able to show that the pre and post results of the SA are closely aligned with the findings by Valtonen et al. (2017). Secondly the scale for the EA items for TPK, TCK, and TPCK are reliable, with $\varkappa = .98$ for closed items and $\varkappa = .85$ for open items. Thirdly, the pre and post increases of the EA were statistically significant with effect sizes ranging from low to high, r=.3-.6 (one-sided Wilcoxon tests). In addition, the scale of the open-ended EA items showed advances in the quantity and quality of the used arguments. Based on the results, future refinements of the EA instrument are planned, with the aim to develop a comprehensive and reliable instrument.

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USING ICONIC REPRESENTATION FOR PROPORTION

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While many studies point out the positive impact of iconic representations on percentage tasks (Walkington et al., 2013), so far none of those studies explores which type of iconic representations students choose themselves to express percentage. Therefore, a qualitative case study was conducted to survey the students' approaches. The research questions are: (1) What types of representation can be reconstructed? (2) Which differences can be identified between those types of representation?

Based on the Design Research approach (van den Akker et al., 2006), tasks have been evolved in four cycles. The sample comprises semi-structured interviews (Edwards & Holland, 2013), with twelve pairs of seventh- and eighth graders. At the beginning, the interviewer presented the fractions $\frac{1}{5}, \frac{1}{20}$ or $\frac{15}{100}$. The students should decide which fraction equals 5%. Afterwards, they were asked to represent the selected fraction visually followed by the task to show its equality with 5% in the chosen representation. The interactionist approach (Voigt, 1995) is used for analyzations.

As a result, three characteristics aspects based on the reconstructed representations are reconstructed and compared. The first characteristic differentiates students' representations into 'percent as fraction' or 'percent as a number' (Parker & Leinhardt, 1995). The second characteristic shows whether the representations underlie a static or dynamic approach. The last characteristic compares the representations regarding either a multiplicative or additive approach. In the presentation the characteristics will be discussed in detail.

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THE MATHEMATICAL THINKING OF CHINESE FILIPINOS THROUGH THE LENS OF LINGUISTIC RELATIVITY

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Linguistic relativity posits that language influences thought. Pavlenko (2014) identified five mediators of mathematical thinking in multilinguals, namely: (1) first language (L1) advantage, (2) language-of-instruction (LOI) advantage, (3) language dominance, (4) language advantage, and (5) language of encoding advantage. This presentation reports from a pilot study connected to the lead author's ongoing PhD project involving Chinese Filipinos who were taught K-10 mathematics in both English and Mandarin, in separate classes. Inspired by Prediger et al. (2019), the study assumes that teaching in two LOI's provides possibly differing mathematical conceptualizations to its learners; hence, the pilot study looked into whether the mediators of mathematical thinking can explain linguistic preferences.

Ten Chinese-Filipino university freshmen (ages 18-19) who completed their K-10 mathematics in both English and Chinese curricula participated in the pilot study. They filled out language profile and preference surveys presented bilingually and were interviewed individually in the language of their choice. The results show stronger preference for English in thinking and working mathematically. In terms of L1 advantage, most learned English at home whereas Mandarin only later in school. Additionally, the participants echoed an LOI advantage due to their extensive exposure to more English-taught subjects. However, in terms of language dominance, most participants still multiplied using Hokkien, a Chinese dialect and L1 of most Chinese Filipinos, but did other operations in English. They attributed this dominance to their memorization of the multiplication table in Hokkien at an early age. Lastly, in terms of language of encoding, most opined that it was easier to remember mathematical terms in English. However, some participants were observed to respond based on the language of the prompt. For instance, they grouped several digits by fours when prompted in Chinese, despite involving an English-speaking person in the hypothetical scenario. In summary, the findings demonstrate the use of the mediators in possibly explaining linguistic preferences. Further details shall be discussed in the presentation.

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STUDENTS' ATTITUDES TOWARDS PROVING IN A UNIVERSITY MATHEMATICS COURSE

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In this study, we analyzed what kind of attitudes first-year university mathematics students have towards mathematical proving, and how these attitudes evolve during an introductory mathematics course. We also investigated how female and male students' attitudes towards proving differed. To achieve this, we developed an instrument that measures attitudes that concern proving. It is known that transition to proving in tertiary mathematics education is difficult for new students (e.g., Selden, 2011), and their attitudes may be a key factor affecting how they approach proving.

The context of the study is the course Introduction to University Mathematics taught in a research-intensive university in Finland. The course is intended to bridge the gap between secondary and tertiary mathematics. The topics of the course include basic set theory, functions, logic and proving. The participants of the study are 75 mathematics students taking this course. Students' attitudes were measured with the new instrument developed for this study. The instrument was based on the ATMI questionnaire that measures mathematics attitudes (Tapia, 1996) and a questionnaire that measures proving related self-efficacy (Iannone & Inglis, 2010). Factor analysis revealed that the instrument measures four attitudes: self-efficacy, anxiety, appreciation, and enjoyment.

The participants answered the questionnaire in the beginning and at the end of the course. We used repeated measures analysis of variance to study what kind of attitudes male and female students had in the beginning of the course, and how their attitudes developed during the course. Our results imply that the course improved proving-related self-efficacy ($p < .01$) and reduced anxiety ($p < .01$). Male students reported higher self-efficacy ($p < .05$) and enjoyment ($p < .05$) than female students. Further studies are needed to understand the causes behind these improvements and differences, as well as to what extent they affect study success.

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EXPLORING AVENUES OTHER THAN HIGH STAKES MATHEMATICS TESTING FOR TEACHERS

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Recently the Ministry of Education proposed the Mathematics Proficiency Test (MPT) as a requirement for all newly licensed teachers (OCT, 2020). This type of test runs directly counter to the research that points to something other than computational knowledge being effective for teaching mathematics (e.g., Holm & Kajander, 2020).

Our research looks at the results from a mixed method survey that was administered to our teacher candidates following the pilot writing of the MPT. The survey consisted of 21 questions with 11 open answer responses to explore the thoughts of the teacher candidates before and after taking the test to better understand their feelings. Thematic analysis (Braun & Clarke, 2006) was used on the open responses, and is the sole focus of this presentation. We had a 38% response rate from our total participant pool with a total of 50 responses.

Results from the survey, unsurprisingly, indicated that majority of participants had feelings of anxiety, nervousness, doubt, fear, uncertainty, and panic prior to taking the MPT. We focus in this research report on the recommendations from the prospective teachers of what would help make them effective teachers of mathematics. The prospective teachers pointed to a need for more specialized understandings and a focus on *how* they teach as being important for raising mathematics scores of students. Recommendations included more math for teaching courses and professional development as being more beneficial for improving teaching than tests like the MPT.

This research is important since teacher candidates will be classroom teachers who teach mathematics, and their insights can inform instructional practices. The amount of stress and anxiety that was caused from taking the MPT may also have a detrimental impact on the individuals who are required to take it, so other ideas are needed to make lasting changes in mathematics teaching.

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USING MATHEMATICS GROUNDING ACTIVITIES TO TEACH DECIMAL: A TALE OF A THIRD GRADE CLASSROOM

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Orchestrating hands-on mathematics games in limited classroom time is a complex undertaking. Teachers not only need to master the progress of games and courses, but also facilitate students to participate in the classroom, experience games, and understand the mathematical concepts. An efficient way to motivate students to learn mathematics is to provide interesting and meaningful activities that offer students not only fun but also meaningful learning (Lin & Chang, 2019). To understand how the teacher combined hands-on mathematics games in teaching practice to facilitate students to think and construct the concept of decimals, we conducted a case study.

In this study, we applied a mixed method. Data collection included observation videos, semi-structured interviews in five lessons, and a learning attitude scale. We used one of the mathematic grounding activity (MGA) modules which is effective to facilitate students' cognitive and affective engagement in mathematics (Lin, Wang & Yang, 2018). We used social interaction perspectives to analyze teaching practices. The results show that teachers' flexible use of questions, prompts, guidance, and peer assessment can help students express answers to tasks and clarify concepts. The specific manipulate of the game helped students explain the problem-solving process and hold a high degree of interest in learning. Also, timely monitoring and use of social mathematics norms help students focus on hands-on mathematics games, especially young learners.

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ALGEBRA DISCOURSE IN MATHEMATICS AND PHYSICS TEXTBOOKS FOR UPPER SECONDARY SCHOOL

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For many students, algebra is one of the most difficult areas of mathematics, and limited algebra knowledge, particularly concerning algebraic symbols, can hinder students' success in other areas, such as physics (Pospiech, et.al., 2019). In a Swedish context, algebra and algebraic symbols are usually introduced in grades 7-9 (age 13-15), and are then taken for granted in grades 10-12, also in physics. Thus, it is important to identify similarities and differences in how algebra is taught and used in the different contexts of mathematics and physics. This pilot study, focusing on the use of algebraic symbols, is part of a larger project contributing to a more holistic view of students' algebraic knowledge in different parts of the educational system. We take a discourse perspective, where the algebraic discourse is characterized by the word-use and visual mediators (e.g., symbols), among other things (Sfard, 2008).

We analysed common Swedish textbooks for the first physics course and the first mathematics course at upper secondary level (grade 10). Content of both textbooks was categorized with respect to, for example, symbols that were used (e.g., Latin or Greek letters), number of different and similar symbols in symbolic expressions, words used to address (part of) symbolic expressions, and overall mathematical context of the symbolic expression (e.g., calculation with a fixed value or derivation of one expression from another). Preliminary results, here delimited to differences, show that on average it is a greater number of different symbols in expressions in the physics textbook compared to mathematics. Calculations with specific values are more common in physics, whereas it is more common that algebraic entities relate to each other in mathematics. In mathematics, more than half of the symbolic expressions are never referred to using words, while the same is true in physics for 18% of the expressions. Commonly used physics words in the mathematics textbook were *time* and *distance*, while *time* was not common in the physics textbook, when referring to symbolic expressions. These differences imply that students meet different algebra discourses in mathematics and physics. Thus, it can be hard for them to identify these discourses as "the same mathematics". By being aware of these differences, teachers can facilitate students' use of algebra, and in the long run, students' learning in both mathematics and physics.

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DEVELOPMENT AND PROGRESSION IN STUDENTS' EXPERIENCE OF FRACTIONS AND PROPORTION

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A basis for students' understanding of and work with fractions, is their apprehension of the fundaments of algebra. Moreover, in order for them to understand and operate with ratio and proportion, they need a thorough understanding of fractions and how to operate with fractions (Behr, Harel et. al., 1992). This is not an easy task. For students, it demands a long-term process of developing adequate knowledge. For teachers, it requires a thorough knowledge of the mathematics they teach (Ma, 1999), what Shulman (1987) calls subject matter knowledge Also other researchers, like (Ball, Bass & Hill, 2004) emphasize the importance of teachers' subject matter knowledge. In a recent study we followed the teaching and learning of fractions and proportion with focus on how students develop their knowledge from grade 4 to grade 8. Another focus was on teachers' ability to identify the object of learning and its crucial aspects (Marton, 2015).

The study includes 25 lessons. The data were mainly collected by by video-taping lessons and interviewing teachers and students, but also by pre- and post-tests. A qualitative analysis of the date shows that the students often failed in understanding crucial aspects of fractions and proportion and for that reason they made systematic mistakes later on. One reason for this appeared to be teachers' lack of subject matter knowledge and an insufficient focus on the object of teaching. This became most evident in grade 8, where students' misconceptions of fractions and extending of fractions, forced the teachers to introduce procedural methods like cross-multiplication. However, such teaching methods caused still more confusion and new misconceptions.

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INITIAL TEACHER EDUCATION PRACTICES FOR PREPARING LANGUAGE-RESPONSIVE MATHEMATICS TEACHERS

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Language-responsive mathematics teaching concerns meeting multilingual students' needs and supporting their languages for developing their mathematical understanding (Lucas & Villegas, 2013; Prediger, 2019). With many pre-service teachers (PTs) being unprepared to meet students' needs, teacher educators (TEs) are challenged to adapt their practices to support PTs deal with issues of teaching and learning mathematics with respect to multilingualism. In this study, I aim to provide empirical insights on initial teacher education practices for supporting PTs in language-responsive mathematics teaching. The study is part of an ongoing participatory action-research project, where I investigate my practices as a TE to prepare PTs for language-responsive mathematics teaching. For this study, I focus on practices related to "identifying classroom language demands" (Lucas & Villegas, 2013) in mathematical modelling activities. Modelling is context of the study as curriculum content, in the mathematics course for PTs of grades 1-7 where the data were collected.

From a content/activity analysis of three audio-recorded lessons, two themes of practices were identified: teaching about supporting communication rather than single language use; and teaching about supporting multimodality related to content-specific mathematical ideas. These practices were associated with PTs' actions of talking, noticing, planning, and applying language-responsiveness. For instance, the TE engaged PTs in actions of noticing by showing examples of students' use of content-specific language of subtraction in a modelling activity in a school classroom.

The study continues with recommendations for further changes of these initial practices and alternative opportunities for improvement. For example, the TE could engage PTs in more explicit actions of talking about language-responsiveness, as well as in further actions of reflecting upon their experiences of language-responsiveness. The findings can potentially support TEs to attend to systematic practices and face possible constraints. Therefore, the study contributes to insights on improving teacher education practice for the language-responsive mathematics teaching preparation of future primary school teachers.

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PERCEPTIONS OF BIG IDEA OF EQUIVALENCE AMONGST MATHEMATICS TEACHERS IN PRIMARY SCHOOLS

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Charles (2005) defined a "Big Idea as a statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole" (p.10). The revised school mathematics syllabuses for primary schools in Singapore (MOE, 2019) reinforces that Big Ideas are central to mathematics as they connect ideas coherently from different strands and levels thereby facilitating a deeper and more robust understanding of individual topics in mathematics. The revised syllabuses list 6 big ideas (Notations, Diagrams, Proportionality, Models, Equivalence, and Measures) for primary schools.

Presently there is a concerted push towards teaching for Big Ideas in mathematics in Singapore schools. A research study, Big Ideas in School Mathematics (BISM) is presently underway in Singapore and a part of it is on professional development (PD) of primary school mathematics related to the enactment of Big Ideas in their mathematics instruction. Research has documented that teachers' lack of relevant content knowledge of Big Ideas in mathematics translates into their lack of explicit attention to Big Ideas underpinning mathematics taught in schools and results in developing isolated compartments of mathematical knowledge in their students (Askew, 2013). The PD of the mathematics teachers began with two introductory sessions. The first engaged them in working through mathematical tasks that involved the big idea of equivalence and during the second they worked in their grade level groups exploring, for a topic they planned to teach in the coming weeks, episodes that would illuminate the big idea of equivalence during their instruction. From the episodes that teachers planned it was evident that teachers were cognizant of equivalent relationships of mathematical objects such as units of measure and space. However, they did not in the past deliberately emphasize such relationships and draw on multiple solution paths for a given mathematical task to engage students in harnessing the "power of the big idea of equivalence" during their instruction.

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YOUNG CHILDREN’S USE OF PREVIOUS MODELLING PROCESSES IN ADVANCING DATA MODELLING

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The advent of big data has led to renewed research attention on children’s *data modelling* (DM; Lehrer & English, 2018). The DM processes begin with posing questions and proceed with the structuring, representation, and interpretation of data to guide decision making and inferences. However, limited research exists regarding the aspects of these processes. Examples of earlier modelling processes and known models are often used as foundations for tackling new modelling practices, in terms of the *emergent modelling* perspective from a *model of* a situation to a *model for* reasoning processes in other situations (Gravemeijer, Cobb, Bowers, & Whitenack, 2000). This study utilizes this theoretical framework to address the following question with the aim of clarifying the dynamic aspects of young children’s DM processes: *How do young children use previous modelling processes in structuring and representing data?* A case study of two children’s data-display progresses was conducted as part of a teaching experiment for 2nd graders (aged 7–8) (Kawakami, 2018). In the experiment, the children completed a task of graphing the distribution of lost milk teeth on tablet screens before and after a two-day lesson sequence, during which they collaboratively experienced graphing of other distributions. This study primarily analyzed the video and audio transcripts from the target children’s on-screen activities in the pre- and post-tasks.

The findings indicate that the two target children were able to produce more formal graphical representations of the distribution, using individual DM processes in the pre-task and/or collective DM processes in the lessons as a *model for* structuring and representing data in the post-task. In the post-task, one of the children adapted classroom DM processes to his own graphing in the pre-task and developed generalized representations that could be used in other situations; the other child analogized data viewpoint, structuring, and representation from the classroom DM processes. These results suggest that young children have the potential to advance dynamic DM processes in a certain teaching/learning environment.

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ANALYSIS OF MODES IN DIGITAL MATERIALS

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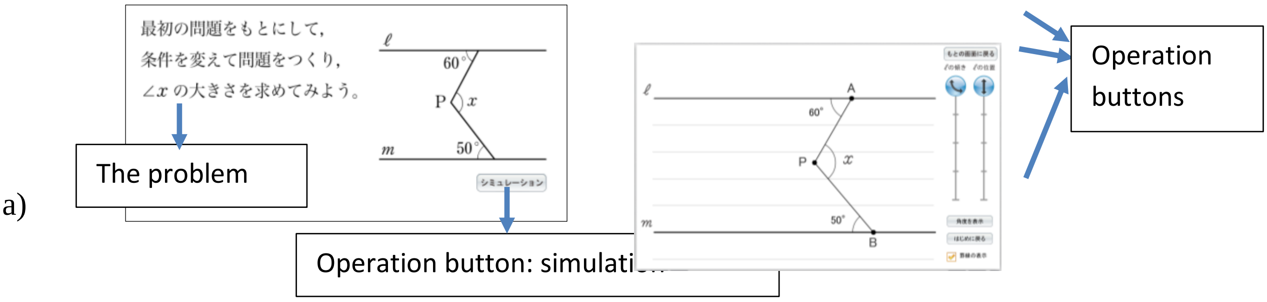
This paper focuses on modes in digital materials of mathematics. Mode is “a socially shaped and culturally given resource for making meaning” (Kress, 2009, p. 54). Many modes make a multimodal environment. The multimodal teaching and learning environment is evolving from using blackboards and pencils to using mobile devices. It is necessary to consider a multimodal perspective in teaching and learning. The research question thus is what modes that exist in digital materials affect mathematical thinking. The analysis object is a digital material for second-grade junior high school students (Tokyo Shoseki Corporation: TSC, 2016). The problem, as displayed in Fig. 1a, is “let’s find the magnitude of ∠x by building on the first problem and changing its conditions”. In digital material, if the user clicks the simulation button on the interface (Fig. 1a), another interface will appear for the user to interact with (Fig. 1b).



最初の問題をもとにして，
条件を変えて問題をつくり，
∠x の大きさを求めてみよう。

Figure 1: a) Initial problem; b) operation screen for creating problems (TSC, 2016).

The analysis includes two steps:1) Identify modes and the interactions among modes in digital material; 2) compare mathematical thinking when these modes are present versus absent. Findings suggested that mathematical thinking is guided by the mode in the background diagram and operation buttons act as a mode of visual and dynamic movement. The modes present in the digital materials thus afford new operations. Therefore, when using digital media for mathematical teaching and learning, it is necessary to consider the influence of modes on mathematical thinking. These modes seem irrelevant to mathematical thinking, yet they indeed make a significant contribution to mathematical teaching and learning in a digital environment.

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DETECTING THE ENCULTURATION FUNCTION OF PROOF IN HIGH SCHOOL STUDENTS' PROOF CONSTRUCTIONS

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When students start to study mathematics on the university level, they meet a bunch of mental obstacles rising from several characteristics of mathematics like the mathematical (symbolic) language, deductive reasoning, a proper understanding of the nature of mathematical "proof" and formal definitions, which are all concentrated in *the* basic mathematical activity, namely in executing mathematical proofs. From a higher point of view, studying mathematical proofWhen students start to study mathematics on the university level, they meet a bunch of mental obstacles rising from several characteristics of mathematics like the mathematical (symbolic) language, deductive reasoning, a proper understanding of the nature of mathematical "proof" and formal definitions, which are all concentrated in *the* basic mathematical activity, namely in executing mathematical proofs. From a higher point of view, studying mathematical proof can be understood as a process of enculturation (into the culture of mathematics). As this process is framed by certain norms (e.g., Dawkins & Weber, 2017), the natural question arises how learners can get enculturated. If we find answers to this question, this might help to link the teaching of proof at school and at university. In order to find such answers, we carried out a project where we examined a bridging course for high school students at the Technical University of Braunschweig. In this course, learners are introduced to the deductive structure of mathematics, to the symbolic language, and to mathematical proof in general. We investigated how and how far that process of enculturation can take place regarding (i) the use and handling of the syntactical aspects of mathematics, (ii) rhetorical aspects, and (iii) logical aspects (cf. Selden & Selden, 2014). E. g. the students were asked to work on the following task: "Prove the following claim: The sum of any two odd numbers is always even". Our qualitative content analysis of the students' proof productions was based on those three a priori categories. We found out that students intensively used solutions of previous proofs. They even *copied* several sections *blindly*, i. e. they did not adapt them to the new context, and thus came to obviously incorrect solutions. For example, some students used the term of divisibility exactly in the way as it was done before in a proof about even numbers, which was fully incorrect in the case of odd numbers. Thus, students just *blindly copied* the use of symbols instead of *rationally adapting* this use to the new context. As a result, we could describe the first step of the enculturation process as simple imitation. Here, the distinction between *blind copy* and *rational adaption* proved to be a helpful category in order to characterize the first step of a richly structured enculturation process.

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PROBLEM SOLVING WITH A PROJECT APPROACH

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Teaching mathematics by problem solving has shown significant effect in the achievement of students (Ali et al., 2010). Based on the authors' experiences, however, students have some difficulty in applying theoretical concepts to related real situation problems. While project-based learning has been known to be a powerful method in creation of students' useful skills (Bell, 2010), the use of it to cover all content of a course is not appropriate. The authors then seek a blended strategy from both approaches to overcome such weakness. The aim of this research is to evaluate the efficiency of implementing problem solving techniques with doing group projects in supporting students' learning outcomes.

The course for this study is ordinary differential equations designed for second-year students in Mathematics. The learning outcomes consist of determining some classifications, drawing integral curves, identifying types, solving for solutions and applying fundamental theory to related scientific problems. The sample group is 28 students enrolled in the course. The learning process consists of lecturing, group activities, group projects. Activities in class are designed for solving mathematical problems and applying the basic knowledge under advice by lecturers. The group project on modelling scientific problems is assigned where each group designs on contents, a presentation, and an evaluation form by themselves under supervision of instructors. A quiz would be given before teaching the next topic in order to represent understanding of the topics taught in the previous class. Summative exams are also given.

For expected passing score of 50% for each learning outcome, the quiz results show good numbers of those students in almost outcomes: 89% (determining classifications), 57% (drawing integral curves), 82% (solving for solutions), and 18% (applications). However, the midterm exam scores show discretion in such numbers of students except the application part. The combined exam evidently affects their performance. This indicates that the implemented approach can improve learning outcome achievement. The group project helps students in applying fundamental theory to scientific problems which are assessed from the presentations, peer evaluation, and students' feedback.

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SEEMING CONFLICTS IN STUDENTS’ SQUARE-ROOTING

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Research has been interested in the learning and teaching of roots. Kontorovich (2019) used a questionnaire to explore the unconventional ways in which 18-19-year-old students in a bridging course extracted square roots from squared numbers and parametric expressions. The study focused on the internal conflicts between students’ responses. In this communication, this analysis is re-interpreted to provide a theoretical account for what may appear as conflicts within students’ discourses.

The commognitive framework argues that communication is a patterned and rule-driven endeavor, which allows people to be efficient in situations that they consider as similar. Lavie et al. (2019) explain one’s capability to act in a new situation by harking back to *precedents* that appear as sufficiently similar to the present one. Identifying relevant precedents occurs within one’s *precedent-search-space*, which can be further deconstructed into internally consistent *pockets of precedents*.

Let us take Anna’s questionnaire as an example. When the radicands were presented as perfect squares, she copy-pasted the prompts preceding them with the ‘±’-symbol and responded with two opposite roots encapsulated under the ‘±’ (e.g., “$\pm\sqrt{169} = \pm 13$”). When the radicands appeared in a squared form, she started with converting the radical to the power of half, followed by reducing the powers to 1, and concluded with the initially squared input (e.g., “$\sqrt{11^2} = (11^2)^{\frac{1}{2}} = 11^1 = 11$”).

To an algebraically versed observer, Anna’s procedures and results may appear conflicting. Yet, it may be suggested that she resorted to two *incommensurable* pockets of precedents: one of perfect squares and one of squared radicands. Possibly unconsciously, each prompt was compared to the relevant pocket and entailed different actions without ever yielding the need to juxtapose them. Within this account, claiming the existence of conflicts in situations that Anna saw as different, would be like arguing that having different breakfasts on the weekdays and the weekends creates a contradiction. This account enriches the existing body of research by offering a student-centered interpretation for common situations where students systematically operate with square roots in noncanonical ways.

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SOME LESSONS–REGARDING INCLUSION AND TEACHER CHANGE–LEARNT FROM DEVELOPING SILENT VIDEO TASKS

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This talk will report on results from a design-based research project aimed at developing silent video tasks and their instructional sequence. In a silent video task, students are asked to prepare and record their voice-over to a short (< 2 min) video clip which has no text or sound and shows mathematics dynamically. The teacher selects a silent video on a previously studied mathematical topic and shows it to students to introduce their task. Students work in pairs to prepare and record their voice-over to the video. Students' task responses become the basis for a whole group discussion lead by the teacher addressing issues such as word use, clarity, meaning, and understanding. The goal is to get aware of the multiple ways to describe and explain mathematics and to reach some common understanding of the mathematics shown in the video.

Staying close to practice, I (the first author) worked with Icelandic upper secondary school mathematics teachers who implemented silent video tasks in their classrooms. In the first data collection phase, four teachers from four randomly selected schools assigned one silent video task to their 17-year-old students. One of the results from this first phase was that silent video tasks might be a valuable tool for formative assessment (Wright, Clark, & Tiplady, 2018). Thus, two schools that put emphasis on the use of formative assessment were purposefully selected for participation in the next data collection phase. Three teachers in these two schools accepted participation.

Working with teachers who had some previous experience with formative assessment and aimed to develop their practice enhanced the task development. For example, the three teachers suggested to place the whole group discussion immediately after receiving students' responses instead of having it day(s) later in a follow-up lesson as had been done in the previous data collection phase. They also suggested that all responses to the task would be listened and reacted to, as opposed to playing only selected student responses, as was done in the first phase. Teachers' emphasis on the importance of immediate feedback and inclusion will be discussed along with a certain recurring pattern in the way teachers reacted to their own (changing) practice after implementing the silent video task for the first time.

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THE ACCURACY OF PRE-SERVICE TEACHERS' DIAGNOSIS AND ITS RELATION TO PROFESSIONAL KNOWLEDGE

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Teachers who are able to diagnose the mathematical understanding of their students, can address their students' needs more individually, as accurate assessments allow for accurate educational decisions (Behrmann & Souvignier, 2013). We assume especially teachers' content, and pedagogical content knowledge to have an influence on the diagnostic accuracy (Philipp & Leuders, 2013). Aiming to investigate pre-service teachers' diagnostic skills, we developed simulated one-on-one diagnostic interviews. Those role-play-based simulations were used to address the following questions:

- How accurate is pre-service teachers' diagnosis of the simulated students?
- Is pre-service teachers' diagnostic accuracy related to their professional knowledge?

To answer these questions, 63 pre-service teachers diagnosed two simulated 6th graders' mathematical understanding (N = 126 observations) in a closed answer format. The simulated 6th graders were played by trained actors. Each interview had a maximum duration of 30 minutes, participants' professional knowledge was measured by a paper-and-pencil test. Data were analyzed by using linear mixed models.

The average diagnostic accuracy was 0.67 (SD=0.15; min=0.22; max=1.00) and above chance level, on average. Contrary to our expectations, a significant relation to accuracy occurred only for participants' content knowledge (B=0.031; F(1,59.10)=5.30; p=.025), but not for their pedagogical content knowledge.

Based on these results, we hypothesize that pre-service teachers struggle to apply the relevant pedagogical content knowledge in an authentic learning situation successfully, even though they have acquired this knowledge. Contrary, they seem to primarily draw on the correctness of students' answers, based on their content knowledge. Teacher education research should investigate approaches to foster diagnostic competences of pre-service teachers in authentic situations, providing support for pre-service teachers to apply their acquired pedagogical content knowledge in real-life situations.

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TEACHING A SECOND COURSE OF CALCULUS USING GEOGEBRA AND KAHOOT!

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The participants of this study were 200 first-year managerial and electronic engineering students. We have chosen differential equations, with particular attention to the phenomenon of resonance, as the subject for this contribution. The research question was the following: how to deal with the phenomenon of resonance in teaching second order linear differential equations to engineering students? The use of quizzes with Kahoot, a Student Response System, was a valid tool to highlight the students' misunderstandings on the phenomenon of resonance and to activate a constructive discussion during the lesson on why and when this phenomenon occurs. We give, as example, the following differential equation $y'' + y = \cos(\beta t)$, the first Kahoot question asked was: For which values of the parameter β the resonance phenomenon occurs; the second one: How we can perturb the solution when resonance phenomenon occur. The 52% gave the correct to the first question and 51% to the second. The 70% answered correctly at least one of the two questions. In the discussion that followed the quiz it turned out that the students had confused having to cancel the characteristic polynomial with the cancellation of the forcing term. For the second one it was very often explained during the lessons that if the forcing term contains the cosine function (or the sine function) as a particular solution it is preferable to take a combination of both. To strengthen the understanding, we invite the students to use a GeoGebra applet (prepared for them) with their smartphone to see as the solution changes when the parameter changes simply by using β as a slider. GeoGebra was a valid help in mastering the consequences from the graphic point of view of the phenomenon. So, to the survey question: Do you think the use of GeoGebra has allowed you to better understand the theoretical notions of the course? The 66% answers Very or Enough. The understanding of the topic was then tested with a multiple-choice quiz on Moodle in which there were 4 out of 8 possible scenarios where the resonance effect could intervene, to the question: “In which of the following differential equations does the resonance phenomenon occur?” only the 8% did find no case. Also, the presentation of a concrete examples such as the RLC circuit reinforce the importance and consequences that the phenomenon of resonance implies, in fact only the 14% did not respond correctly in a resonance problem applied to the RLC circuit by a multiple-choice quiz on Moodle.

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IS THERE A 'GOOD' APP FOR THAT? DEPENDS WHO YOU ASK

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Research on touch-screen technologies in early math education offer optimistic results on the influence of educational apps on numeracy understanding (Baccaglini-Frank & Maracci, 2015; Moyer-Packenham, et al., 2015), engagement and motivation (Schacter & Jo, 2017).

This ethnographic case study aimed to capture the experiences of integrating numeracy apps into kindergarten classrooms. For two years, I worked with five teachers in two low SES schools (Canada) to examine app design features that best engaged young learners. I visited classrooms twice monthly for 1-2 hours each visit, collecting video data through observation, informal discussions with students / teachers, and 1-on-1 interviews. I compared the children (n=78) and teachers' perspectives on the features they deemed most important for 'good quality' early numeracy apps. Of the criteria the children and teachers highlighted, three patterns emerged.

Both groups highlighted the importance of *Adaptability* and *Ease of Use*. Specifically, apps must quickly differentiate tasks to accommodate learning needs; and the touch-screen technologies must respond in a timely manner to children's imprecise movements. The students' evaluation focused on the quality of the gaming experience: *Meaningful Feedback* and *Control*. In particular, frequent positive verbal feedback and rewards for task completion were essential. Children also want control over the content and activities explored. The teachers' evaluation focused on *User Controls* and *Content/Curriculum*. Teachers want features that allow them to track student progress, control playing time and differentiate content. Teachers favoured apps with a limited content focus, allowing for direct alignment with content explored in class. The study contributes to research on evaluating the quality of educational software. This research provides an alternative perspective by illustrating some commonality in how young children and their teachers evaluate the quality of early numeracy apps.

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THE INTER-FUNCTIONING MODEL BETWEEN EMOTION AND COGNITION OF MATHEMATICS LEARNING

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The practice-based research program, Just Do Math, in Taiwan shows strong cognitive and affective engagement (Lin, Wang, Yang, 2018). In the 1930's, Vygotsky considered the separation of affect and cognition is a greatest defect, and Hannula (2002) also shows that students' affect is better viewed holistically, so the researchers proposed an inter-functioning model between emotion and cognition of mathematics learning, which comprises cognitive ring and emotion ring. The cognitive ring comprises making sense, realization, and competency; and the emotion ring comprises triggering, sustaining, and applying. Based on the proposed model, the article will present a case study on how emotion can be utilized to trigger mathematics thinking in class via a narrative analysis on a video, Skemp's rectangular numbers in class.

In the video, after students play the game manipulate pieces of goals to form a rectangle shape, the teacher starts to ask students about their feelings of numbers. He asks a student that if someone gives you the number twenty-three, what do you feel? The student says that he feels a little bit down. How about twenty-two? He replies that "Yeah! So happy about it!" Although twenty-two and twenty-three are numbers only, students can make a rectangle with twenty-two to score one point, so they feel great; on the contrary, they cannot do it with twenty-three, so they have no score and get negative feelings. In the process, the teacher sustains students' feelings triggered by the game via questions. Next, the teacher asks students to classify the numbers and name the classification. In the beginning, students give the name such as "evil numbers and kind numbers," with their emotions, but in the process, one of the students calls the numbers that can't form rectangle "line-dot numbers," and the numbers that can form rectangle "ordinary numbers." This is very surprising due to that although he classifies numbers with emotions but names the numbers with the shape observed during game played. Thus, the emotion starts to support the competencies in mathematics. Here is the evidence of the inter-functioning model. After, the "line-dot numbers," another student also names the number that can form rectangle "square-rectangle numbers (pronunciation as normal in Chinese)." Here, the researchers find that students' engagement of the game will trigger their emotions and then applying this emotion will trigger students' awareness and competence, so students will name the numbers based on their observations and aware the isomorphism between {dot, line, rectangle} and {one, prime, composite numbers}.

Besides this example on the inter-functioning between emotion and cognition, more examples in the program's video shall be presented to elaborate the model in the future.

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THE USE OF VIRTUAL CLASSROOM SIMULATIONS TO DEVELOP PRE-SERVICE MATHEMATICS TEACHERS' NOTICING SKILLS

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Virtual classroom simulations are widely considered promising for next-generation teacher education, and the subject of a growing body of education research. In particular, they have been found to help bridge the gap between theory and practise, by enabling pre-service teachers to practise their teaching repeatedly in a controlled and structured environment, featuring teaching scenarios of reduced complexity that fulfil teacher educators' or researchers' training aims (Dalinger et al., 2020).

These simulations have been used to develop a range of teacher competences (e.g., teacher questioning or classroom management skills). However, they have rarely been used to build pre-service teachers' noticing skills, despite those skills being both critical to instructional decision-making and difficult to develop through traditional coursework (Theelen et al., 2019). As such, this study attempted to develop pre-service teachers' noticing skills through the use of an easy-to-setup virtual classroom environment, Cartoon Class (see a sample video at https://youtu.be/s4mF5vvZfL8). The creation of this environment by the author did not require much computer-programming expertise, as it was achieved simply through the application of existing software (Skype and Adobe Ch). Because it was self-built rather than licensed, this platform was very low in cost and could be used whenever it was needed.

Twenty-seven (9 males and 18 females) pre-service teachers were enrolled in this study. After engaging in a semester-long mathematics methods course that involved three weeks of classroom simulations, the participants reported significant improvement in their noticing skills, both in terms of noticing more classroom events (from M=4.54 to M=6.12) and of shifting their classroom focus on mathematics thinking (from 36.7% to 46.3%). In addition, the participant group had significantly positive perceptions of the Cartoon Class experience, with all mean ratings in five dimensions being above the mid-point.

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LONGITUDINAL STUDY ON THE TEXTBOOK USE: A STUDENT'S PERSPECTIVE

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Textbooks, digital or printed ones, play an important role in mathematics education. Their content and exercises may influence students' opportunities to learn. How students take those advantage of these opportunities has been studied sporadically (Rezat, 2013). The utilization of mathematics textbooks can be examined using socio-didactical tetrahedron (SDT). Rezat and Sträßer (2012) developed this model from the original didactical triangle (student, teacher, content), which they further modified by adding the fourth vertex to get a tetrahedron. The fourth vertex is an artifact, i.e., humans' production, made with the precise aim to accomplish a particular task (textbooks, digital tools, tasks, language, etc.). The didactical tetrahedron, together with social and cultural influences, models the use of the artifact in the classroom.

The study reported here is a longitudinal case study of one average achieving student. The study aimed to investigate his textbook utilization inside and outside of school over the years. Data on the textbook use were collected at three-time points: at the end of lower secondary school (2017), in the middle of upper secondary school (2019), and at the end of upper secondary school (2021). Data set include student's interviews in all three-time points and diary entries the students kept for a month on his textbook use in all three-time points. The SDT was used as the lens to analyze and interpret results, particularly triangles student-textbook-mathematics and teacher-student-textbook.

The results show a change in the way the student used the textbook at home. In 2017, the student rarely used the textbook for self-learning. In 2019, while in upper secondary school, the student relied heavily on the textbook for self-learning. In 2021, the student used the textbook primarily as a source of tasks for exam preparation. In all time points, the teacher was the mediator of the textbook use in the classroom. Moreover, the student explained that the teacher's relationship with the textbook affected his interaction with the textbook at home. Various social and institutional parameters (e.g., preparing for the state graduation exam) influenced student's decision to use/not use the textbook for self-learning.

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AFFORDANCES INFLUENCING PROOF IN STEM'S GEOMETRY

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Advances in artificial intelligence and materials science, in digital and information technologies, and in the biological sciences are driving a Fourth Industrial Revolution (Schwab, 2017) that is ushering in technology-driven *change*. To prepare students for everchanging problem-solving contexts, Learning Scientists advocate for developing *Adaptive Expertise* in STEM, and reference Hatano's (2003) insight: "flexibility and adaptability seem to be possible only when there is some corresponding conceptual knowledge to give meaning to each step of the skill and provide criteria for selection among alternatives" (p. xi). Research question: How do learners utilize the unique *affordances* that differently combined instructional representations provide for grounding conceptual knowledge linked with procedural knowledge, in a learning intervention designed to develop Adaptive Expertise in STEM's geometry, specifically formulation of deductive proof schemes for the Centroid Theorem of Triangles? Based on the theoretical framework of Grounded and Embodied Cognition (Barsalou, 2008), arguing that meaning making is facilitated by engaging perceptuomotor modalities with *concrete* affordances in instructional representations exhibiting a targeted concept, this learning intervention incorporates: (a) Enactive representations, object-shapes for manipulating spatial-evidenced proof; (b) Iconic representations, picture-shapes for drawing Euclidean-evidenced proof; (c) Symbolic representations, symbol-numbers for writing formula-evidenced proof. Methods: post-secondary non-math majors (N=8), between-subjects design, random assignment to 4 conditions that manipulate instructional representations, Enactive-Iconic-Symbolic (n=2), Enactive-Symbolic (n=2), Iconic-Symbolic (n=2), Symbolic-only (n=2); procedures pre-test, training, post-test, novel post-test; outcome measure was deductive proof, percent of correctly communicated constructions (procedural knowledge) and justifications (conceptual knowledge) compared to optimal; qualitative coding videoed think-alouds during post-test and novel post-test. Significance: Oral Communication will focus on 3 participants' reliance on unique affordances of concrete instructional representations, informing "selection of some alternative" (Hatano, p. xi) solution proofs at critical moment of perceived problem-solving impasse, while formulating deductive proof on challenging novel problem – demonstrating cognitive *flexibility* of Adaptive Expertise.

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PROMOTING PRE-SERVICE TEACHERS' EVOLUTION IN RADFORD'S LEVELS OF GENERALIZATION THROUGH THE CORE CONCEPTS

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We present an experimental research aimed at understanding the potential of the Core Concept (CC), intended as a generative and transdisciplinary element that cyclically recurs in a discipline and has a structuring value for the learning. The study is in tune with recent trends calling for synergies between Mathematics Education and other disciplines, especially for teacher professional development (Bakker et al., 2021). The CC, as a boundary object between Mathematics Education and Teaching Education, was used as an operative tool in a pedagogical device involving pre-service teachers (PTs); our goal was to explore at what extent the CC fosters the PTs' evolution of cognitive processes and (re)construction of mathematical meanings. PTs faced a learning path consisting of: Solving an arithmetic-algebraic task; reviewing their solution after the introduction of the CC; metacognitively reflecting on the experience done. We collected the PTs' productions and used the levels of generalization (Radford, 2001) as a lens to analyze the development of the PTs' cognitive processes. Our qualitative analysis of the PTs' solutions, supported by their reflections, allowed us to highlight the role of the CC in favoring the PTs' transition from the factual level of generalization (operative solution of the task), to the contextual one (emergence of structural regularities as mathematical objects), to the symbolic one (elaboration of mathematical meanings). Moreover, previously unforeseen aspects arose: a dynamic interplay emerged between the CC and the mathematical activity. The CC revealed its value as a structuring and structured element: it gave structure to the learning path, promoting the PTs' transition towards higher levels of generalization; conversely, it was structured by its instrumental use within the mathematical task. The obtained results have implications for teachers' education, suggesting using the CC as a trigger in design for learning.

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VISUALIZING THE COLLECTIVE MATHEMATICAL UNDERSTANDING PROCESS IN A SMALL GROUP

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Pirie and Kieren (1994) proposed a transcendental recursion theory and suggested the possibility of describing the Mathematical Understanding Process (MUP) of individual learners. Based on this theory, proposals such as consideration of the mathematical understanding process of small groups and extension of models have been made. (Cf, Martin and Towers, 2014, Nakamura and Koyama, 2018) This is because when students learn mathematics, lesson is practiced collective and several mathematical representations with different cognitive methods are used. On the other hand, Duval (2017) categorized mathematical representations based on differences in cognitive methods and transformation rules and called them registers. This perspective shows the possibility of describing the learner's MUP described in more detail. Therefore, the purpose of this paper is to explore how to visualize learner's MUP in small groups by incorporating the perspective of Duval's Mathematical Representation Register (MRR) into a Cross-Tools PK model (CTPK model).

The method is to classify learners' actions, utterances, and expressions in a small group according to two axes, Mathematical Understanding Level (MUL) and MMR, and to plot them in a CTPK model over time. From the aspect of collective MUP visualized by this method, some features related to collective understanding are considered.

In the case of groups, it is significant that the range of movements between the levels of the mathematical understanding process is small. This is because it means that there is a collective understanding based on the consensus building of the participants. Furthermore, the folding back is motivated by learner's group mind which seeks to form consensus towards a common goal. The effectiveness of the mathematical understanding process of a small group can be known by visualizing it.

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INTEREST AND SELF-CONCEPT AS DETERMINANTS FOR THE USE OF READING STRATEGIES FOR PROOFS

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Students in advanced mathematics courses spend a lot of time reading proofs. It is assumed that the use of specific reading strategies, like strategies for elaboration, helps students to gain information from proof. Nevertheless, it has been observed that only a few students use such strategies (e. g. Weber, 2015). A reason for this may be a lack of interest or self-concept regarding proving but only a few studies address this topic. In this study we examine the relationship between interest and self-concept regarding proving and the use of cognitive reading strategies for proofs of N = 119 mathematics students of a German university. We focus on the following research question:

- RQ 2: How do interest and self-concept regarding proving predict the use of reading strategies for proofs?

We developed a questionnaire to assess students' use of reading strategies for proofs as self-reports, covering different kinds of cognitive reading strategies assigned to three different scales: rehearsal, organization, and elaboration. The reliability was only acceptable, but as this was a first attempt to measure reading strategies for proofs for university students scale-based, we are convinced that this approach is advantageous. We used established scales to measure interest and self-concept regarding proving (Ufer, Rach & Kosiol 2017). The scales of the individual characteristics had good till very good reliabilities.

Against our expectations the motivational characteristics only correlate significantly with the use of strategies for elaboration. The regression analysis shows that at least 16 % of the variance of using elaboration strategies is predicted by individual characteristics and only interest is a significant predictor ($\beta = .33, p < .01$).

Despite some limitations, our study gives a good insight into which strategies students use and the relationships between the use of reading strategies for proofs and motivational characteristics. Additional results will be discussed in the presentation.

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ROUTINES AT THE HORIZON: WHEN AND HOW A TEACHER CAN GO BEYOND THE MATHEMATICS OF THE MOMENT

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The purpose of this study is to explore in-service mathematics teachers' discourse that cross the limits of the curriculum and teaching instructions. Drawing on the literature around *Horizon Content Knowledge* (Ball & Bass, 2009) and building on the theory of commognition (Sfard, 2008) and the theoretical construct of *Discourse at the Mathematical Horizon* (Cooper & Karsenty, 2018), I study one teacher's communication patterns that go beyond the topic of the day.

Horizon Content knowledge was initially described as "an awareness […] of the large mathematical landscape in which the present experience and instruction is situated" (Ball & Bass, 2009, p. 6). The attempts to use and elaborate this idea in research led to a range of often conflicting narratives about horizon. However, a characteristic that remains relatively consistent throughout these different narratives is the ability of the teacher to recognize elements of key mathematical ideas and practices in students' contributions and addressing them in the classroom. Thus, I propose that shifting the attention to teachers' mathematical and pedagogical discourses (Sfard, 2008) could complement the original approach. Cooper & Karsenty (2018) describe *Discourse at the Mathematical Horizon* as the "patterns of mathematical communication that are appropriate in a higher grade level" (p. 242). I extend this definition to encompass an elementary perspective on advanced mathematics. The preliminary commognitive analysis of data from lesson observations and interviews with teachers, indicate some patterns that facilitate or hinder teachers' opportunities to engage in conversation beyond the topic of the day with their students. Here, I report on data from one teacher to exemplify how Discourse at the Mathematical Horizon is operationalized. I focus on her teaching routines, addressing specifically the *procedure*, *initiation*, and *closure* of the routines (Sfard, 2008). The analysis indicates when and how a teaching practice can be identified as characteristic of the discourse. I see significant potency of the commognitive analysis in the analysis of teaching practices 'at the horizon'.

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A POSSIBLE COMMUNITY OF PRE-SERVICE MATHEMATICS TEACHERS IN DISTANCE EDUCATION IN BRAZIL

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In 2017, when I started my PhD, financed by FCT, I proposed to develop an e-research (Wishart & Thomas, 2017) to study distance education (DE) in the context of mathematics teacher education in Brazil. Due the pandemic, there is now a strong tendency in the educational research to debate DE and, especially, tools used in this context. This study is part of a PhD in education and technology. The research main goal is to characterize students' practices of a program of pre-service mathematics teacher education in DE. For that, we use the theoretical lens of the Communities of Practice to develop an e-research in a mixed method approach. Before describing practices, it was necessary to find a group of students to interview. For this we carried out an online survey about student's perception in DE courses, adapted from Owston, York and Murtha (2013). The survey had four dimensions classified by us as: learning, satisfaction, engagement and difficult. Therefore, the objective of this phase was to find the student group that show the best perception with the DE program: students that feel more engaged and satisfied, and, at the same time, believe that this model allows to learn more and to have less difficult if compared with the face-to-face model.

The sample consists of 144 students from seven different university programs in Brazil. Due the Brazilian diversity, we picked up at least one university from each geographic region in Brazil. We run the one-way ANOVA (teste *post-hoc*: LSD). Outputs show that students from the UFT feel more engaged ($p = .002 < .05$) and satisfied ($p = .000 < .05$) in DE model than students from the others six universities. They also believe that learn better in DE model ($p = .000 < .05$). Moreover, 84.93% ($n = 73$) students from UFT answered that they agree or strongly agree with the question 11 ("… I feel more engaged in this course") and with the question 1 ("Overall, I am satisfied with this course"). On the "learning" dimension, 76.71% ($n = 73$) of the UFT students said that they agree or strongly agree the question 23 ("...this course has improved my understanding of key concepts"). From these and other analyses performed, which it will present orally, we chose the UFT group to interview in the second phase of research data collection.

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KINDERGARTEN AND PRIMARY PROSPECTIVE TEACHERS' SPECIALISED KNOWLEDGE OF PARALELISM BASED ON VISUALIZATION

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To develop teachers' knowledge in geometry, the central topics of visualization, localization, and the relative positions of elements and objects (in particular parallelism) were considered. Teachers' knowledge is considered in the scope of the Mathematics Teachers' Specialized Knowledge – MTSK (Carrillo et al., 2018) and since such knowledge does not merely develop over time simply by having more teaching experience (Ribeiro, Mellone, Jakobsen, 2013), conceptualizing tasks aiming at promoting the development of such geometrical knowledge in teacher education are needed. To accomplish this, we explicitly considered the need to assess the aspects of teachers' knowledge that make it specialized (in terms of nature and particularities), which can be used to establish a foundation for the future preparation and implementation of mathematical practices that will allow pupils to understand what they do and why they do it.

Data collection corresponds to 24 prospective kindergarten and primary teachers (PT) productions to a task designed to access and develop their specialized geometrical knowledge having as a starting point a mathematical critical situation concerning learning and teaching of relative position of lines and objects in 2D and 3D.

We will present and discuss the nature and focus of the task design and the knowledge revealed by PT in a set of mathematical critical situations from school practice which led to a set of teachers' specialized knowledge descriptors focusing on the specific topic. Knowledge of this Topic. The obtained list of specialized knowledge descriptors highlights crucial points to focus on teacher education – both the problematic aspect in PTs revealed knowledge as well as the descriptors obtained – and enhance the central role of the tasks for teacher education in and for developing teachers' knowledge.

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STUDENT ACHIEVEMENT EMOTIONS: EXAMINING THE ROLE OF FREQUENT ONLINE ASSESSMENT

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The rapid inclusion of online assessment in higher education has left a void in investigating its relationship with achievement emotions. Limited research suggests students experience fewer negative emotions during computer-based assessment. This study adopts the lens of the control-value theory (CVT) (Pekrun, 2000), which considers achievement emotions as products of an individual's appraisals of the subjective control and value of activities and outcomes. We examine university students' emotions around frequent online assessment. The research questions are:

- How do students' emotional perceptions of an online quiz compare to the benchmark of a traditional test?
- What interactions exist between assessment emotions, prior achievement, and gender?
- Through the lens of the CVT, how can we explain differences in students' emotional perceptions around frequent online assessment and invigilated forms of assessment?

The study was conducted at a New Zealand university in a second-year service mathematics course, featuring 31 online quizzes and an hour-long invigilated test. An adapted portion of the Achievement Emotions Questionnaire (AEQ) was distributed to 94 students in attendance at the end of semester to measure their emotions before and during taking a quiz and the test. Qualitative data was collected by asking students to identify what they perceive to be the main difference between a quiz and the test.

Enjoyment, hope, and pride were reported significantly more in a quiz (d = 0.31, 0.47, and 0.31), while anxiety, anger, hopelessness, and shame were reported significantly more in the test (d = 0.88, 0.65, 0.58, and 0.52). Thematic analysis revealed students commonly identified the main difference between assessments to be aspects of a quiz that allow them control over succeeding, such as being open book and time per question, as well as the comparative importance of each assessment. Further, many students reported the main difference to be their emotional experience.

This study demonstrates potential for frequent online assessment to interrupt habitualised negative assessment emotions, build positive assessment experiences, and change how students view assessment through improving their affective experiences.

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DELAYING DEMANDS FOR FORMAL VERBAL ARTICULATION IN EARLY ENCOUNTERS WITH MATHEMATICAL IDEAS

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This presentation will share data on interactions between the second author (hereinafter referred to as ‘the teacher’) and four Grade 3 learners (9- to10-year-olds) in the course of an after-school Maths Club session. The pedagogical goal of the club session was to guide learners towards an understanding of the need sometimes to go beyond whole numbers to ensure the equal division of certain quantities. The teacher sought to highlight the importance of fractional units as useful measures of quantity through challenging the learners to share 24 candy bars equally between 5 people.

The Maths Club is part of the South African Numeracy Chair Project. A key mandate of the Project is to contribute towards enhancing the numeracy achievements of primary school learners from disadvantaged backgrounds. Language has been identified as a significant contributory factor in poorer South African children’s lower levels of mathematics achievement. English, a second language (L2) for the majority of the country’s learners, is the main language of teaching and learning within the schooling system. Most of the country’s poorer children, including the four Club learners introduced in this presentation, are thus learning mathematics in a language in which they are not yet adequately proficient. What L2 acquisition literature indicates, however, is that L2 learners’ receptive language skills develop in advance of their productive skills: L2 learners generally comprehend considerably more than their ability to actually articulate such comprehension might suggest. The data we share show how communicative acts of a largely non-verbal sort (use of physical objects, production of visual representations, gesturing) were instrumental in helping the Club learners work out how to divide 24 objects into five equal measures. While the teacher provided substantial verbal input, mainly in English; learner verbalization, despite prompting, was limited. The learners did however communicate their thinking through their drawings and gesturing. While we acknowledge the importance of classroom talk for mathematical sense-making (Moschkovich, 2018), we contend that, in combination with the learners’ receptive L2 language skills, the various non-verbal cues used sufficiently guided them towards conceptual fraction understanding for the equal sharing of certain quantities. Pushing L2 learners to verbalize such understanding in formal terms is a necessary further step. In some contexts, however, it may be productive to temporarily delay this in the early stages of the sense-making process.

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PROSPECTIVE PRIMARY TEACHERS' ABILITY TO GENERALISE AND KNOWLEDGE OF GENERALISING PROCESS

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To succeed in mathematics, students must be able to reason mathematically in a fluent way. To help students to develop this ability, teachers need to develop their own mathematical reasoning, as well as their knowledge about reasoning. The generalizing process is a central mathematical reasoning process and it consists of inferring statements about a set of objects from the analysis of a subset of these objects (Jeannotte & Kieran, 2017). Although there are many studies that address the ability to generalize in prospective teachers, there is a lack of evidence about their knowledge of this reasoning process. In this communication, we aim to discuss both the ability and the knowledge of generalizing process among prospective primary teachers.

A teacher education experiment with 31 prospective primary teachers was implemented as the 1st cycle of a Design-Based Research project. The experiment was developed over six lessons, one per week, each lasting two hours and 30 min and focused on mathematical reasoning addressing specialised mathematics knowledge for teaching. The data were collected through participant observation of the lessons using audio and video recordings, and documents collection.

Prospective teachers were able to generalize, having the exemplifying process as support. There were no difficulties in this process, but there was a concern about the meaning of the variables expressed symbolically. Our results also show six levels of knowledge of generalizing process (confusing it with the justifying process; taking on the meaning of the term in everyday language; recognizing it though considering only 'correct' processes; fitting the definition presented, (i) but explicating it only through illustrative example(s), (ii) and enunciating its properties, and (iii) including its relationship with the justifying process). Moreover, the relationship between the ability and the knowledge of generalizing process needs further research.

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STUDENTS’ PREFERENCES IN NUMERICAL FORMATS FOR QUANTIFYING PROBABILISTIC SITUATIONS

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In secondary school, probabilities are generally represented as *fractions* (e.g., ¼), *percentages* (e.g., 25%), or *decimals* (e.g., 0.25). However, *ratios* (in an explicit way) are also used to describe probabilistic situations. In lotteries and everyday life, *odds ratios* are a popular numerical format (e.g., *1 to 3* or *1:3*). Furthermore, ratios in the form of *natural frequencies* (e.g., *1 out of 4* or *1 in 4*) have been shown to be a comprehension-enhancing way to handle Bayesian situations (Gigerenzer & Hoffrage, 1995). Therefore, the present study investigated the following research questions: (RQ1) What numerical formats do secondary students choose to quantify probabilistic situations? (RQ2) Do the chosen numerical formats depend on grade level?

An item (“A 20-sided die has 20 equal sides. What is the probability that you will roll a 6 with this die?”) accompanied with a picture of the die was presented to $N = 261$ students in Grades 8, 9, and 10 in a paper-and-pencil test. The students could freely decide in which numerical format they wanted to quantify the probability. Answers with multiple formats (e.g., “$\frac{1}{20} = 5\%$”) were assigned to multiple categories.

Of the 246 answers with a numerical format, 40% contained fractions, 37% contained percentages, 34% contained ratios, and 0.4% contained a decimal. The numerical format differed significantly between grades (homogeneity test, $\chi^2(6) = 12.622$, $p = .049$), but only the use of fractions showed a monotonic increase with grade. However, fractions did not replace other numerical formats because their proportion remained almost constant. Instead, students answered with a fraction in addition to other answer formats in higher grades.

As expected, the study showed that fractions and percentages play a significant role in quantifying probabilities. Surprisingly, a third of the students used ratios, although ratios are scarcely used in teaching probabilities. Therefore, it appears that many students connect their intuitive probabilistic thinking with the ratio concept. This raises the question of whether formats such as *1 to 3* or *1 in 4* should be incorporated more into teaching probability. Potentially, this could help students to connect their intuitive probabilistic thinking with the formal description as fractions and percentages.

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TOPIC-SPECIFIC LITERACY IN THE SCHOOL-UNDERGRADUATE MATHEMATICS TRANSITION

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Language research on the school-undergraduate mathematics transition approaches the problem from various perspectives: language proficiency, mathematical discourse, multilingualism, etc. Our focus on *topic-specific literacy demands* responds to the manifestation of this stubborn problem at an English-medium university in multilingual South Africa. Performance is poor overall, with students not recognising their school 'functions' and 'proofs', but also inequitable by declared race and – in poorly understood ways – by measured literacy practices in English (Shay et al., 2020).

Sfard's (2008) *commognition* recognises the key role of multimodal language in *routines* with and endorsing *narratives* about mathematical objects. For the specific problem in our context, we supplement Sfard's *word use* and *visual mediators* from literature on topic-specific mathematical language (e.g., Prediger & Hein, 2017). *Literacy practices*, constituted by registers, language modes, and genres, are practised in one or more named *languages*. Eleven students with diverse language repertoires volunteered to answer six function questions during the 2020 'remote' course. We analyse their written solutions and prompted, audio-recorded talk about these.

In our preliminary analysis students perform accurate routines with a given function but may not act with a function as mathematical object. Their narratives about proofs and definitions, in which conditional language (words and symbols) is not used productively, are generally not mathematically endorsed. Students are unsure what language modes constitute a valid proof and may be convinced by visual appearance. These results point to how, by supplementing Sfard's language tools, we are surfacing the detail of students' *literacy practices* for *specific topics.* The results and our perspective are informing first-year course design and lecturer education development in our multilingual context and could potentially be used in other such contexts.

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DECORATIVE AND REPRESENTATIONAL PICTURES IN MODELLING PROBLEMS – AN EYE TRACKING STUDY

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Pictures often accompany modelling problems and can strengthen the extent to which the problems are linked to the real world. In combination with text, pictures can serve different functions for learners as they solve a modelling problem (Böckmann & Schukajlow, 2018). Pictures with a decorative function (decorative pictures) are primarily aesthetically appealing, whereas pictures with a representational function (representational pictures) represent the content of the problem. Despite the widespread use of pictures, research has not yet documented how individuals process decorative and representational pictures while solving modelling problems. Therefore, we addressed the following research questions:

- Do individuals look at the pictures that are presented with mathematical modelling problems?
- How do eye movements on pictures differ between decorative and representational pictures when solving mathematical modelling problems?

61 preservice secondary school teachers of mathematics were randomly assigned to two groups. The first group solved modelling problems that were accompanied by decorative pictures, and the second group solved the same modelling problems accompanied with representational pictures. We tracked participants' eye movements with a remote contact-free eye-tracking device.

Only 2 of the 61 participants (3%) did not fixate on any of the four pictures that were presented. As expected, there were significant differences in fixation times on the pictures ($t(31) = 5.27$, $p < .001$, $d = 1.33$) and in the number of alternating eye movements between text and picture ($t(32) = 4.50$, $p < .001$, $d = 1.17$).

We conclude that representational pictures receive more attention than decorative pictures and seem to play an important role in the solving of modelling problems by supporting the construction of a model of the situation.

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INVESTIGATION OF STUDENT-TEACHERS' VIEWS OF TEACHING PRACTICE IN CLASSROOM

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This study was aimed to investigate student-teachers' views of teaching practice in classroom. The didactic triangle (Inprasitha, 2014) was used as a conceptual framework composing of Teaching Process, Learning Process, and Thinking Process. A participative research design was employed for research methodology. Seven fourth year student-teachers were a purposed group who had been teaching practice in schools for two weeks in mathematics teacher education program, Lampang Rajabhat University. They are voluntarily participated in schools using Lesson Study and Open Approach (Inprasitha, 2011; 2015), and collaboratively designed lesson plans for elementary levels coached by researchers. After two-week of the teaching practice in schools, they were asked to reflect about their teaching practice covering 1) their teaching practices are accomplished or not, show with some evidence of students' ideas 2) identify problems found in your teaching practice 3) identify improving aspects of teaching practice for next lessons improvement.

Results of the study showed that the student-teachers' views of their teaching practice are as follows. 1) Teaching Process, the students' teachers reflected about how to engage the students to have their own problematic from problem situations and prefer to use semi-concrete aids such as blocks to extend the students' ideas occurred in the classroom and how to connect them, moreover, teachers' role in encouraging with questions. 2) Learning Process, the student-teachers reflected about the students are able to solve the problem situation in various ways and also share their ideas with their friends. 3) Thinking Process, the student-teachers reflected about students' ideas of current lessons formed by using 'how to' from previous lessons to solve the problem situation. Findings from the study were found that the student-teachers' views can relate each component of the didactic triangle from their teaching practice in a context of classroom.

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STUDENTS' CONCEPTIONS OF SUBSTITUTION

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Substitution is a key idea that is woven throughout the mathematics curriculum. In secondary school, substitution is described as an interchangeability of equal numbers, and then as a method for finding solutions of systems of equations. In university, substitution is used as a means to recognize familiar structures in Integral Calculus. Despite its prevalence in mathematics, there is little research on substitution, especially on students' understanding of substitution. This work aims to investigate students' meanings for substitution, and how they use it.

We draw on Tall and Vinner's (1981) ideas of concept definition and concept image to explore students' meanings of substitution through their personal definitions of substitution, what they identify as substitution, and how they perform substitution. In this presentation, we report on elementary algebra students' responses to questions about substitution. Data comes includes written responses to multiple-choice and open-ended questions and transcripts from clinical interviews across multiple semesters at a community college.

Through a combination of thematic and conceptual analysis, we categorized students' thinking about substitution and what features appeared to impact how they enact it. We found that students often identify substitution as a process of replacement of one mathematical object for another but differ in the generality of the mathematical objects that they consider (e.g., strictly as the replacement of a number for a variable versus replacement of any expression for another expression). Students further differed in whether or not they thought that substitution entailed equivalence of the objects being replaced. When performing substitution (e.g., substituting $x + 1$ for y in $2y^2$), we found that students' activity was heavily based on their understanding of the structure of the expression where the substitution is taking place (the unified 'pieces' of $2y^2$). In addition to other findings, we elaborate on the mental processes that students engage in when performing substitution and synthesize our findings with the notion of substitution equivalence (Wladis et al., 2020).

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TAIWANESE STUDENTS' PERSPECTIVES ON THE VALUES OF MATHEMATICS LEARNING

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The findings of TIMSS indicated that Taiwan has the most students who do not see the values of mathematics learning among all 39 participating countries (Mullis et al., 2020). PISA's results revealed that low percentages of Taiwanese students consider learning mathematics helping they are getting a job, work, and improving career prospects (OECD, 2013). This study aims to investigate students' perspectives on what learning mathematics should bring to let making an effort in mathematics is worth.

A questionnaire with 30 open-ended items was employed on 83 students who have just entered one senior high school from different junior high schools. The questions included "what abilities do you think mathematics should help you develop?" "What abilities do you think mathematics ha helped you develop?" and "what should teachers do to let you feel learning mathematics is worthy and why?" A content analysis was conducted on the responses of the students.

The students' perspectives on what learning mathematics should bring to make the learning worthy can be categorized into 6 groups containing 66 items. The first group contained the most items (24). This group is pertinent to cultivating students' abilities required in real-life or future work, such as how to communicate with various groups of people, to analyze situations objectively, to inquire and deal with things flexibly, calmly, and persistently, and to apply mathematics in solving problems. The second group is about the integration of mathematics and technology, such as how mathematics is used in technology. The third and the fourth groups are about thought-oriented and content-oriented mathematical competencies respectively. The fifth group is about the affective facet toward mathematics, such as developing students' interests in mathematics and the willingness to tackle challenging problems. The sixth group is about preparing students for future study, job, and life (e.g., financial investigation).

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THE ORRSEM PROJECT: OBSERVING, RECORDING & REPORTING STUDENT ENGAGEMENT IN MATHEMATICS

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The influence of motivation and affective factors on student engagement for learning mathematics is considered as being crucial and complex (Eccles, 2016). Although behavioral and overtly emotional engagement are more readily observed by teachers, more subtle emotions and cognitive engagement are harder to identify and more difficult to clearly describe (Skilling et al, 2016). One way to elicit teacher engagement and motivation beliefs is through a research based and teacher informed tool, which provides a mechanism for noticing, articulating and communicating all types and levels of observed student engagement in mathematics classrooms and connecting theory to practice—and important for shaping instructional choices that act to promote student engagement in mathematics learning. The Engagement framework proposed by Fredricks, Paris and Blumenfeld (2004), which delineates behavioral (participation), emotional/affective (feelings, values, attitudes and interest) and cognitive (self-regulation and metacognition) types of engagement and underscored by several important motivational theories, provides a clear conceptual framing for connecting engagement and motivation and underpins the development of the main tool described in the ORRSEM Project. Using a design-based approach the ORRSEM Project involves five phases over 16 months, including iterative cycles to elicit individual teacher engagement beliefs and practices through survey and interviews, collaborative workshops to refine the innovative ORRSEM tool, trailing the tool in mathematics classes, (ages 12-14years) and developing an engagement intervention. The participants include 15 teachers from nine secondary schools who report pre-trial expectations about the tool: "a protocol like this would be useful for gauging and reflecting on engagement in mathematics…to pick up on any trends and changes in engagement…and used to make more informed interventions" (M6) and "For observation of precise behavioral, cognitive or emotional engagement characteristics and ways to enhance my practice to help improve student engagement" (F8).

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CONTRADICTION IN TEACHING MATH ACTIVITY TRIGGERED BY EDUCATIONAL ASSESSMENT POLICIES

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This study analyzes the learning of four teachers when dealing with the regulation on teachers' work in K-12 policies in the state of Tocantins, Brazil, which are guided by the Organization for Economic Co-operation and Development (OECD). In line with the perspective of Ethnography in Education, using participative observation in an elementary school located in an underprivileged region, during one semester. The empirical material included registries of the classes, planning meetings, and interviews with the participant teachers. We adopted the perspective of expansive learning under the theoretical scope of Cultural-Historic Activity Theory, positioning the analytical lenses either on the micro level of the classroom or the macro level of teachers' work relations with Tocantins educational system. The teachers faced a contradiction in the educational activity: teaching for a type of learning that is seen as human development *versus* teaching to fulfill the goals of efficiency and excellence of the educational system indicators. This provokes constant restructuring, changing everyday life, and creating work overload. On one hand, teachers reinforce official discourse emphasizing Mathematics to reach good evaluation indexes and the distinction of their school within local educational system. On the other, teachers show some resistance to the impositions of neoliberal reproduction model, showing empowerment when collectively organized and creating alternative ways of teaching that are not hierarchized by the system. To do so, they adopt collective actions and share knowledge and values to work towards an education that comprises two domains that make up Pedagogical Knowledge for Teaching: *Knowledge of content and students* and *Knowledge of content and teaching*.

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RANDOM TRANSFER JUDGMENTS IN PROBABILISTIC TASKS

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This study explores the judgments of participants in probabilistic tasks which require comparing two probabilities either with or without introducing an additional manipulation of uncertainty. The novelty of this research is in exploring the probabilistic reasoning in tasks that involve an additional condition of uncertainty.

According to dual-process theorists two general types of reasoning processes tend to be used in making probabilistic judgments. On the one hand, people use analytic processes. On the other hand, people develop heuristics, intuitive rules for analyzing the probabilities. (Babai et.al., 2006). Inspired by many studies of understanding probability we used tasks of binary comparisons (e.g., Stavy et al., 2016). A typical probabilistic task presents the participants with two containers of white and black balls: Urn A and Urn B. The first task, a comparison of probabilities (CP), asked participants to compare probabilities of randomly drawing a black ball from two different urns. The second task, an additional condition of uncertainty (ACU), asked participants to decide how the probability of randomly drawing a black ball from Urn B would change after transferring a randomly selected ball from Urn A to Urn B (Tal & Kukliansky, 2020). These tasks differ in their content but can be solved analytically in the same way. The aim of the study was to explore if there is any difference in the accuracy and reaction time of responses between the CP task and the ACU task and to learn about the influence of the probability of transferring a black ball from urn A to Urn B on the responses. The participants were 66 college students (29 males and 37 females) who had previously studied a course in probability. They were presented with 80 computerized tasks. A paired samples t-test, revealed significantly higher accuracy and higher response time in the CP task (p <.01), meaning that heuristics was used in the more difficult ACU task. Regarding the ACU task participants' responses were affected by the composition of Urn A, regardless of the composition of Urn B. To overcome this bias toward a deterministic thinking, we developed a gradual method that would build better intuition for the students.

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STUDENTS' PERCEPTION OF CHANGE IN GRAPHS: AN EYE-TRACKING STUDY

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Working with functions requires perceiving two different quantities and in particular, how these quantities change in relation to one another. A distinction is made between chunky and smooth images of change (IoC): In a chunky IoC, change is perceived in completed intervals, in a smooth IoC it is perceived as continuous (Castillo-Garsow et al., 2013). Yet, little is known about how students' approach when they perceive change in graphs. To shed light on these processes, we used eye tracking, as this method promises to provide insights into individual cognitive processes when processing visually represented information (Schindler & Lilienthal, 2019).

This study pursues two goals: Methodologically, (a) we intend to find out whether it is possible to infer students' perception of change in graphs from their eye movements. If this turns out to be possible, (b) we aim to gain initial insights into the approaches students use when perceiving graphs and their change in relation to their IoC. To pursue these aims, we conducted an exploratory case study with two university students (E. & G.) who worked with graphs connected to three situational contexts. Data collection and interpretation followed Schindler and Lilienthal (2019): While E. & G. individually worked on tasks to perceive the change in the graphs, their eye movements were recorded using eye-tracking glasses. Then, in stimulated recall interviews using gaze-overlaid videos (individual gaze was visualized as dot wandering around in the video), the students reported about their cognitive processes during their work on the tasks. We used qualitative content analysis to analyze gazes and cognitive processes.

Results show (a) that it was possible to infer students' perception of change from the eye movements and (b) that the approaches of the two students differ in relation to their IoC: E. follows the graph with his gaze and shows indication of smooth IoC, whereas G. shows indications of both, chunky and smooth IoC by focusing on prominent parts/reading values alongside with a description of increase/decrease in intervals.

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DIGITAL TECHNOLOGY SUPPORTING FORMATIVE SELF-ASSESSMENT

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Formative assessment can be conceptualized as “all those activities undertaken by teachers, and/or by their students, which provided information to be used as feedback to modify the teaching and learning activities in which they are engaged” (Black & Wiliam, 1998, pp. 7-8). Digital technology can support formative assessment for example by means of adaptive real time feedback, new types of interactive tasks and dynamic visualizations (e.g., Olsher et al., 2016). However, little is known about how technology might support students' formative SELF-Assessment (Ruchniewicz & Barzel, 2019). Self-Assessment requires learners to make judgments about aspects of their own performance and puts learners in a central position where they have to take responsibility for their own learning which is crucial for developing self-regulatory competencies. In order to support formative assessment of learners we developed a digital tool targeting the content area of basic arithmetic competencies. The tool draws on different theoretical frameworks and comprises three key elements: (a) an interactive task (Olsher et al., 2016), (b) a self-checklist which provides a list of characteristics of a correct solution (Ruchniewicz & Barzel, 2019) and (c) adaptive elaborated feedback on students’ self-assessment (Shute, 2008; e.g., in the form of visualizations, prompts, hints). In particular, rather than providing adaptive feedback on task performance (as it is mostly done in digital formative assessment environments) the tool provides students with adaptive feedback on their self-assessment. In the presentation we elaborate on the design principles of the tool and the theoretical underpinnings as well as on a design-based research study investigating students learning pathways with respect to mathematical competencies as well as self-regulatory competencies when working with the digital tool.

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TEACHERS' CHALLENGES IN IMPLEMENTING MATHEMATICS BY INQUIRY

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Learning mathematics through inquiry is promoted as central for students to develop mathematical thinking and deep understanding of mathematics (Jaworski, 2015). But teachers tend to find it challenging to foster inquiry-based learning and implement inquiry-based tasks. One reason for this is that they were taught mathematics differently and need inquiry experiences. However, teachers may experience specific challenges that are important to be aware of and address to help them to transform their practice. This study explored teachers' perspectives of the challenges they experienced in implementing inquiry learning resources to promote inquiry in learning mathematics. The study is based on *reSolve: Mathematics by Inquiry,* an Australian Government funded project with a focus to promote a spirit of inquiry in school mathematics. The theoretical framework of the project consisted of: (i) mathematics is purposeful; (ii) tasks are inclusive and challenging; (iii) classrooms have a knowledge-building culture. The project included the development and dissemination of a coherent suite of inquiry teaching/learning resources for Foundation to Year 10 grades.

About 300 teachers from across all states and territories of Australia participated in the project. Their participation over 12 months included attending workshops on learning and using the resources, implementing the tasks in their teaching, and providing feedback on their experience. Data for this study were based on their feedback provided at the final set of workshops they attended. In groups, they reflected on prompts regarding their experience in implementing the resources. The focus here is on the prompt regarding the challenges they encountered or became aware of that would be helpful to support other teachers in implementing the tasks. Their key thinking were summarized and categorized to produce themes of potential challenges teachers could encounter in implementing inquiry-based tasks. Findings consist of 10 themes of the teachers' collective challenges. The two most dominant themes are *pedagogical skills* and *time*. For pedagogical skills, the teachers highlighted challenges related to strategic intervention to support productive struggle or stimulate thinking while students worked on the tasks. For time, they highlighted challenges related to pre-lesson, in-class, and course schedule time constraints/pressures. Content knowledge was least mentioned as a challenge. The findings provide a landscape of possible challenges from the teachers' perspectives which can inform teacher professional development.

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GESTURAL NUMBER SENSE: SUBITISING, ESTIMATING, COMPOSING, AND COUNTING

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Number sense is a foundation of arithmetic and algebra, and it includes constructs such as counting, subitising, composing, and estimation (Sarama & Clements, 2009). Despite acceptance of fingers as relevant to developing number sense, predominant conceptualisations of number sense inconsistently account for embodiment. From an embodiment perspective, thought is modal and involves "operating on, with, and through actual or imagined objects" (Abrahamson et al., 2020, p. 2). Embodiment can involve gestures. Conceptually congruent gestures, where the qualities of the enacted physical movement and the concept align, can support learning (Segal et al., 2014). Some multi-touch technologies, such as the iPad app Fingu, afford developing number sense while using conceptually congruent gestures. To help expand conceptualizations of number sense, this study explored children's gestures during interactions with Fingu.

Data was collected across three consecutive projects in the USA involving 4-6-year-old pre-schoolers or kindergartners (66 total) interacting with Fingu for ≅5-15 minutes ≅3 times weekly for 3-5 weeks in classroom centres. Interactions were video recorded on the first day and at the end of each week. Iterative qualitative analyses were conducted for studies focusing on subsets of this data, primarily using microgenetic learning analysis, analytic memoing, and eclectic coding of video and memos. This study used similar techniques to re-analyse extensive portions of the video data, analyse existing memos and codes, and generate new memos, codes, and themes.

Four distinct types of gestural number sense were present: gestural subitising, gestural estimating, gestural composing, and gestural counting. Subtypes and combinations also emerged. All are conceptually congruent as embodied versions of number sense. Although context (e.g., app constraints) influences enactment, versions of these and other types of gestural number sense might apply in many contexts. Gestural number sense supports expansion of conceptualizations of number sense to account for embodiment, with implications for learning, teaching, and technology design.

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THE COMPARISON OF THE PROVING PROCESSES IN THE FIELDS OF GEOMETRY AND ALGEBRA BASED ON HABERMAS' CONSTRUCT OF RATIONALITY

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Proving is an activity with rational, conceptual, social and problem-solving dimensions and it is used to ensure the validity of expressions in mathematics (Nardi & Knuth, 2017). In recent years, Habermas' construct of rationality has been used as a tool to evaluate the proving process of students from an epistemological perspective and in terms of the proving strategy they use and the communication they establish within the context of the mathematics culture they have (Morselli & Boero, 2009).

This study aimed to reveal the difficulties that pre-service mathematics teachers experienced during their proving processes in the fields of geometry and algebra. The effect of the interaction between the rationality components on the proving process was investigated. The proving processes of 22 freshman students in the mathematics education program were analyzed based on the criteria for the rationality components. Each student was interviewed to clarify the unclear parts in the proving process.

The results showed that the rationality components at which the students were strong or weak affected their performance in other components and the proving process. It was also found that the interaction between the rationality components shows some important changes depending on the field of proving. New subcomponents were needed in both fields within the scope of communicative rationality and the modeling requirements of epistemic rationality. Hence, an elaboration has been proposed in Habermas' construct of rationality which has been adapted to mathematics education.

*This study has been adapted from the Doctoral Thesis of the first author.

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POSTERS AN USEFUL TOOL TO PROMOTE VISUAL SOLUTIONS AND DISCUSSIONS THROUGH CHALLENGING TASKS

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An effective teaching approach implies the orchestration of productive discussions, giving learners opportunities to communicate, be creative, think critically, solve problems and understand mathematical ideas. In this sense, tasks being the basis of all students' learning assist the introduction of fundamental mathematical ideas, particularly those that allow different approaches and challenge students. Students have different types of thinking, visual or nonvisual, that influence the way they understand and solve a mathematical task. For this reason, we are interested in tasks with multiple solutions/approaches, with a strong visual component, because we believe that there is a strong relationship between visualization and mathematical problem-solving abilities. Posters are connected with the nature of knowledge used in our high-tech daily lives, based largely on images full of information, and enable visualization, fostering students' learning. They help engage students, encouraging them to share and reflect on learning during collaborative work, and enable them to learn from other students' ideas (Zevenbergen, 1999). This study aimed to identify preservice teachers (aged 6-12 years) resolutions when solving multiple-solution challenging tasks, in particular to identify the visual ones, and characterize their reaction during their engagement in the creation and discussion of posters, in a Didactics of Mathematics classes. We opted for a qualitative methodology and data was collected through observations and written productions. The participants solve two tasks in groups, then created a poster, with their resolutions, displayed the posters around the classroom, having the opportunity to give and receive feedback by their peers and collectively discuss ideas. We identified the strategies used by the participants: Although they privileged routinized formulas and procedures, they also used visual resolutions. They engaged in peer resolutions and classroom discussions clarifying doubts and increasing their individual repertoire of resolution strategies. They reacted positively to this experience by expressing interest and recognition of its importance in mathematical learning at any level.

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COVID-19’S EFFECT ON HIGHER EDUCATION STUDENTS’ MATHEMATICS SELF-EFFICACY

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Self-efficacy has an influence on students’ transition to higher education and their academic performance. On the other hand, students’ self-efficacy may decrease during the first year in higher education. Particularly, mathematical self-efficacy can decrease. However, since self-efficacy is responsive, it can also be increased (Zimmerman 2000).

Our longitudinal study aims to examine the development of higher education students’ self-efficacy in flipped and lecture-based engineering mathematics courses. The most crucial visible difference between flipped and more traditional courses was that, on the lecture-based courses, there were lectures for large classes of approximately 250 students but, on flipped courses, there were no lectures but events for small groups. Fundamentally, it is a question of how efficiently the students use learning resources.

In this sub study, we are interested in determining how the changes in teaching practices caused by the coronavirus disease (COVID-19) affected students’ math self-efficacy in the spring 2020. The analysis made using the mixed between-within subjects ANOVA is based on three successive self-assessment measurements that were collected from 179 students (95 in intervention group, 84 in control group) in January, March and May. For assessing self-efficacy, the Self-Efficacy for Learning Form – Abridged instrument (Zimmerman & Kitsantas 2007) was used with the four strongest loading items.

In the first measurement, there was no statistically significant difference between groups’ mathematical self-efficacy, but a statistical difference appears in the second measurement ($M_{\text{flip}}=3.7$, $M_{\text{trad}}=3.3$, $p<.001$). Between the second and the third measurement (i.e. when changing to distance learning due to COVID-19), self-efficacy of both groups decreases ($M_{\text{flip}}=3.5$, $M_{\text{trad}}=3.2$, $p=.001$). In conclusion, although the change was similar in the two groups, the intervention group’s self-efficacy scores were significantly higher than those of the control group.

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STUDENTS' MATHEMATICS PROOF COMPREHENSION AND PROOF CONSTRUCTION: A CORRELATIONAL STUDY

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Many studies conducted by researchers related to mathematics proving indicate the difficulties experienced by students in the construction of a proof. Various factors can influence this, one of which is the students' lack of understanding of mathematical proof. The students get the understanding from their teachers, written learning sources such as textbooks and other sources. Therefore, the ability to read and understand proof from written sources is essential. By understanding proof, students are expected to have a picture of the construction process. Hence, they learn how to use various proof schemes and learn how to construct valid and correct proofs. However, empirical studies on the relationship between two skills are still limited. Therefore, this study aims to examine the relationship between mathematics reading proof comprehension and the students' skills in constructing the proof.

This research involved 380 students of preservice mathematics teacher from the early to the third year. The mathematics proof comprehension instrument developed is a multiple-choice test with an assessment model in Mejia-Ramos et al. (2012). In contrast, the instrument of proof construction used open-ended problems of algebraic word problems. Furthermore, the results of both scores were analyzed using correlation analysis.

To measure the mathematics proof comprehension instrument's reliability, the researcher used the test-retest reliability using a smaller sample size. The coefficient correlation between the first and the second administration is $r(53)=.72$, $p<.001$, indicating acceptable reliability. Meanwhile, the reliability of the proof construction score uses the interrater reliability and interclass correlation coefficient. A good agreement between the two raters' judgment is indicated by statistic Cohen $\kappa = .883$ and the ICC value by .914. The correlation analysis between mathematics reading proof comprehension and proof construction shows a significant value of $r(380)= .265$, $p<.001$. Even though it shows a low level of correlation, reading proof comprehension correlates proof construction significantly to be used as a reference in analyzing the factors affecting students' mathematics proving skills.

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THE MESSINESS OF LEARNING TRAJECTORIES: AN EXAMPLE WITH INTEGER ADDITION AND MULTIPLICATION

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We present some of the unexpected nuances and messiness of thinking and learning about integer addition and multiplication. It is expected that when a whole number addition strategy or conceptualization works, these ways of thinking can be extended to multiplication (Sherin & Fuson, 2005). The mess emerges when conceptualizations for addition work but do not readily extend to multiplication (situation 1) or vice versa (situation 2), as with integers. Integers are especially messy given the complexity of conceptions of integers (Gallardo, 2002).

Illuminating situation 1, a fifth grader solved -1 + -7 using a rule that adding two negative numbers should result in a negative number, “I know that seven plus one is eight. So—but they have a negative, so I added a negative with it.” In contrast, a different fifth grader solved -4 × -2 and shared an incorrect solution of -8:

> It wouldn’t really make sense for a negative to be multiplied by a negative to equal a positive. … Because if a positive would equal a positive, then I would assume that it would be the same for a negative. And it would be negative times a negative would equal a negative.

Illuminating situation 2, one fifth grader interpreted the negative sign as a subtraction sign, as in -1 + -7: “So I just did one plus seven equals eight. So I took seven away from it, and I took one away from it. So that would be zero.” The inclination to leverage the binary or subtraction meaning of the minus sign had more success for students doing multiplication. A different fifth grader solved -3 × 4 “by multiplying 3 times 4 and… subtracting it from zero.” The reality of negative integers is that students are developing their conceptions of integer addition and multiplication simultaneously or in close proximity, so teachers need to look forward from addition and backward from multiplication to understand and support students’ conceptions.

Additional information

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AN ANALYSIS OF MULTIPLICATION AND DIVISION WORD PROBLEMS IN THREE SOUTH AFRICAN TEXTBOOKS

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Multiplicative reasoning is the process of thinking about multiplicative relations. Siemon, Breed and Virgona (2008) explain that multiplicative reasoning consists of three characteristics, that is, the ability to: (1) work with multiplication and division of whole and rational numbers in flexible ways; (2) solve a variety of multiplication and division word problems; and (3) communicate one's thinking. Our research focuses on the second characteristic and asks the question: What is the nature of the multiplicative reasoning word problems in three South African textbooks?

We examined the nature of the word problems highlighted in three popular Grade 4 textbooks. Drawing on the literature, we developed a typology as the analytic framework for classifying whole number multiplicative reasoning word problems. Our typology included: equal groups, multiplicative comparison, unit rate and Cartesian product. We extracted all the whole number multiplication and division word problems from each of the texts and entered the data onto a spreadsheet using this typology. We calculated the number, the types and subtypes of word problems in each of the texts.

The three Grade 4 textbooks were not consistent in the number of word problems, and the types and sub-types of word problems. The limited number of word problems in all three texts suggests that there is little opportunity to relate the mathematics learned to real-life situations. Most word problems across all three texts were unit rate problems and the dominants subtype was product unknown. In a situation like the recent lockdown brought on by Covid-19, where learners are given textbooks and required to learn on their own, the dearth of word problems of various types and subtypes for developing an understanding of multiplicative relations, suggest that learners will, at best, be learning rules to apply to context-free calculations.

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A MODEL OF STUDENTS' CONCEPTIONS OF EQUIVALENCE

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In mathematics education, much research has focused on studying how students think about the equals sign, but equality is just one example of the larger concept of equivalence, which occurs extensively throughout the K-16 mathematics curriculum. Yet research on how students think about broader notions of equivalence is limited. We present a model of students' thinking that is informed by Sfard's theories of the Genesis of Mathematical Objects, in which she distinguishes between operational versus structural thinking (e.g., 1995), which we conceptualize as a continuum rather than a binary categorization. Sfard also describes a *pseudostructural* conception, in which the objects that a student conceptualizes are not the reification of a process. We combine Sfard's theory with a categorization of the source of students' definitions, where *stipulated* definitions are given a priori and can be explicitly consulted when determining whether something fits the definitions, while *extracted* definitions are constructed from repeated observation of usage (Edwards & Ward, 2004). We combine these theories with inductive coding of data (open-ended questions, multiple-choice questions, and cognitive interviews) collected from thousands of students enrolled in a range of mathematics classes in college in the US, to generate categories of students' thinking around equivalence. We see this model as a tool for analysing students' work to better understand how students conceptualize equivalence. With this model we hope to begin a conversation about how students tend to conceptualize equivalence at various levels, as well as the ways in which equivalence is or is not explicitly addressed currently in curricula and instruction, and what consequences this might have for students' conceptions of equivalence.

	Operational Thinking ←→	Structural Thinking
Extracted Definition of Equivalence	**Pseudo-Process View:** Students see equivalence as a computational process, and their approaches come from extracted rather than stipulated definitions. Definitions of equivalence are typically non-standard, ill-defined, and/or unstable.	**Pseudo-Object View:** Students are able to identify/generate equivalent objects by drawing on structure, instead of reverting to explicit computation; but criteria for equivalence are not the reification of a process. Objects are typically extracted rather than based on stipulated definitions; definitions of equivalence are typically non-standard, ill-defined, and/or unstable.
Stipulated Definition of Equivalence	**Process View:** Students see equivalence as a process governed by stipulated rules. Such students can often calculate in the correct order, but this may not translate to an ability to use stipulated definitions to recognize equivalent objects.	**Object View:** Students are able to think about equivalent objects by considering syntactic structure, without reverting to explicit processes to determine equivalence, by drawing on stipulated rather than solely extracted definitions of equivalence.

Figure 1: Model of Students' Thinking about Equivalence of Mathematical Objects

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THE TASKS REQUIRING COMPUTATIONAL THINKING IN MATHEMATICS EDUCATION RESEARCH: CRITICAL REVIEW

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Computational thinking (CT) and mathematical literacy are emphasized as core competencies for living in the digital society. However, few studies investigate how the tasks requiring CT (CT-tasks) in mathematics education research can support the enhancement of students' CT and students' learning of mathematics. The aim of this review study is to synthesize literature on what and how mathematics is integrated with CT-tasks in mathematics education research. It helps us to understand the correspondence between the types of CT-tasks and mathematic content as well as the approaches to integration CT and mathematics.

We used the keywords in terms of the three categories: CT, Mathematics and Task, to search the relevant studies by exploring Web of Science Core Collection database. and assessing appropriateness of articles for inclusion in this study. After screening the total of 84 articles, the 19 articles were identified as being appropriate and included in this review study. There were forty tasks in the 19 articles. The type of each task was classified based on two elements: unplugged or plugged, and three task levels of tinkering, making as well as remixing (Kostopolou et al., 2017). Because CT and mathematics are two different disciplines, we referred to the four approaches to integration in STEM education, including *Disciplinary, Multidisciplinary, Interdisciplinary* and *Transdisciplinary* (Vasquez, Sneider, & Comer, 2013) to analysis the relationship between CT and mathematics underlying each task.

The review results include (1) there are more the plugged tasks than the unplugged tasks; (2) there are more the tasks of the tinkering and the making types than the tasks of the remixing type; (3) the disciplinary, interdisciplinary and transdisciplinary approaches have similar number of tasks and more than the multidisciplinary approach; (4) the disciplinary and multidisciplinary approaches tend to the progress from mathematics to CT, whereas the interdisciplinary and transdisciplinary approaches tend to the progress from mathematics to CT, then back to mathematics. How the approaches to integrating CT and mathematics, in addition to types of tasks, can contribute to the task design will be discussed.

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EXAMINING STUDENT PARTICIPATION IN COLLABORATIVE MATHEMATICS PROBLEM SOLVING THROUGH THE LENS OF COMMOGNITIVE FRAMEWORK

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CONCEPTUALIZATION OF THE STUDY

This study examined the interactive problem solving by four Chinese students through the lens of commognitive framework. Drawing on the framework (Chan & Sfard, 2020), student participation profiles, participation structures and their roles-in-activity were examined. We report the use of the framework in analyzing a four-student group discussion, and how the framework contributes to explaining student participation during collaborative problem solving.

Methods, results, and discussion

The analyzed data came from the Australian government-funded research project: The Social Essentials of Learning. One group of four Chinese students, consisting of two girls and two boys in Grade 7 (12 to 13 years old), working collaboratively to solve an open-ended task, was examined. We particularly focused on one student in the group who seemed to actively participate in group activities at the beginning but turned out mostly engaged in self-talking activities by the end of the discussion.

From the analysis, we identified that the focal student in the group played the role of a follower-in-mathematizing, in contrast with the role that another two students played as leaders-in-mathematizing. This could be because as the other group members communicated about mathematical objects, the focal student did not appropriately react to and engage in their discussions, and he made more subjectifying utterances himself which made him gradually moved away from the group communication.

As revealed from this study, the changing of the focal student's participation structures allows us to look at student participation at a more micro level across the stages of problem-solving task. This study serves as a starting point of applying the commognitive framework to analyze four students' interactions, and it has demonstrated the possibility and complexity of applying the commognitive framework in analyzing a larger group's social interactions.

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MULTI-AREA TEACHING FOR PROMOTING MATHEMATICAL COMPETENCE IN SCHOOL MATHEMATICS

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Mathematical competence is internationally cultivated as the common goal of mathematics education. In 2017, the new high school curriculum standards in China divided ***the core competence of mathematics*** into six areas: mathematical abstraction, mathematical computation, mathematical modeling, intuitive imagination, logical reasoning, and data analysis (Ministry of Education, China, 2018, pp. 4–7). Although the connotation of mathematical competence has been clarified (Niss & Højgaard, 2019), the anxiety of teachers has not been resolved in China.

Six teachers were interviewed after teacher training. Among them, four believed that the six areas of mathematical competence were separate and difficult to teach in school. The teachers also mentioned that it is tough for students to master sixty competencies in ten school subjects. To solve the confusion of teachers, the correlation of various mathematical competencies was explained with 189 students surveyed. As a result, the correlation between all mathematical competence was significant at the 0.01 level, and it is argued that they are strongly related to and affect each other. The knowledge applies to the superposition rule of numbers, but not mathematical competencies. Likewise, they cannot be separated.

An example based on a statistical background but diverging from the others will be given. It is expected to show that multi-area joint teaching data analysis for improving mathematical competence is possible even under the current mathematics curriculum system. The subject matter selected here is statistical content, which is distinguished by many teachers from other mathematics areas. The goal of the statistical investigation is to learn in the context sphere (Wild & Pfannkuch, 1998), and its foundation is classification. Learning other knowledge content in a statistical environment also promotes the connection between data analysis and mathematical abstraction or modeling. Moreover, the process of summarizing by students is uncomplicated to achieve the re-creation of mathematics.

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PME44 VIRTUAL

THE 44th CONFERENCE OF THE INTERNATIONAL GROUP
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POSTER PRESENTATIONS

COMPARING PLANNING ABILITY FROM GRADE 5TH TO 7TH BY USING TOWER OF LONDON TEST

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Solving a problem requires a plan, in which a child takes decisions, selects strategies, and uses them for a successful score (Kroesbergen, et al.2010). To investigate the non-routine problem-solving planning abilities in strategic moves by using the Tower of London Freiburg (TOL-F) version test has been done in grades 5, 6, and 7 in this study.

The main assessment of the TOL-F test for standard neuropsychological evaluation is the planning ability of a person. It describes a person's ability to plan in a specified context using clearly defined rules and thus to arrive at a correct solution in the optimal way. The TOL-F test aim is to convert a starting state into a defined finishing state in as few moves as possible. It is considered the number of correct solutions (CS) which are either achieved optimally with a minimal number of moves (modeling the planning ability (PA)) or the non-optimally solved problems (NOS), where more than the minimal number of solutions is needed. The numbers of CS, thus demonstrating the level of the participants' planning and problem-solving ability, in this project are fully in accordance with what is known to the measurements of the TOL-F test**.** There is a significant difference concerning the PA ($p = 0.008$), CS ($p<0.001$), and NOS ($p<0.001$) of fifth graders and seventh graders. The difference between grades 5 and 6 is also significant in CS ($p<0.001$), and (NOS ($p<0.001$). There is a significant shift in the performance in terms of CS between the 5th and 7th grades. Interestingly, this shift is mainly obtained from grade 5 to grade 6 with both a significant difference and a strong effect size. Comparing the extreme grades 5 and 7, we see improvement in all measurements done. This suggests progression in planning abilities from year to year, which is too small for significant results in the analysis.

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FIRST DESIGN CYCLES TO DEVELOP A CONCEPT OF LEARNING ANALYTICS AS A FEEDBACK PROVIDER

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BACKGROUND

At universities, many courses take place in very large groups of students, especially in the early stages of studies. Such groups are often very heterogeneous in many ways. These circumstances make it difficult to provide students comprehensive support in the learning process, especially in giving them adequate feedback. However, feedback can be considered one of the most important building blocks of good teaching (Hattie 2015). Fortunately, new techniques and research approaches today make it possible to give feedback to students automatically. In this context, the research field of learning analytics aims to use large, learning-related data sets to not only generate knowledge about learning but to directly improve learning processes (Siemens & Long 2011).

METHOD

In this context, the project presented here aims to develop a concept using a design science approach that provides students of large courses individual feedback. As a basis, a technical system was developed that supports students in all aspects of learning, and that provides the necessary database. As a first preparatory cycle, lecturers and teaching assistants were asked which information from the data would be useful to provide constructive feedback. In the first application cycle, these data were used and compared with the subjective feedback of the teaching assistants for one semester. This resulted in a revised concept that is to be the basis for the next design cycle.

RESULTS

The underlying technical system was also developed in several cycles to offer a starting point for learning analytics. For the first cycle, especially the completion frequencies and solution rates of formative assessments were used for feedback, but also the completion time was considered as useful information. Already with these data, meaningful support for further teaching could be achieved. At the same time, however, it was determined for the next cycle that an evaluation according to task types and also an increased evaluation of material downloads would be constructive.

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AUGMENTED REALITY CONTRIBUTIONS FOR THE TRAINING OF TEACHERS IN THE REMOTE CONTEXT

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During pandemic period, it is essential that technologies get associated with pedagogical practices in remote context and, in particularly, with the Mathematics teaching and learning process. The digital transformation in teaching practice has demanded the technological artifacts to become pedagogical instruments and, consequently, training and qualification are necessary for the teachers to get security handling the technology and in their teaching activities. Among the countless technologies that can be used in Mathematics teaching, Augmented Reality (AR) can be an option in the training of Mathematics teachers, as its potentialities relate theory and practice, even in the remote context, by integrating virtual elements with real-world views through a camera. The training of teachers to use AR, with focus on Spatial Geometry content, is the object of study of this project that is under development. The main objective is to identify, from reflections and observations presented by the participants, during remote workshops, contributions in teacher´s formative process and how the technological instruments are perceived and used by them. The Activity Theory and the aspects of the expansive cycle, from the perspective of Engeström, were considered as theoretical support of the project. The workshops were designed according to the levels indicated by van Hiele model and using the Design Experiments research methodology, which can be understand as a progressive improvement of the investigation. Two workshops were proposed remotely and each one was designed with six interactive and practical classes using GeoGebra 3D (GeoGebra, 2021) tool with the AR feature. Workshop 1 took place from August,2020 to September,2020 for 10 Mathematics teachers. Workshop 2, an improvement of Workshop 1, started in March,2021 and will end in April,2021 with the participation of 5 Math teachers and 7 students of Mathematics major. The reflections exposed by Workshop 1 participants and observations collected in Workshop 2, so far, show that AR arouses the interest of teachers for the ability to teach in a fun and comfortable way for the student. The participants affirm that the AR resource can contribute to overcome the difficulties of visualization and abstraction in the context of Spatial Geometry, because the possibility for the student to interact with virtual objects allows a visual dive in their characteristics and properties. Practices in workshops have shown that the use of technology can play an important role in understanding mathematical content and allows students to engage in the learning process even in a remote context.

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DEVELOPMENT AND VALIDATION OF MEASURERING META-SCIENTIFIC KNOWLEDGE ABOUT MATHEMATICS

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One aim of upper secondary schools or introductory university courses in mathematics is that students shall become acquainted with fundamentals of scientific ways of working, they shall develop a scientific attitude and be able to think about the nature and the limits of scientific findings. This aim can be called “scientific propaedeutics“. Theoretical approaches to conceptualize scientific propaedeutics suggest a model consisting of three hierarchical levels: meta-scientific knowledge, methodological awareness, and meta-scientific reflection (Müsche, 2009). In this presentation, we focus on the first level of this model which includes the knowledge about the basic concepts and characteristics of mathematics as a deductive and proving science.

We aim to conceptualize meta-scientific knowledge about mathematics and to investigate how this knowledge relates to motivational variables, e.g. interest in mathematics. We developed an instrument to measure meta-scientific knowledge according to the suggested themes of the Nature of Science research (Ledermann, 2006). Face validity was established by conducting an online-survey ($N = 13$) and a group discussion with 14 mathematicians. The test and questionnaires for self-concept and interest (Ufer, Rach & Kosiol, 2017) were implemented in an online survey.

The sample comprises 313 university students (42.8% female, 63.6% 19-22 years) who enrolled in a Bachelor’s degree program with at least one course in mathematics. The findings indicate an acceptable reliability of the test. Meta-scientific knowledge relates small to mathematical self-concept ($r = .28$, $p < .01$) and shows a moderate correlation to interest in mathematics ($r = .35$, $p < .01$), providing evidence for the construct validity of the test. In the presentation, we will discuss further results and implications.

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ENHANCING TEACHER EDUCATION WITH CARTOON-BASED VIGNETTES: THE DIVER TOOL

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Providing prospective teachers with the opportunity to engage in representations of practice, so-called vignettes, has proven to be an effective approach in teacher education and corresponding research (Herbst et al., 2011). Questions regarding possible designs of vignette-based learning and testing environments have become essential in this context and the potential of cartoon-based vignettes has gained increased attention. Cartoon vignettes combine numerous advantages ascribed to video and text vignettes: Cartoons allow the systematic, theory-based design and variation of classroom situations whereas individual characteristics that are important to comprehend a classroom situation can easily be added (Friesen & Kuntze, 2018). In the project coReflect@maths, a digital tool for creating cartoon-based vignettes is developed and programmed. The so-called DIVER tool (Developing and Investigating Vignettes in teacher Education and Research) will not only allow the creation of cartoon vignettes (e.g., by arranging student and teacher characters in classroom environments, adding speech bubbles, etc.), but will also enhance the collaborative reflection and exchange of vignettes within the learning platform Moodle. The poster presents the features of the DIVER tool and displays a cartoon vignette inviting colleagues via QR code to analyse and to exchange responses with the authors.

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EARLY NUMBER SENSE STRATEGIES AND PATTERNS IN MULTI-TOUCH INTERACTIONS WITH A DIGITAL GAME

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Early number sense, including quantification through subitising, composing, and counting, is integral to arithmetic (Clements & Sarama, 2007). Embodiment, which frames thought as modal and involving engaging with actual or imagined objects (Abrahamson et al., 2020) implies that fingers and gestures, as objects and enacted actions, can be significant for learning and knowing. Interacting with certain multi-touch technologies can foreground fingers and gesture. To inform understandings of development of early number sense, this study explored children's quantification strategies while using the multi-touch iPad app Fingu.

Data collection occurred near the end of the school year at an early childhood centre in an urban location in the USA. Eighteen 4-5-year-old pre-school students and their classmates were regularly offered opportunities to interact with Fingu during centres for five weeks. Video recording of participants' interactions occurred on the introduction day and weekly thereafter. Qualitative data analysis focused first on the video, with researchers engaging in multiple rounds of generating descriptive field notes and analytic memoing. Researchers also coded the video, memos, and notes, supporting identification of recurring patterns.

Initial findings include: 1) Quantification strategies (subitising, composing, counting) and their corresponding finger patterns (semi-decimal, direct mapping); 2) Patterns of subitising-dominant and composing-dominant strategy use, with subtypes related to development and to encountering challenging configurations. Some subitising-dominant participants moved towards conceptual subitising and composing. When challenged by greater quantities, some subitising-dominant participants counted. When challenged by unfamiliar configurations, composing-dominant participants rarely counted, but often applied perceptual subitising with direct mapping. Participants usually applied their dominant strategy once a task was no longer challenging. Ongoing work based on the preliminary results focuses on analysing and exploring implications for improving children's mathematical abilities.

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MATHEMATICAL MODELLLING WITH EXPERIMENTS – SUGGESTION FOR AN INTEGRATED MODEL

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Mathematical modelling is a mathematical key competence. According to Niss (1998), the validation of a model is “the single most important point related to mathematical modelling” (p. 369). One possibility for realistic modelling that fosters students’ validation-competence is the use of real data conducted via (physical) experimentation. In the mathematics education literature there are many best practice examples for modelling with experiments. However, little is known about factors influencing modelling with experiments, related effects and the process itself.

In this poster I present the suggestion for an integrated model of mathematical modelling with experiments (figure 1) that is based on the modelling-cycle by Blum and Leiß (2005) and the cyclical experimentation-model by Ganter (2013). Two assumptions are central for the suggested integrated model: 1) Since every experiment involves idealisations, the experiment itself can be seen as a real model in the sense of Blum and Leiß (2005). 2) The notation of data (e.g. in a table), collected in the experiment, is a first step of mathematization in the modelling cycle.

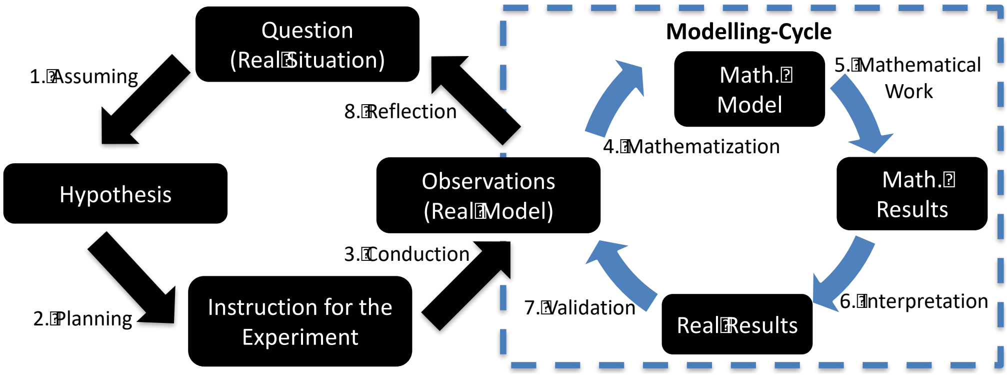
Question (Real Situation)
1. Assuming
Hypothesis
2. Planning
Instruction for the Experiment
3. Conduction
Observations (Real Model)
8. Reflection
Modelling-Cycle
4. Mathematization
Math. Model
5. Mathematical Work
Math. Results
6. Interpretation
Real Results
7. Validation

Figure 1: Integrated Model of Modelling with Experiments

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INTRODUCING PRE-SERVICE MATH TEACHERS TO RESEARCH AND DEVELOPMENT INSTITUTES AS A WAY TO DEVELOP PEDAGOGICAL KNOWLEDGE

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INTRODUCTION AND BACKGROUND

Studies have shown that students often fail to see the relevance of Mathematics and how it is used in everyday life. Students often believe that the mathematics knowledge they are taught at school is useful only in Math classes and for academic tests (Musto, 2008). These attitudes may be affected by pedagogical design capacity. Pedagogical design capacity is the capacity to perceive and mobilize existing resources to construct instructional episodes (Brown, 2011).

RESEARCH QUESTION

How does the experience at industrial facilities and research institutions contribute to the pre-service teachers' use of authentic instructional methods in Math?

PROGRAM DESIGN

The pre-service Math teachers each held a bachelor's degree in some sort of science or in Math. The pre-service teachers accompanied researchers and engineers from industry and non-profit research institutions as part of their practical training. The pre-service teachers interviewed researchers/engineers and wrote about their experiences in the field. They were then asked to design related activities suitable for secondary-school students.

FINDINGS AND DISCUSION

Our findings show that the integration of researchers and engineers into the training program opened a new world to the pre-service Math teachers. The pre-service teachers got ideas about relevant and meaningful instruction. As one representative pre-service teacher noted, "The process exposed me to possibilities for integrating real-life questions and processes from research and industry into the learning process." The pre-service teachers' exposure to the world of industrial R&D allowed them to acquire tools for authentic instruction in their classrooms, including inquiry-based and project-based learning, as well as ideas of how to spark their students' curiosity.

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VIGNETTES AS A MEANS TO FEEL DIFFERENT EXPERIENCES OF MATHEMATICS AND ITS TEACHING

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Students' mathematical performances can be affected by the ways of working that are supported in the classrooms in which they study. The different tools and representations through which they are encouraged to express their mathematical knowledge enable and disable different learners in different ways, especially if teachers have difficulty in feeling mathematics in ways that are congruent with their students' strategies (Fernandes and Healy, 2020). Vignettes offer ways of representing both the mathematical strategies that emerge in classroom practices – be they typical or atypical – and the pedagogical framing of such activity, providing virtual experiences of different classrooms realities through which prospective and practicing teachers can reflect upon mathematics and its teaching (Buchbinder, & Kuntze, 2018).

Part of the coReflect@maths project involves the design of vignettes that encourage teachers to focus on the challenge of developing inclusive pedagogies, to explore how different ways of feeling and doing mathematics can enrich and expand learning opportunities, and to notice how normative classroom expectations rather than individual characteristics can contribute to the marginalisation of particular groups of students. The poster displays an example vignettes and conference participants are invited via QR code to exchange responses with the authors.

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CONSIDERING THE MATHEMATICAL EXPERIENCES OF FUTURE TEACHERS

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Wilkins (2008) noted that not just teacher beliefs, but attitudes about mathematics, had an impact on teaching practices. Research has noted that teachers tend to teach the way they have been taught when experiencing difficulties in their mathematics classroom (e.g., Cross, 2009), so past experiences have an impact on practice. Since feelings and experiences in mathematics have a lasting impact on what teachers bring into their classrooms, it is helpful to learn more about where and how the negative or positive feelings about mathematics begin in order to support effective teachers in mathematics.

This research examines the past experiences of future teachers in mathematics through examining personal narratives about the future teachers' experiences in mathematics, as well as their feelings (positive and negative) about the subject. A thematic analysis (Braun & Clarke, 2006) was used to look for themes between the participants in order to determine when and why their perceptions of mathematics were created. Data was collected from three different years of future elementary (K-8) school teachers in their teacher education program.

Results showed that many of the teacher's ideas about mathematics were shaped by specific moments in their stories. Although some of the individuals noted their feelings changed in elementary school, most individuals felt a shift towards a negative perception of mathematics began in the secondary school years (grade 9-12). When discussing what they loved or hated about mathematics, there was also not a clear divide: some individuals cited things like competition and timed tests as reasons to hate mathematics, while others noted this was why they loved mathematics.

The variability in the data points to larger conversations being needed in order to support individuals in seeing mathematics in a more positive light. This poster shares the contradictory results in order to start this conversation about supporting changes in the field of mathematics in order support better perceptions for our future teachers.

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DISCERNING ASPECTS OF NUMERATE BEHAVIOUR

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World-wide, too many citizens lack the necessary numeracy competencies to participate autonomously and effectively in our technologized and number-drenched societies and consequently many citizens are overlooked for certain jobs and have problems in their daily life, dealing with the fast-growing abundance of number-related situations. In literature, these numeracy competencies are mentioned specifically in studies on 21st century skills, global competences, and skills for the 4th industrial revolution (Schwab, 2016). Numeracy is about how people deal with the quantitative and multidimensional phenomena in the world around us, both in daily-life situations and professional contexts. In the latest and most state-of-the art definitions of numeracy, it is described as a broad and multifaceted concept and as a social practice. It manifests itself in a plethora of observed numerate practices of people, showing that numerate behaviour is affected by cultural, social, personal, emotional traits, and societal power relations. In short, numeracy takes the person and his/her relationship with the world as a starting point.

In 2019 funded by the European Union, an Erasmus+ project started under the name Common European Numeracy Framework (CENF) to create an overview of the relevant aspects which matter in the quality of numerate behaviour of citizen. This was based on a literature review on emergent themes in numeracy, a wide-scale European Numeracy Survey, and expert consultations. The main categories of aspects which were discerned, are: Content knowledge and skills, Context, Higher order skills, and Dispositions.

In the poster we will show the underlying subcategories of each of the main categories. In the CENF, for each subcategory (e.g., Quantity and Number, Self-efficacy, Mathematising) corresponding descriptions of observable numerate behaviour were developed. These descriptors are formulated on six levels, so that they also give indications of possible learning trajectories. By this, teachers and learners in adult education can establish which aspects of numerate behaviour can be addressed and improved in educational settings.

In a later stage of the project professional development modules were developed to implement the framework across many countries, starting in Europe.

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DESIGNING VIGNETTE-BASED COURSES FOR TEACHER TRAINING

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Representations of practice (so-called vignettes) understood as a depiction of a classroom situation (e.g. a transcription of students’ answers to an activity, or a cartoon showing a teacher-student interaction) promote pre-service teachers’ reflection and discussion of authentic classroom situations (Buchbinder, & Kuntze, 2018). Therefore, vignettes provide pre-service teachers with real-life contexts to analyse and interpret aspects of the teaching and learning of mathematics and with opportunities to relate theoretical ideas with examples from practice (Ivars, Fernández, Llinares, & Choy, 2018). One of the objectives in the coReflect@maths project, is the design of digital vignette-based courses for pre-service and in-service teachers. With a focus on developing pre-service primary school teachers’ competences, such as noticing students’ understanding or planning a lesson, the poster presents the structure and design principles of a vignette-based course related to fractions. Particularly, the course consists of four vignettes where pre-service teachers have to reflect on classroom-situations (answering some guiding questions) using a theoretical document with information regarding the teaching and learning of fractions. Furthermore, the poster displays one of the vignettes as an example, inviting the audience via QR code to exchange responses with the authors.

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AN ANALYZING STUDENTS' MATHEMATICAL COMPETENCE: ORDER OF OPERATIONS (PEMDAS)

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The main point for teaching mathematics is that the learner is truly learning by solving problems by themselves, not by memorizing rules or formulas. It is argued that the condition of order of operations (PEMDAS), is merely providing the learner with the sequence of steps (Wu, 2007). This contradicts Lampert (1986) and Merlin (2008), who think that the condition for the order of operations are very important in the context of mathematics and reflects the importance and depth of action. In addition, using the condition of PEMDAS is the foundation of mathematics, science, technology and programming. This study aimed to analyze students' mathematical competence in order of operations. Thirteen items of the Suken test in level 6 was an instrument. The target group were 139 students. The data analysis consisted of students' answer sheet, test item analysis and interview.

The results showed that; 1) 17.80%of students had the ability to perform mathematical order of operations (PEMDAS). Based on the item analysis, it was found that students were able to correctly use the condition of PEMDAS. 2) 80.58% of students lack the ability to perform mathematical order of operations. Based on the item analysis, 47.48% of students were unable to correctly use the condition of PEMDAS. They performed operation from left to right regardless of the use of the condition of PEMDAS, and 33.10% of students were unable to solve the problem.

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JAPANESE ESL STUDENTS' PERFORMANCE IN ENGLISH MATH WORD PROBLEMS IN QUADRATIC EQUATION

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International Baccalaureate (IB) is currently spreading all over the world and Japanese government is also promoting it. In terms of IB Diploma Programme (DP), Japanese public schools have also started the programme and the number of math classes where English is used as an official language is also increasing. Thus, it is important to clarify the difficulty in ESL students' math learning. It is known that the understanding of math word problems consists of two processes: "Problem representation" and "Problem solution" (Mayer, 1987). In this study, what kinds of stumble occurs in math word problems in English are to be clarified. The research questions is:

- What are the difficulties when students solve English math word problems in Japanese ESL classroom?

In August 2020, three math word problems were presented to 24 IB students in Japanese public high school. After that, the students' reflections were analyzed by inductive analysis (Thomas, 2003) which categorize qualitative data through five steps.

From the analysis, the "Problem solution" was carried out based on the correct English understanding, but the student doubted their own understanding of English and returned to the process of "Problem translation" then reconsidered it with incorrect interpretation. From this, it became clear that one of the students has doubted their own process of calculation because of insufficient understanding of English. Also, one of the students said that they could have not understand the situation because of one unknown word. From this, it has become clear that not being able to understand one word influences the whole English reading comprehension.

Finally, I argue that math teachers need to know these students' stumbling and approaches to prevent them from the stumbling in future classes should be discussed in the context of Japanese classroom. Thus, we should expand this research more in mathematics education research.

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TEACHING SYMBOL SENSE TO GRADE 11 STUDENTS

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Students have serious problems with giving meaning to algebraic formulas and reading through these formulas, which are aspects of symbol sense. It has been suggested to give meaning to formulas via linking them to other representations of a function that are more accessible, e.g., graphs (Arcavi et al., 2017). In the current study, we investigated how teaching graphing formulas by hand could foster essential aspects of students' symbol sense, i.e., identifying the structure of formulas and key features, and reasoning with and about formulas. In an intervention of five lessons of 90 minutes, 21 grade-11 students were taught to graph formulas by hand through qualitative reasoning and recognition, using whole tasks with student support (GQR-design). The whole tasks reflected different levels of recognition and included teaching a repertoire of basic function families that are used as building blocks of formulas, the habit of questioning the formula (what do I notice?), and qualitative reasoning about the global shape of a graph, using global descriptions, and ignoring what is not relevant.

The 21 students made a written 45 min pre-test, post-test and, four months later, retention test with graphing 14 formulas. Six months after the retention test, a written symbol sense was administered to the 21 students and to 93 students from five other schools. This test included 12 non-routine algebra tasks, e.g., 'Give the number of solutions of the equation $5\ ln\,ln\ (x) = \frac{1}{2}x - 10$' and 'Can the y-value become larger than 70 when $y = -0.1(x-3)(x-10) + 40/(x-3)$'. Six students were asked to think aloud during the pre-test, post-test, and symbol sense test. The protocols were transcribed and analyzed on the use of symbol sense, i.e., recognition of function families and key features, and qualitative reasoning. The scores showed that the 21 students significantly improved their graphing abilities: mean score in pre-test 2.4 (SD=2.4), in post-test 9.2 (SD=2.6), in retention test 7.0 (SD=3.4). In the symbol sense test, the 21 students used more symbol sense strategies and scored significantly higher than the 93 students from other schools (mean score 5.3 (SD 2.2) versus 2.5 (SD=1.6)). The thinking aloud transcripts showed that the six students used similar symbol sense strategies in all tests after the intervention. This suggests that aspects of symbol sense, like identifying structure and qualitative reasoning, can be taught via the GQR-design.

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BUILDING UP PROFESSIONAL KNOWLEDGE FOR FOSTERING STUDENTS' ARGUMENTATION IN THE MATHEMATICS CLASSROOM – A VIGNETTE-BASED APPROACH

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Fostering students' argumentation is often emphasised as a central aim of the mathematics classroom in all grades. Teachers, therefore, should have corresponding professional knowledge on how to foster students in this respect, including, for instance, an awareness of possible learning opportunities which allow students to further develop their argumentation, or knowledge on common difficulties of students in requirement contexts related to argumentation. However, empirical studies imply that teachers often lack of such professional knowledge (e.g. Stylianides et al., 2016; Krummenauer & Kuntze, submitted), which should thus be built up in university teacher education. For this, vignettes – understood as representations of profession-related requirement contexts (cf. Kuntze & Buchbinder, 2018) – show a high potential, as they allow teacher students to reflect on particular profession-related contexts and to connect them with relevant theory elements. In the European project coReflect@maths, a vignette-based intervention approach for building up teacher students' (and also in-service teachers') professional knowledge on fostering students' argumentation is being developed, which is presented in the poster, including a sample vignette. The poster also contains a QR code, which allows access to a vignette-related online activity and to submit feedback on the vignette to the authors.

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PERFORMANCE OF 4TH GRADE STUDENTS ON MATHEMATICS SENSE LEARNING STRATEGIES

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Fractions are abstract but important concepts for mathematical learning. In 2004, 199 4th-grade students(Leu, Lee, Liu, and Wu, 2009) was tested in Taiwan and found that only 24.5% of students can present conceptual understanding by drawing or text about $1\frac{1}{4}+2\frac{1}{4}$ is $3\frac{2}{4}$.

Author wants students to learn the ability of lifelong learning, he proposed five learning strategies, that is giving examples, simplifying, drawing, asking why, and rethinking. This study is to survey the performance of 4th-graders' learning strategies when their teacher has participated in the mathematics sense program. The mathematics sense program has three years for grades 3-6 in Taiwan. During the study period, we held a full-day workshop to help teachers understand how to use the five learning strategies for students, and a half-day workshop was held each month to solve teachers' teaching problems.

The research method is a questionnaire survey method. The research object is 13 classes, 283 students of the 4th grade in Taiwan. The questionnaire was written by a mathematics educator based on 4th-grade mathematics content and mathematics sense learning strategies and was examined by three 4th grade teachers for the suitability of its words and terms to students.

The survey found that teachers' participation in the program over an average of 10 months. More than half of the units in the last semester used five learning strategies in teachers' teaching, and the average of the five-point scale are 1.30, 1.10, 2.00, 1.50, and 1.60 (1 is very helpful, 5 is very unhelpful) that teachers think it is useful for their teaching. It shows that the teachers participating in this program for about one year, and they think they are helpful and often use five learning strategies in teaching.

The study found that 4th-graders are more likely to use drawing to present conceptual understanding of fraction concepts (49.9-80.2%), use words to give examples or explain why (33.6%), and rethink the relationship between concepts (26.9%). Compared with the research of Leu et al.(2009), it is found that more students can use drawings or text to explain their conceptual understanding (49.8-80.2% vs 24.5%).

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HOW AN ELEMENTARY SCHOOL TEACHER APPLIES MATHEMATICS IN DEALING WITH FINANCIAL ISSUES: THE EXAMPLE OF MORTGAGE LOAN

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The financial literacy or financial capabilities is considered as an important issue in latest education, and the financial capability is considered as one important and connected to numeracy. In Gerardi, Goette, and Meier's (2013) study, they found that numerical ability can especially predict mortgage default. The connection among finance, mathematics, and quantitative reasoning undoubtedly could be viewed as the demands for financial literacy. Moreover, teachers' financial capabilities are considered crucial and as a prerequisite to the development of students' financial literacy.

This study explored an elementary school teacher's mathematical capabilities of financial issues through continual discussions between a discussant D and the teacher. Each discussion of the learning group lasted 30 to 40 minutes once a week and lasted for one year. The discussion contents were then categorized into ten different themes. In this study, we focused on the theme of mortgage loan which was raised by T.

In the preliminary stage, T revealed her avoidance to discuss the accumulation of shopping points when such issue was raised by D. She asked D "should we need to spend so much energy to calculate for so small portion of money?". Later, T raised a financial issue of mortgage loan that was cared by her. For this issue was what she cared about and she was wondering how the amount of payment for housing loan is calculated by the bank. The task was set with a loan of 5 million New Taiwanese Dollars, the annual interest of 1.7%, and to be made in 20 years. T started to interpret the payment of mortgage loan with simple interest. In order to make T to reflect on her misconception of simple interest, D posed a simplified mortgage loan problem to T. The problem was simplified into the loan of one million and the (annual) interest rate of 1.7% to be made in 3 months. Later, she checked her answers by inputting the quantities into the online system to see whether the values were the same. D then encouraged she to write the solution process in algebraic, that is to generalize the calculation into the algebraic formulae. In later discussion, T reflected that "I finally understand how the formulae of mortgage loan come".

After the teacher experiencing the exploration with the mortgage loan, she showed her enthusiasm in solving the daily-life finances.

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MOVE IN MATH AND MOVE WITH MATH: THE TRIANGLES

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Learning mathematics outside the classroom has the potential to facilitate students to understand the concept of mathematics and specially geometry. It also provides students practically involve with the learning process through solving problem tasks. It lends itself to the Inquiry-based mathematics education a student-centered paradigm of teaching mathematics and science. This means they must observe phenomena, ask questions, look for mathematical and scientific ways of how to answer these questions, interpret and evaluate their solutions, and communicate and discuss their solutions effectively. The main feature of cooperative learning is the opportunity to discuss and reason with others and justify one's mathematical thoughts on how to solve different mathematical problems. Cooperative outdoor learning in mathematics gives the possibility to observe that a task at hand can be solved in more than one way and that more than one "right" solution to the problem may exist. The sensorimotor experiences arising from the environment also play a paramount role in learning. The activity (see https://www.youtube.com/watch?v=lGJbz_d7OUs&t=80s for other examples) takes place in the "Classroom under the sky", so called the outdoor amphitheatre that was inaugurated this school year after more than a year of work (thanks to funds of "Italian Teacher Prize" as best teacher). Here we give an example of an outdoor lesson on triangles and some notable points to 17 sixth grade students. Students were randomly divided into four teams of 4/5. A rope as a compass, some wooden stakes and red discs are their tools. The square is the same one used by the ancient Egyptians 5 thousand years ago: a rope divided into 12 equal parts. Each of the 12 segments has nodes as ends. Three students take the knots every 3, 4 and 5 segments and tend the thread to find that is formed is a magical, perfect, stable figure: a right triangle. They started discovering some properties to build a triangle: the length of one side cannot exceed the sum of the lengths of the other two. After getting a triangle with wooden stakes, each group is asked to identify the midpoint of each side. One group used a piece of string as a unit of measurement to measure one side of the triangle and then calculates the half, while the other used the string to measure one side, joining the two extremes to obtain the half, arriving so at a more precise result. In this way they build the axis, the median, the circumcentre, and the centroid. The activity was tested with a series of exercises on axes and medians and the results compared with similar exercises on bisectors and heights, explained exclusively in the classroom. The average percentage of improvement of the performance with outdoor activity was around 17%, even reaching a maximum of 54%. With Mentimeter, the most voted word was "funny" followed by "cooperative learning".

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USING MUSIC NOTE VALUES TO SUPPORT THE TEACHING OF FRACTIONS

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RESEARCH QUESTIONS AND THEORETICAL PERSPECTIVE

The key aim of the study, currently in its preliminary stages, is to explore the value of integrating mathematics and music to support teachers in teaching fractions to enhance conceptual understanding. Helping teachers (and, through them, their students) more clearly recognise mathematics as an elegant and creative human activity could help improve participation in school mathematics. The study will look at ways for integrating musical note values into the teaching of fractions to students in Years 4, 5 and 6. Research questions guiding the study include: What are some ways teachers could integrate music note values into their teaching of fractions to support their students' conceptual understanding? What challenges might teachers encounter through such integration? Realistic Mathematics Education (RME) principles inform the study. RME recognises mathematics as a human activity which gives learners opportunities to view and reinvent mathematical concepts through mathematical encounters in real-world contexts (Cobb, Zhao & Visnovska, 2008), the real-world context in this study being music.

METHODS AND ANALYSIS

This study will be a qualitative, participatory, design research study. The researcher will set up a Community of Practice comprising a small group of Years 4, 5 and 6 mathematics teachers. The researcher will work with the teachers to develop, interrogate, and trial teaching resources and strategies for integrating music note values into the teaching of fractions. Data, collected via both individual and focus-group interviews, will be analysed using Karsenty and Arcavi's Six Lens Framework (2017), a framework which supports teachers in reflection on their actions and decisions.

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CURRICULUM NOTICING: A CASE OF LOWER SECONDARY TEACHER

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When teachers interact with curriculum materials, they require and use a special type of noticing. Curricular Noticing Framework (CNF) describes how teachers recognize opportunities within curriculum materials, understand their affordances and limitations, and use strategies to act on them (Dietiker et al., 2018). Curriculum noticing is a process that involves sets of skills that unfold in the phases of curricular attending, curricular interpreting, and curricular responding. The process of curriculum noticing is not linear. Curricular attending serves as the starting point of curricular noticing because teachers can interpret only what they attend to. The teacher might respond to what is interpreted, or the teacher's interpretation may raise a question that directs the attention to another part of the material, leading the teacher to the phase of attending once again. The study reported here is a longitudinal case study of an expert mathematics teacher from the lower secondary school (grades 5 to 8). The study investigates whether teacher's noticing practice changed over the years with respect to the official curriculum (i.e., the official textbook and curriculum outlines) and in what way. Data were collected at three time-points: in 2013, 2017, and 2021. Data set includes teacher's interviews about lesson preparation in all three-time points and lesson plan for the topic Corresponding angles. The longitudinal approach showed that the teacher's curriculum noticing has changed over the years, and more specifically, it showed how it changed. In all three-time points, the teacher used monthly or yearly plans, created at the beginning of the school year from national curriculum outlines, as the starting point for lesson planning. She attended the same textbook elements, but each time she responded differently. Responding became more aligned with active learning strategies. The CNF shows that teacher's interpretation of the material also changed. Overall, the results indicate that teacher's curriculum noticing changed due to the growth of knowledge related to mathematical ideas embedded in curriculum material, representations and connections among these ideas, problem complexity, and mathematical learning pathways (Remillard, & Kim, 2017).

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STUDENTS' PERCEPTION OF THE QUALITY OF MATHEMATICS TEXTBOOK

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Mathematics is a core subject in the secondary school curriculum. Textbook plays a critical role in learning mathematics. It has the potential to be powerful tool to help students develop an understanding of mathematics (Weinberg & Wiesner, 2011). However, textbook is not perfect. It may contain errors (Betts & Frost, 2000). Insufficient examples and examples containing errors were indicated (Huang & Cribbs, 2017). Students' learning may be affected by the errors in textbook. The situation may be getting worse if the teacher cannot detect the error. Thus, the research question is:

1. How the errors in textbook affect students' learning of mathematics?

The study was conducted at a public high school in Mainland China. One class in Grade 12 was selected by random sampling. The sample size is 33. Questionnaires were distributed to the students. Open-ended questions are used. The questions were written in Chinese so they can understand the questions better. They can answer the questions by both Chinese and English. The data was verbatim transcripted and processed by coding. The findings indicate that students' learning is affected by the error in the textbook. They said that they are confused by spelling mistakes. More time is used for solving the problems. In addition, they are misunderstood by the wrong answer. Thus, their academic performance may be affected. Furthermore, most of the students have a high expectation to Mathematics textbook. Accurate, well-organized, well-illustrated, rigorous, rational and easy to understand are their requirements. Also, worked solution for the exercises is highly preferred. Interestingly, some students hope that there is some space for them to take notes on the textbook. In summary, mistakes or errors in textbook affect students' learning of mathematics. In the presentation, further results will be discussed in detail.

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UNDERSTANDING THE APPROPRIATION OF GEOMETRIC KNOWLEDGE OF STUDENTS FROM THE EARLY YEARS

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This work presents partial results of a research with students from a Brazilian public school. The research aims to analyze the possible signs of transformation of students in the early years in the process of appropriation of geometric knowledge. The question of the research is: what are the signs of transformation of students in the process of appropriation of geometric knowledge in a teaching organization based on Cultural-Historical Activity Theory (CHAT)?

The Teaching-Orienteering Activity (TOA) (Moura, 2016) is assumed to be a theoretical-methodological principle that guides the action of the teacher. It is realized as a mediation between the teacher activity and the student's learning activity. Thus, the cognitive development of children occurs in a process of qualitative changes, in the course of their development. The appropriation process allows subjects to create new skills, which in this case refers to geometric thinking.

The methodology was a didactic-formative experiment (Davidov, 1988), composed of a set of Triggering Situations of Learning (TSL) involving geometric knowledge, based on the TOA (Moura, 2016). The TSL was developed for two consecutive years with 24 children aged 9 years old. The data collection instruments used were audiovisual recordings of the meetings.

The analysis was based on the concept of units of analysis, proposed by Vygotski (1995). As initial conclusions, in view of the teaching organization based on CHAT, we show qualitative changes in the subjects, in the quality of geometric thinking, through their collective experiences in the process of conceptual appropriation.

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DO PRE-SERVICE TEACHERS ATTAIN THE EVALUATION STANDARDS OF COMPULSORY EDUCATION?

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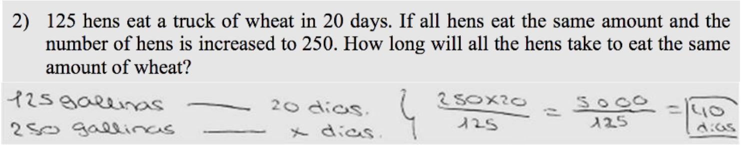
Forming capable teachers is a challenge. Spanish investigations suggest that pre-service primary school teachers lack of math subject-matter knowledge (SMK), and it may be also the case for pre-service early childhood teachers (Ortiz-Laso & Diego-Mantecón, 2020). Pre-service teachers should at minimum reach the SMK required at compulsory education (Leung et al., 2015). SMK is not only desirable for instruction but also to underpin the pedagogical knowledge taught at higher education. Thus, our study aims at analysing the extent to what pre-service early childhood teachers reach the evaluation standards of compulsory education.

A battery of mathematical tasks was administered to 165 subjects in North Spain. The analysis of their responses and the cognitive interviews revealed pre-service teachers' difficulties in: selecting adaptive strategies; managing numbers; applying algebraic reasoning; and establishing result consistency. The image exemplifies some of these difficulties. It is a missing-value problem with an inverse proportional relationship between quantities, which was unsolved by 80% of the subjects.


2) 125 hens eat a truck of wheat in 20 days. If all hens eat the same amount and the number of hens is increased to 250. How long will all the hens take to eat the same amount of wheat?

125 gallinas —— 20 dias.
250 gallinas —— x dias.

$\frac{250\times20}{125} = \frac{5000}{125} = \boxed{40\ \text{dias}}$

The analyses revealed that the pre-service teachers did not recognize the inverse relationship, and applied the *direct rule of three* often used at school. They did not consider an adaptive strategy and applied a low level of algebraic reasoning, driven by a rote learning approach. They did not attempt to establish result consistency despite obtaining that more hens will eat the same amount of wheat in more time. Just 20% of the teachers achieved the correct solution. Conclusions about the study in general as well as teaching implications will be deeply discussed in this poster.

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EFFECT OF ATTENDANCE AND ASSESSMENTS ON STUDENTS' PERFORMANCE IN LEARNING MATHEMATICS

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Student attendance is an important issue in higher education which is studied that there exists a positive correlation between class attendance and academic performance (Guleker & Keci, 2014). Meanwhile, assessment is an essential element in the educational process that uses measurements of the extent to which the learning experiences of students in order to achieve the learning outcome of the course of study (Balogun et al., 2017). In view of these, the study aims to investigate the effect of attendance and assessment of content knowledge on student's performance. The research question, what are the affecting factors to final examination score?

The sample group for this study is the 32 undergraduate students of Computer Sciences department, King Mongkut's University of Technology Thonburi, Thailand, who enrolled mathematics module in academic year 2/2018. The correlational analysis is performed on the data collected: number of attendances, scores of assessments obtained by in-class assessments, take-home assignments, and subtests.

Based on these results, the most affecting factor is in-class assessments and followed by subtests, take-home assignments, and attendance, respectively. Further, it is found that in-class assessment is the most affecting factor to students' performance. This is verified that the learning process should be improved continuously including new other activities for attracting a student to attend a class. Moreover, the results indicate that subtests and take-home assignments are also related to student's performance even they are not the most affecting factor because home-works are returned to students with feedback in class, as well as subtests are used to assess prior knowledge which is also done in class. Although student attendance is not a direct factor, it is an indirect factor affecting students' performance.

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ENHANCING TEACHER EDUCATION WITH CARTOON-BASED VIGNETTES: THE CASE OF CONCEPT CARTOONS

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Representations of school practice have an irreplaceable role in professional preparation of teachers. The representations might have various appearances based on video-recordings, audio-recordings, pictures, texts or their combinations. In professional preparation, they serve diagnostic or developmental purposes (Buchbinder & Kuntze, 2018). Educational vignettes called Concept Cartoons are based on a combination of pictures and texts and belong to such representations. Recent research has offered a methodology for the use of Concept Cartoons in diagnosing subject-matter knowledge and pedagogical content knowledge of prospective mathematics teachers in a paper-and-pencil form (Samková, 2019). In the coReflect@maths project, we develop a digital instrument called DIVER (Developing and Investigating Vignettes in teacher Education and Research) that would allow us to transfer the paper-and-pencil form to an interactive electronic form and thus broaden the possibilities of the use of Concept Cartoons in teacher education. Such an interactive form would facilitate the process of creating, modifying and reflecting Concept Cartoons and also the process of collecting data on knowledge and its development. The poster presents a sample of a Concept Cartoon and a set of indicative questions, and invites colleagues via QR code to exchange responses with the authors.

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MATHEMATICAL COMMUNICATION IN REMOTE LEARNING

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During the COVID-19 lockdowns mathematic teaching and learning in Germany were mostly held by online meeting tools (e.g. https://bigbluebutton.org). Considering the classification of traditional classroom interactions with digital technologies (e.g. Trenholm & Peschke, 2020), these situations are best described as digital face-to-face scenarios because they represent the digital counterparts of analog conversational situations. While learning mathematics with technology, communication can be distinguished: *Communication through technology* refers to the simple use of technology in mathematics classrooms (e.g. beamers can be used to display and share the generated learning products). *Communication of technology* refers to the interaction evoked by the output of the technology (e.g. the discussion about the underlying mathematical concept while using a dynamic geometry software) (Drijvers et al, 2016).

In our Teaching and Learning Lab, digital face-to-face meetings have been conducted and recorded. Analyzing two scenes with an interpretative perspective (Voigt, 1995), observations regarding interactions, indications of particularities in and about learning mathematics with technology are explored. Two research questions are examined in detail: (1) To what extent do the interactants manage to establish successful communication about mathematics? (2) And how can this be mapped with existing frameworks for communication about mathematics with the help of digital media?

By comparing the interviews, it is possible to reconstruct the use of digital features to generate mutual attention and interpretations as an element for successful communication between the interactants. In relation to the second research question, the digital face-to-face communication in the analyzed meetings can be classified as “communication through”. However, a differentiation of this category with regard to the mathematical negotiations between the interactants will be discussed.

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PARTICIPATION IN MATHEMATICS CLASSROOMS – A QUALITATIVE EXPLORATORY INTERVIEW-STUDY IN A COMPARISON BETWEEN THE USA AND GERMANY

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The political inclusion discourse began in the USA in the 1970s with the Political Law (PL) 94-142: Education for All Handicapped Children Act (U.S. Department of Education, 2010). This often leads to the assumption that the USA is "further" with regard to the implementation of inclusion and can thus serve as a model for implementation in German-speaking countries (cf. Johnson, 2016). This model character may therefore also apply to inclusive mathematics classrooms.

The aim of our empirical case study is to survey the realizations of inclusive concepts in mathematics classrooms in the USA and in Germany. To gain first insights, a pilot study with two partner schools has been conducted. The research questions are: (1) How do teachers organize their lessons in order to do enable participation for all learners? (2) To what extent can similarities and differences be reconstructed?

The sample comprises interviews with four US and four German mathematics teachers from our partner schools in Colorado and North-Rhine-Westphalia. The interviews were conducted as semi-structured interviews (cf. Edwards & Holland, 2013). Impulse questions were asked on the subject areas "good mathematics teaching", "typical mathematics teaching", "heterogeneity" and "inclusion". The collected data is analyzed via qualitative content analysis (Mayring, 2000).

As a result, the difference between "good" and "typical mathematics teaching" in the interviews with the German teachers is particularly striking. In contrast, the analysis of the interviews with US teachers did not reveal such a large discrepancy between aspiration and reality. In the presentation further results will be discussed in detail.

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MATHEMATICAL ACHIEVEMENT THROUGH COOPERATIVE, CROSS-COMUNITIES STUDY

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INTRODUCTION AND BACKGROUND

Schools focus mainly on academic skills and there is a lack of social-emotional competencies (SEL) among schoolchildren (Blum, Libbey, Bishop & Bishop, 2004). Researchers have argued that education should foster not only academic development, but also social, political, and emotional development (Bekerman, 2005; Durlak, Weissberg, Dymnicki, Taylor & Schellinger, 2011). Skills in these areas are part of a broad range of 21st-century skills, such as communication, collaboration, teamwork, and empathy (Pellegrino & Hilton, 2012).

In Israel, Arabs and Jews usually attend different schools and live in different areas (Smooha, 2010).

THE ACTIVITIES

The teaching unit was developed by Jewish and Arab apprentice teachers from Beit Berl Academic College as part of the Living Together program. The apprentice teachers planned a series of lessons to be taught to Jewish and Arab elementary-school students who would be studying together. It was decided that the teaching unit would include mathematics and art, as those two disciplines are “languages without words” that can bridge the distance between cultures and overcome language difficulties. We chose math topics that combined visual elements with outside-the-box thinking. The tasks were based on comparisons (as opposed to mathematical procedures), to facilitate meaningful and experiential learning and provide the children with an opportunity for a shared mathematical discussion. The lessons that integrated mathematics with art included topics such as polygons, series and transformations. Those topics were chosen because they are taught throughout elementary school, in several different grades, and because they spark mathematical discussion and can be addressed in research activities, games and experiential tasks.

GOALS OF THE ACTIVITIES

The activity program was guided by two main goals. The first goal was to improve mathematical achievement through cooperative, cross-sector study. The second goal was to cultivate social and emotional skills that are important for life in a multicultural society.

QUALITATIVE FINDINGS FROM STATEMENTS PROVIDED BY THE APPRENTICE TEACHERS

The creative activities in the groups helped the children to start conversations, cooperate with one another and form interpersonal and intercultural connections. Learning words in the foreign language also helped form bonds. The creative games that were characterized by a

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sense of belonging, a connection to daily life and/or a common subject led to fruitful cooperation among the children, which was enjoyable and moving.

During the lesson, there was full cooperation among the children and that was very noticeable. The learning was active and enjoyable and combined math games, art and the students getting to know each other.

A game that included familiarity with math concepts in the two languages was a very special experience. It was clear that the children enjoyed the game, sufficient time was allotted to it and the game included higher-order thinking, as well as strengthening and expanding vocabulary in a foreign language. We could see that everyone was participating and learning.

DISCUSSION OF THE QUALITATIVE FINDINGS

The findings show that co-learning within a divided society benefits social and emotional development, as noted previously by Durlak et al. (2011). This co-learning also enhances empathy at the intergroup level, as noted previously by Batson and Ahmad (2009).

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CONSTRUCTING VIGNETTES TO STIMULATE THE PROFESSIONAL KNOWLEDGE OF PRE-SERVICE TEACHERS

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Vignette activities are valuable ways to prompt critical examination and personal reflection of pre-service teachers as they develop their professional knowledge. Well-crafted vignettes (in text, cartoon, video formats) are important for two reasons. First, they succinctly represent fictitious 'situations' which are discernible by participants, allowing them to form interpretations of familiar experiences and provide opportunities for linking theoretical elements to practice via transformative experiences (Nind & Pepin, 2009). In addition, purposefully designed vignettes provide credible research methods for eliciting value laden constructs and are therefore effective for identifying and challenging the beliefs of pre-service teachers in mathematics education courses. Designing and constructing vignettes according to a *vignette framework*, such as that developed by Skilling & Stylianides (2019), connects underlying theories to the vignette narrative, providing credibility of this method for research purposes. The poster portrays how two image/text vignettes are used in teacher training courses in mathematics education and outlines their contribution to the coReflect@maths project. A QR code is included for exchanging responses with the authors.

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AN INVESTIGATION ON USING TASKS TO PROMOTE STUDENTS' ABILITIES IN SOLVING MATHEMATICAL PROBLEMS: UTILIZING OPEN APPROACH METHOD IN LESSON STUDY PROCESS

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This research aimed to investigate 24 students' abilities in solving mathematical problems by setting up tasks through Open Approach in the Lesson Study process. The case study method was employed to explore students' abilities in solving mathematical problems. A total of five undergraduate students were purposively selected as they have been trained through a teaching professional development internship program using Open Approach in Lesson Study process (Inprasitha, 2010). The five undergraduate students set up a Lesson Study team and conducted classroom that using tasks to promote students' abilities in solving mathematical problems (Isoda, 2012). In each Lesson Study cycle, the Lesson Study team members would plan to design tasks, teach, and observe the research lesson. Qualitative data was collected through participatory observation, reflection sessions, collection of artifacts, and interviews based on the six open-ended problems derived from the topic of addition and subtraction in the 6th Grade mathematics textbook. Data were analyzed using protocol analysis and analytic description.

Qualitative results revealed that there are several pieces of evidence from the four phases of the Open Approach to support their problem-solving abilities. The evidence of posing open-ended problems showed that students read and listened to the teacher's instruction, asked for clarification if they were unclear about the given tasks. This is followed by the second phase about students' self-learning whereby students are expected to search for the solutions to solve the mathematical problems by understanding the sequence to complete the tasks. The results of the third phase about whole-class discussion and comparison indicated that students were able to describe, compare several alternatives, and answer questions confidently. Finally, students showed that they were able to connect their mathematical ideas emerged in the classroom and making a summary by showing what they have learned and shared their ideas with their peers. In conclusion, the results of this research contribute significantly to the importance of Open Approach method in the Lesson Study process in promoting students' problem-solving abilities.

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SIXTH-GRADERS’ MATHEMATICS SELF-EFFICACY, LEARNING MOTIVES, AND PROBLEM-SOLVING SKILLS WITH DIFFERENTIATED INSTRUCTION

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Previous studies showed that two psychological states, i.e., mathematics self-efficacy belief (MSE) and mathematics learning motives (MLM) are significantly correlated with students’ learning achievement and performance (You, Dang, & Lim, 2016); for instance, various strategies used for learning, positive learning behaviors, higher-level thinking and mathematics problem-solving skills (MPSS). Nowadays, leaners with mixed ability, this is specifically true for mathematics, sit in every classroom. The differentiated instruction (DI) accordingly provides a balanced solution for a teacher to conform to every leaner’s needs by furnishing him or her proper learning tasks and essential social interactions. Regarding mathematics teaching and learning, empirical evidence indicates that implementing DI is beneficial for both promoting mathematics teachers’ professional development and increasing their students’ learning outcomes, interest, and confidence (Tomlinson & Moon, 2013).

Consequently, it is logical to expect that this favorable DI learning environment promotes the positive growth of students’ two psychological states and their academic learning performance (i.e. MPSS). There are two main purposes in this study: First, under the DI learning context, by means of a longitudinal design with pre- and post-tests, it is to determine the effectiveness of this DI intervention in promoting 6th graders’ MSE, MLM, and MPSS, where the DI intervention will be briefly introduced in this presentation. Secondly, it is to examine the relationship among 6th graders’ MSE, MLM, and MPSS after one year DI intervention. The targeted 6th grade teacher and her colleagues formed a “mathematics teacher learning community” and received the DI professional development in mathematics, where she designed and implemented 2 research lessons every semester in her classroom. Her 25 students were the main participants in this study. Three instruments for evaluating students’ MSE, MLM, and MPSS were used to collect the data, while corresponding statistical analyses were applied to address the objectives.

The findings showed that the application of the DI learning environment was significantly beneficial for advancing 6th graders’ MSE, MLM, and MPSS. Besides, MSE significantly predicted MLM and MPSS as well, such that the mediating impact of MLM on the effect of MSE on MPSS was partial. This finding shows that the higher a 6th grader’s mathematics self-efficacy beliefs, the better her/his own mathematics learning motives are, which in turn advance her/his mathematical problem-solving skills in school. Finally, a discussion of the findings and recommendations for future study and further improvements was proposed, as well as the limitations of this study.

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COLLOQUIUM

TEACHER PROMOTING STUDENT MATHEMATICAL ARGUMENTS THROUGH QUESTIONS

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In this paper we focus on how a teacher promotes the construction of arguments and encourages the development of student's argumentation skills through questions in the context of collective argumentation. In the frame of Global Argumentation Structure (GAS), we reconstructed the argumentation that occurred in a mathematics class with fifth graders (K-5). Findings provide four types of questions that teacher used to promoted collective argumentation and posed to students: an answer, a warrant, an evaluation of partners' arguments and a drawing as backing.

INTRODUCTION

Research has documented that teachers play an important role in mathematics argumentation, because they support collective argumentation in various ways: posing problems to be solved, contributing various parts of arguments, and asking questions (Singletary & Conner, 2015), also their interventions promote students to share mathematical ideas (Mueller, Yankelewitz, & Maher, 2014), and increase the opportunities of students' participation (Graham & Lesseig, 2018). The purpose of this paper is to reveal how teachers' questions promote the construction of arguments in the mathematical classroom, encourage the development of argumentative skills in students, the confrontation of arguments and the refutation of conclusions. To do so, we use the analysis of the Global Argumentation Structures (GAS), they describe larger structures of argumentation, contain students' arguments and reveal a complete overview of the argumentation that took place in the classroom (Knipping & Reid, 2015). The following research question guides the study: What teacher's questions promote the construction of arguments in the collective argumentation?

COLLECTIVE ARGUMENTATION

It is widely recognized that refutation and argumentation are important in mathematics classroom activities (Komatsu & Jones, 2019; Cervantes-Barraza, Cabañas-Sánchez, & Reid, 2019). We understand collective argumentation as the social and rational activity that occurs in the interaction between students and teacher with the purpose of building a final conclusion based on evidence (data) and valid reasons (warrants). In this activity, the teacher manages the participations of students and the refutation of partners' arguments through questions. In this paper, we refer to a refutation as a mathematical statement that denies completely one part of the argument (see Figure 1). Three ways in which

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an argument can be refuted have been reported: The *data* of the argument can be refuted, leaving the conclusion in doubt. The *warrant* of the argument can be refuted, again leaving the conclusion in doubt. Or the conclusion itself can be refuted, implying that either the data or the warrant is invalid, but not saying which (Reid, Knipping, & Crosby, 2011, p.3).

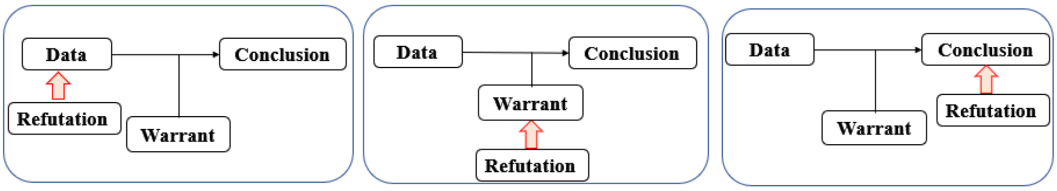


Figure 1: Ways of refuting an argument. (Source authors)

PROMOTING STUDENTS’ CONSTRUCTION OF ARGUMENTS

The teacher plays an important role in collective argumentation because s/he contributes part of the argumentation (e.g., data or warrants), asks questions to support the construction of arguments (Singletary & Conner, 2015), increases the opportunities of students’ participation through instructional activities and promotes their disposition towards argumentation (Graham & Lesseig, 2018).

Extending the teacher’s contributions reported in the literature in order to support argumentation in classroom, in this paper we focus on how the teacher promotes the construction of arguments and encourages the development of the students’ argumentation skills through questions. By argumentation skills we refer to the students’ capabilities of carrying out a particular activity in the frame of argumentation such as: justifying, evaluating, and refuting. Based on empirical results, we provide a category of questions that teachers ask in order to manage the confrontation of arguments and refutations of conclusions. In Table 1 we present the categories of questions, their description and examples of teacher’s questions used in a mathematics class with fifth graders in the context of triangle classification according to angle measure.

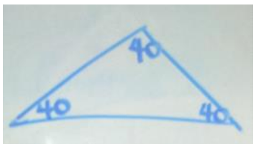
Category	Description	Example
Request an answer	Ask students to present a closed answer (conclusion) of “yes” or “no” type.	**Teacher:** Are there equilateral triangles with one angle greater than 90°? Justify your answer [T2] **Teacher:** What is your answer? [T3]
Request an evaluation	Ask students to evaluate partners’ arguments. Answers of this type of question call on students to refute conclusions, valid them, or show the sufficiency of warrants.	**Teacher:** What can others say? [T3 & T6] **Ulises:** It is wrong! (Refuting) **Teacher:** All of you agree? [T5] Julieta: Yes! (validation) **Teacher:** Bigger than what? [T1] **Agustin:** It is going to be bigger than one hundred and eighty! (Sufficiency of warrant)
Request a warrant	Ask students to justify their answers (conclusions).	**Teacher:** Why do you say that they do not exist? [T3] **Teacher:** Why the conclusion of your partner is wrong? [T5]
Request a drawing as backing	Ask students to explain their conclusions and warrants based on drawings on the board.	**Teacher:** Can you go to the board and explain to your partners that it is not possible? [T6] **Manu:** It is not possible!... because the sum of angles is not one hundred and eighty… [the student went to the board and drew an isosceles triangle with three angles of forty degree] 

Table 1: Types of questions that promote the construction of arguments. (Note: T1, T2, T3, …T9 means task 1, task 2, and so on.)

METHODOLOGY

The analysis presented in this paper is part of a wide study developed with fifth graders (K-5) with the aim of develop students' argumentation abilities in the context of collective argumentation through teacher's questions. In this paper we focus on students' arguments and teacher interventions during the development of one of the nine mathematics tasks.

Mathematics Task

The design of the tasks considered design principles in order to promote collective argumentation in classroom (Cabañas-Sánchez & Cervantes-Barraza, 2019). The mathematics content of the tasks refers to triangle classification based on internal angle measures (see Table 2). The tasks were designed with the aim of requiring students to construct arguments about the existence of equilateral, isosceles and scalene triangles with an angle less, greater or equal to ninety degrees.

T1: Are there equilateral triangles with a 90° angle? Justify your response.	**T4:** Are there isosceles triangles with a 90° angle? Justify your response.	**T7:** Are there scalene triangles with a 90° angle? Justify your response.
T2: Are there equilateral triangles with an angle greater than 90°? Justify your response	**T5:** Are there isosceles triangles with an angle greater than 90°? Justify your response	**T8**: Are there scalene triangles with an angle greater than 90°? Justify your response
T3: Are there equilateral triangles with an angle less than 90°? Justify your response	**T6:** Are there isosceles triangles with an angle less than 90°? Justify your response.	**T9**: Are there scalene triangles with an angle less than 90°? Justify your response.

Table 2: Tasks about triangle classification

Participants and Classroom Activity

The participants were 22 fifth grade students from an urban school located in southern Mexico. The authors of this paper played the role of teacher, developed all the tasks with the collaboration of two auxiliary researchers and the teacher responsible for the group participated in the role of observer. The activity in the classroom took place in two different phases: individual and collective. In the first phase, students answered each mathematics task on their worksheets. In the second phase, the teacher promoted the construction of collective arguments and fostered the students' development of argumentative skills like refuting, justifying and arguing.

Analysis and Data Collection

The qualitative analysis of data is based on Knipping and Reid's (2015) proposal for the reconstruction of mathematics argumentation in classrooms in terms of local and global arguments. According to Knipping and Reid (2015) the reconstruction of argumentation consists of a three stage process: 1) Reconstructing the sequencing and meaning of classroom talk, 2) analyzing arguments and argumentation structures, and 3) comparing argumentations to reveal their rationale. We reconstructed and present here some findings from the argumentation that occurred during Task 5, because this task contains all the types of questions, which allows us to provide evidence of each type. Interactions in the classroom were video recorded, transcribed and the students' arguments reconstructed.

Types of teacher questions that guided the construction of arguments were identified from transcripts of the sessions. Questions were classified as requesting: an answer, an evaluation, a warrant and a drawing as backing. We added teacher questions as a part of the local and global argumentation structures, because they guide the students in the construction and confrontation of arguments. In the local argumentation the questions are presented in *italics* located in the moments when the teacher made them and, in the global argumentation structure we represented them with question marks (¿?).

PROMOTING ARGUMENTS CONSTRUCTION THROUGH QUESTIONS

Questions that Request an Answer

Task 5 required the students to argue about the existence of isosceles triangles with an angle greater than ninety degrees (see Figure 2). The global argumentation structure of this task is made up of three argumentation streams (AS). In the first argumentation stream (AS-1) the teacher, in order to promote the construction of arguments about the existence of the triangle under study, read the question of the task and provided the initial information (D1). Based on the task question, the teacher asked: *Who said not*? And, *Someone else*? requesting students to present "yes" or "no" answers as their conclusions (C1 and C2).

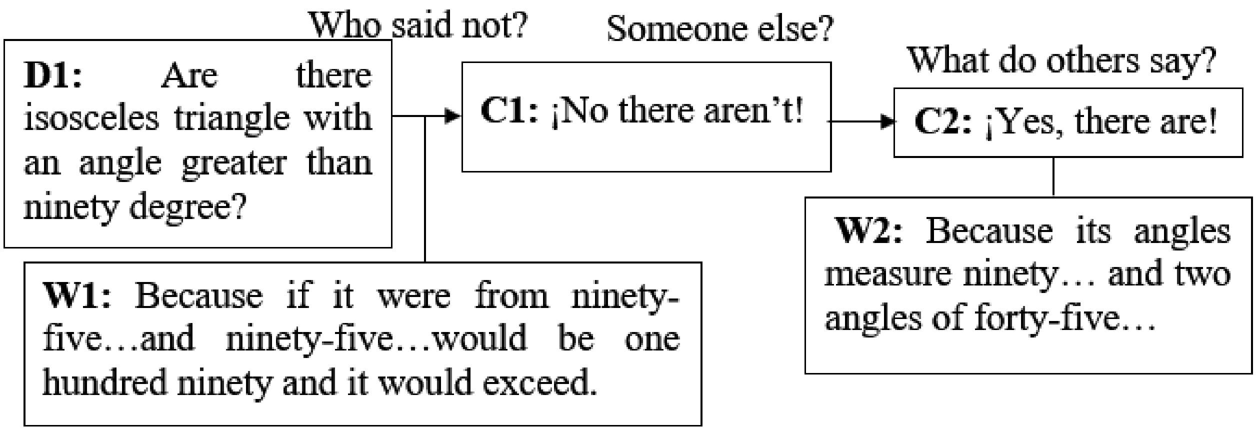

Who said not?
Someone else?
What do others say?
D1: Are there isosceles triangle with an angle greater than ninety degree?
C1: ¡No there aren't!
C2: ¡Yes, there are!
W1: Because if it were from ninety-five…and ninety-five…would be one hundred ninety and it would exceed.
W2: Because its angles measure ninety… and two angles of forty-five…

Figure 2: Teacher's questions that request an answer in AS-1.

Questions that Request an Evaluation

This type of question encourages students to evaluate their peers' arguments. The answers of this type of question force students to refute conclusions, validate them or give evidence for the sufficiency of warrant. In the case of (AS-1), the teacher asked *What do others say*? with the aim of involving students in providing an evaluation about conclusions (C1 and C2) and promoting that peers refute them (see Figure 3).

Questions that Request a Warrant

Following the argumentation flow, in the second argumentation stream (AS-2) two students refuted (R1 and R2) the conclusion (C2) (see Figure 3) and presented warrants (WR1 and WR2) in response of the teacher's question: *Why is he wrong*? with the objective of requesting a justification of the refutation presented by the students and convince the students that presented the conclusion (C2).

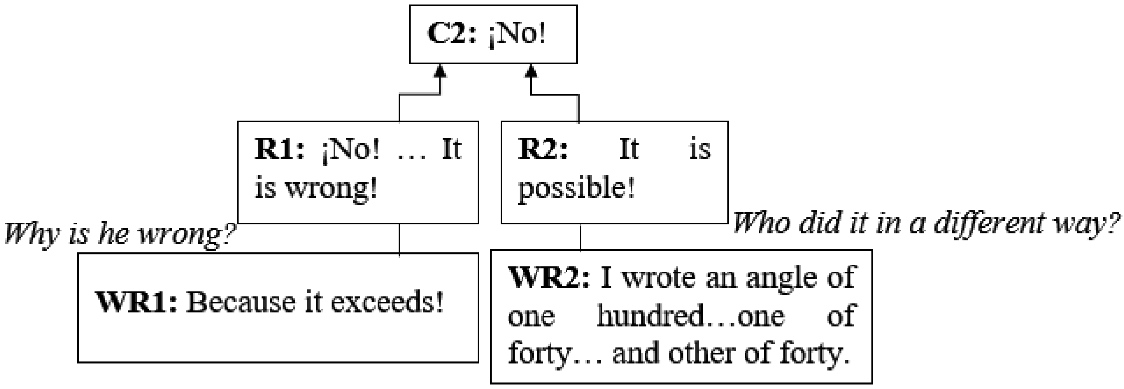
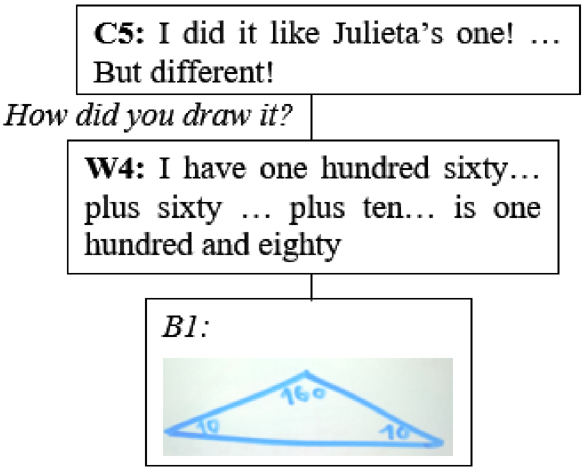

C2: ¡No!
R1: ¡No! … It is wrong!
R2: It is possible!
Why is he wrong?
Who did it in a different way?
WR1: Because it exceeds!
WR2: I wrote an angle of one hundred…one of forty… and other of forty.

Figure 3: Refutation of conclusions C2 and C1, and teachers' questions in AS-2

This type of question promoted that students offered warrants with the function of justifing the refutation of conclusions (C1 and C2). The warrant (WR1) implies the property that the sum of the internal angles of any triangle is 180 degrees, and the warrant (WR2) contains measures of angles that satisfy the property and make valid the refutations. We also recognize that the teacher question *Who did it in a different way*?... *Where it is possible?* requested to the students an evaluation of the argument, particularly about the conclusion (C2) and (W2) in figure 2.

Questions that Request a Drawing as Backing

This type of question asked students implicitly to provide a drawing of particular cases mentioned, backing up the warrant that justifies the students' conclusions. In argumentation stream (AS-3) the teacher guided the students in the construction of the final conclusion of the task. To do so, he made different questions with the purpose of validating the existence of an isosceles triangle with an angle greater than ninety. The question *Someone did it in a different way*? (see Figure 4) motivated the participation of Agustin. This student drew a triangle on the board to support the conclusion (C5) and the warrant (W4).



C5: I did it like Julieta's one! … But different!

How did you draw it?

W4: I have one hundred sixty… plus sixty … plus ten… is one hundred and eighty

B1:

Figure 4: Argument of Agustin in AS-3 with a drawing of an isosceles triangle drawn on the board

Global Argumentation Stream of Task 5

In the global argumentation structure of the task (see Figure 5), we recognized different teacher's questions that guided students in the construction of arguments through the final conclusion "They do not exist" (C7). We also documented questions that implied conclusions, warrants, drawings as a backing and an evaluation of other's arguments in (AS-1) and (AS-2). About (AS-3), teacher asked different questions such as: *Do you agree*? *Someone did it in a different way*? *How did you draw it*? implied conclusions (C4, C5, C6, C7) and encouraged students in providing warrants (W4 and W5) that justified the conclusions.

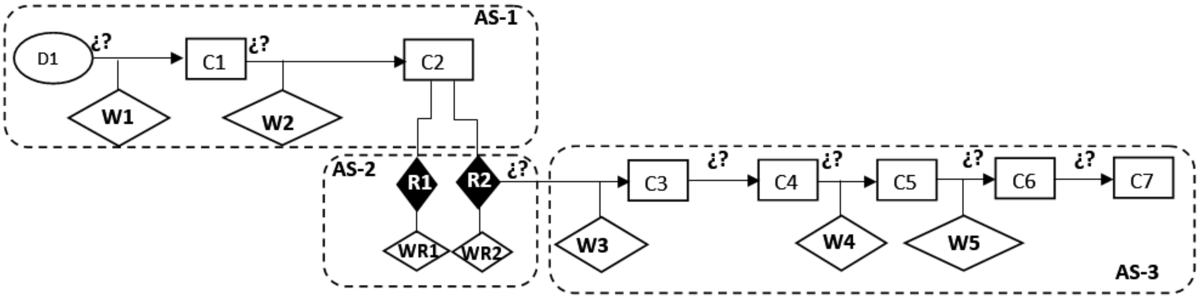


Figure 5: Global argumentation structure of the task 5

This global argumentation structure of the task is made up of three argumentation streams that show the interaction between the students and teacher, who guided them to construct a final conclusion in the task. Contrasting findings of this paper with research reported about argumentation structures, we recognized that the structure of this task is similar to source-structure documented by Knipping and Reid (2015). This consists of several data and conclusions that support the same final conclusion. As a contribution of this paper, we included teacher interventions in the global argumentation structure exactly in the moment where teacher fostered the construction of arguments through questions (¿?).

CONCLUDING REMARKS

In this paper we revealed how the teacher promotes the construction of arguments from questions in the mathematics classroom. The teacher poses questions that request from students an answer, an evaluation, a warrant and a drawing that supports the conclusions presented. In general, the teacher's questions fostered the development of argumentation skills, guided the construction of arguments, refutation of conclusions and promoted the presentation of necessary arguments elements (e.g., warrants, backings, conclusions). Also, we highlight that these types of questions can be helpful for future and in-service teachers in fostering argumentation taking them as a part of their classes.

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CHARACTERIZING PROOF AND PROVING IN THE CLASSROOM FROM A CULTURAL PERSPECTIVE

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This theoretical paper proposes a new perspective on identifying and characterizing the cultural specificities of proof and proving in the classrooms of a given country. To this end, based on the related literature, researchers propose "structure", "language", and "function" as a triplet of aspects that constitute proving activities. Researchers then exemplify each aspect in an example case of proving activities in a Japanese classroom and discuss how it allows us to characterize the cultural specificities of proof and proving.

INTRODUCTION

What is this thing called "proof"? The meaning of proof and proving is still a subject of debate among researchers in mathematics education (e.g., Stylianides, Bieda, & Morselli, 2016). Some previous studies have shown how proof and proving are differently situated in the curricula, textbooks, and classroom practices of different countries (e.g., Jones & Fujita, 2013; Knipping, 2004; Miyakawa, 2017). These studies imply that proof and proving in mathematics are culturally embedded activities. However, the number of earlier studies on proof and proving from a cultural or international perspective is relatively small (Reid, Jones, & Even, 2019); therefore, further theoretical, and empirical studies on this topic are needed.

Since the diversity in the definitions of proof and proving may constitute an obstacle to international communication among researchers from different countries (Reid, 2015), it is important to take into account researcher's epistemology on proof (Balacheff, 2008) as well as the cultural dimension of proof (e.g., Shinno et al., 2018). Although the expression "cultural" is rather ambiguous, researchers understand it as "institutional", based on *The Anthropological Theory of the Didactic* (hereafter the ATD; Chevallard, 2019). Within the ATD, a mathematical object (proof and proving in our case) exists in each *institution* under the influence of several factors of different origins. According to this tenet, cultural factors determine the way in which students relate to proof and proving and produce diversity according to the institution to which the object belongs. To gain deeper insight into the cultural specificities of proof and proving in a given institution, researchers need to develop a theoretical lens to analyze and explain such specificities. Therefore, the research question in this research is as follows: *What are the critical aspects that allow*

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researchers to identify the cultural or institutional specificities of proof and proving in the mathematics classrooms of a given country? To answer this, we propose a new theoretical perspective and then exemplify it in an example case of proving activities in a Japanese middle school classroom.

THEORETICAL PERSPECTIVE

Related Literature: Different Aspects of Proof and Proving

What counts as a principal aspect of proof depends on the theoretical perspective adopted for the research being undertaken. Mariotti et al. (1997) proposed the notion of *mathematical theorem* that consists of a system of relations between a *statement*, its *proof*, and the *theory* within which the proof makes sense. Balacheff (1987) proposed a framework composed of *knowledge*, *formulation*, and *validation*. These works certainly deal with crucial aspects when analyzing and understanding the complex nature of proof and proving, especially in relation to the mathematical knowledge behind it.

When discussing proof and proving in the classroom, another important aspect is its relationship with argumentation, which directs us to the discourse or rhetorical means to convince others (Stylianides et al., 2016, p.316). Duval (1991) nicely characterized the functional aspect of mathematical proof with respect to argumentation in terms of *the epistemic* and *logical values* attributed to the statement to be proved: The mathematical proof provides the logical value (true or false), while the argumentation changes the epistemic value that is the degree of certainty the collocutor has with the statement (certainly, probably, etc.). Knipping (2008) adopted Toulmin's argument model to describe the argumentation structure in the proving process. This offers the *argumentation analyses* that make it possible to compare and infer the rationale of the argumentation in both local and global structures through classroom talk (Knipping & Reid, 2013). It is also important to note that argumentation is the act of persuading someone with a claim in ordinary life, since proving often plays this role even though it is not an ordinary practice in some countries (Sekiguchi, 2002; Sekiguchi & Miyazaki, 2000).

Although different theoretical approaches have been used to characterize proof and proving (or argumentation) in mathematics education so far, cultural, or institutional perspectives are rarely considered. Nevertheless, some ideas from the existing frameworks mentioned above can be reconsidered and integrated into a new theoretical perspective to characterize what constitutes proof and proving from a cultural perspective. Our proposition is that proving activities in a given institution can be characterized by three aspects: *Structure*, *language*, and *function*. Although these aspects have already been mentioned in different ways in previous studies, researchers intend to reconceptualize them as a triplet to identify the cultural specificities of proving in the classroom. Let us briefly explain each aspect, and then provide some examples.

Proof and Proving as a Triplet: *Structure, Language,* and *Function*

Structure denotes here the organization of reasoning or arguments showing how different statements consisting a proof are connected. The structure required in a proof may differ according to the institution. For example, in our everyday life, arguments are given one after another to persuade the collocutor of the validity of a statement (Duval, 1991). In contrast, proof and proving in school geometry often requires a basic step consisting of "given statements", a "theorem/axiom/definition", and a "conclusion", as well as the chain of steps in the propositional logic. At a more advanced level, it may be more appropriate to grasp the reasoning for the universal proposition in the predicate logic including quantification (Durand-Guerrier & Arsac, 2005). The argumentation analysis (Knipping, 2008) is a way to identify such a structure that may be implicit in classroom interactions.

Language is the semiotic representation, *register* or not (Duval, 2006), used in a given institution to express the arguments and structure of reasoning. Gestures and oral discursive representations are also languages that may express arguments and the structure of proof. In the classroom, different representations are used such as gestures, oral and/or written discourse, diagrams, and so forth (Chen & Herbst, 2013). Since the formulation of proof is concerned with ordinary language, this aspect shows strong cultural effects at both the grammatical and semantic levels.

Function has been extensively studied in the mathematics education research field (e.g., Hanna, 2000). However, the function attributed to the proof differs according to the institution and is not reserved to the ones often mentioned in the literature (verification, illumination, communication, systematization, and discovery). In other words, what is called "proof" may differ according to the function attributed to the justification. For example, Miyakawa (2017) showed that in French lower secondary schools, proof is a means to justify a statement without relying on perception or visual information.

EXAMPLES: PROVING IN JAPANESE LESSONS

To exemplify how and to what extent our theoretical perspective allows us to account for the cultural specificities of proof and proving in the classroom, researchers will provide empirical data to be analyzed in the next section.

Lessons in a Japanese Middle School

Researchers collected data on ordinary mathematics lessons in a public middle school. A series of five mathematics lessons given to a Grade 8 class (13–14 years old; 35–40 students) were videotaped. A teacher was asked to give ordinary lessons for a unit on "conditions for a parallelogram", right after teaching proofs and congruent triangles in geometry lessons. This middle school teacher had around 20 years of teaching experience.

Fig. 1: Oral proving by the teacher

One of the specificities in the proving activities through the five lessons is that the written proof was given only twice, while there were 10 oral proving activities for the true statements (and five others for the false statements). Below, researchers describe two example cases of proving: One given orally, and another given orally then in a written form.

Case 1: Oral Proving

Case 1 is taken from the third lesson, in which the class proved one by one different statements on the conditions for a parallelogram. The statement was that a quadrilateral with two pairs of equal opposite sides is a parallelogram, which is considered as an important theorem in the textbook used in this class. The teacher reviewed the proof of the statement as follows:

> Teacher (T): I will tell how we proved it. Pay attention. Well, by drawing [a diagonal], two triangles appear. A pair of sides are equal. The second pair, [are] equal, the third pair, overlapping ones are equal [T draws a round mark on the diagonal (Figure 1)], so [they are] congruent. Since [they are] congruent, this angle and this angle are equal [T draws marks for equal angles]. Well, since [they are] congruent, this angle and this angle are equal [T draws marks for equal angles]. Well, next. Since this angle and this angle are equal, Z appears, and [they are] alternate-interior angles [T draws marks for parallel lines], [they are] parallel. Well, [they are] parallel. Then, here, and here, since the angles are equal, Z appears, and [they are] alternate-interior angles [T draws mark for parallel lines]. Well, it [the diagram] is now a mess, but we can say [it is] a parallelogram. Okay, we can say it.

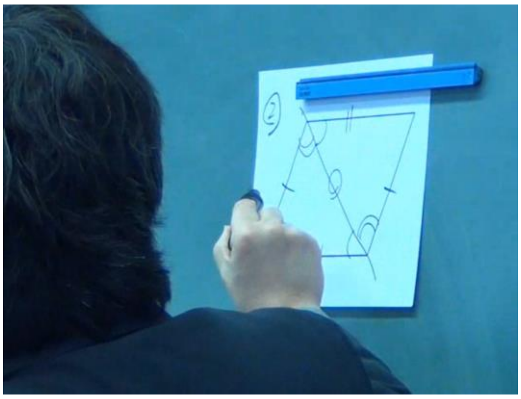
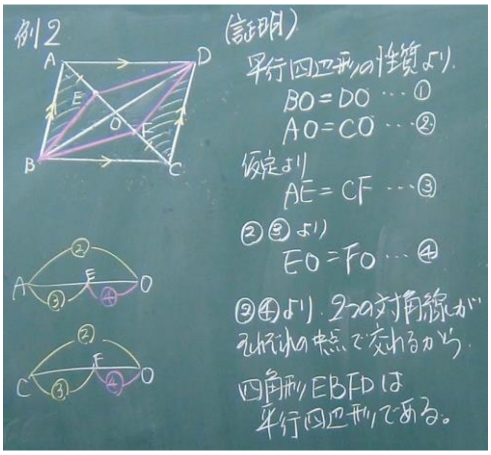


Figure 1: Oral proving by the teacher

Case 2: Written Proof

In the fourth lesson, the teacher proved the statement "Like the right diagram, when taking two points E and F so that AE = CF on the diagonal AC of the parallelogram ABCD, prove that the quadrilateral EBFD is a parallelogram," which is an exercise in the textbook. The statement and proof were first given orally through interactions with students, and then the proof was given in written form on the blackboard (Figure 2). The proving process was therefore divided into two phases: Oral proving and its formulation as a written proof, as in the following transcript:


例2
A
B
C
D
E
F
O
②
③
④

（証明）
平行四辺形の性質より.
BO = DO …①
AO = CO …②
仮定より
AE = CF …③
②③より
EO = FO …④
②④より. 2つの対角線が
それぞれの中点で交わるから.
四角形EBFDは
平行四辺形である。

(Proof)
Based on the property of a parallelogram,
BO = DO ... (1)
AO = CO ... (2)
From hypothesis
AE = CF ... (3)
From (2) and (3)
EO = FO ... (4)
From (2) and (4), since the two diagonals intersect at their midpoints, the quadrilateral EBFD is a parallelogram.

Figure 2: A written proof given on the black board and its English translation

Teacher: We are going to write what I said now. At first, we say that BO and DO are equal, and AO and CO are equal. This is like what we have done so far. This is not new. Well, as the quadrilateral at the very beginning is a parallelogram, what we can use is this [writing on the board]. Ok? Well, [it is] the property of parallelogram.

EXEMPLIFYING PROOF AND PROVING AS A TRIPLET

Researchers will now analyze these two cases together from our theoretical perspective of proof and proving as a triplet, by identify the *structure* of reasoning given in these cases, the *language* used to describe the structure, and the *function* played by proof and proving in the classroom.

Structure of Reasoning

The proving in Case 1 consists of the explanations of the congruent triangles and parallel lines by the alternate-interior angles. One could identify the structure of reasoning from the teacher’s speech and gestures on the diagram in the video (whereas there are many implicit points): The first step is to prove that the three pairs of sides of the two triangles are respectively equal; the second is to prove the congruent triangles; the third is to prove the equal angles; the fourth is to prove the parallel lines, and the last is to prove the parallelogram. This structure is based on propositional logic, since the teacher never mentions quantifications such as “any” or “all”, and the statements are connected from the hypotheses to the conclusion by means of the geometrical properties (definition and theorems).

In Case 2, the class is seeking the reasoning structure that connects the hypotheses to the conclusion. The process of proving goes backward from the conclusion to the hypotheses. This is different from Case 1, in which the teacher proved forward like a written proof from hypothesis to the conclusion. However, the reasoning structure one may identify in the oral proving and written proof is similar to Case 1. The statements are connected by the properties in each step, and the intermediate conclusion is reused in the next step as a given statement until the target conclusion. This structure of reasoning is more explicit in the written proof. In addition, quantification is not mentioned in this case either.

Language

What kinds of language are used to express the structure of reasoning? In the oral proving of both cases, the language used by the teacher is an amalgam of oral discourse, diagrams, and gestures. What he says could not be a written proof per se and would not make sense without gestures and diagrams (see Case 1). Further, in Case 1, the teacher never uses labels (e.g., A, B, C, etc.) when referring to the points or angles. Instead, he often uses “demonstrative words” (e.g., “this” or “they”). This implies that the teacher sees the diagram and gestures as parts of the proving.

In Case 2, the teacher provides a written proof after the oral proving (Figure 2). It is given in Japanese with many symbols. However, the proof consists of a list of symbolized statements with properties, but not of proper Japanese sentences, except for the last line, because each statement does not include a subject and verb and there is no *dot* at the end of the line, which there should be in proper sentences. Unlike English, expressions like “BO = DO” are not abbreviated sentences (as in Nesselmann’s *Syncopated algebra*) in Japanese, but symbolic statements that do not preserve Japanese grammar (since the verb should come at the end of the sentence). The teacher says when writing the last line, “The conclusion must be written in Japanese. This becomes a little bit long”. This utterance supports our interpretation that the other lines are not given in proper Japanese.

Function of Proving

One may identify some functions of proving in the two cases. In Case 1, the proving is to provide a *logical value* (Duval, 1991), allowing them to reuse that statement in other proofs, and to systematize geometrical knowledge that has been learned in the previous grades (e.g., parallelograms in primary school). Notably, this was not done to convince someone. The students already knew this statement from the previous lesson and there was no discussion on its truth. One cannot see, therefore, the function of argumentation that changes a student’s *epistemic value* (Duval, 1991) of the statement, and there was no argumentative activity in the observed lessons. This was also the case in Case 2. The class never discussed the truth of the given statement but took it for granted. In fact, the teacher introduced the statement after drawing the diagram, as follows:

> Teacher: Well, this is the problem to prove that the red quadrilateral becomes a parallelogram. It looks like a parallelogram, this red one. It also looks like a rhombus. But this becomes a parallelogram. Why is it? This is a problem.

One may see here that another function of the proof is to explain why the statement holds. This role was accomplished by orally exposing the structure of reasoning that connects the hypotheses to the conclusion. In other words, the main function of proving was to create a deductive chain reaching to the conclusion from the hypotheses and show the logical structure. The teacher conveyed this logical structure through oral discursive language with diagrams and gestures.

DISCUSSION

What is Proof and Proving in Japanese Middle School?

One could find in both cases that the teacher did not accord importance to the written proof. It was given only twice in the sequence of five lessons. This implies that what counts as “proof” in this classroom is not necessarily formulated as a written entity. Instead, the oral proving with diagrams and gestures was considered as a proof. This may cause some ambiguities in the connections between statements. One may not explicitly see how the hypotheses are connected to the conclusion by means of a theorem, because the discursive language is not well organized and the “if-then” form is not used to describe the properties. Further, even in the written proof, although it complements some logical connections of statements in the oral proving, there are still some ambiguities due to the non-use of the “if-then” form and the Japanese language.

So, what is the basis to conclude that a statement has been proved in a Japanese classroom? The proving activity is not an argumentative activity to convince someone of the truth of a statement (this is aligned with Sekiguchi’s [2002] claim), but an activity to establish the structure of deductive reasoning from the hypotheses to the conclusion in the propositional logic with a specific level of linguistic rigor (as described above) and to explain why the statement is considered to be true. Further, the proven statement should be universal, as mentioned by Miyakawa (2017), but quantification is not explicitly dealt with in the structure of reasoning.

Theoretical Reflection: Proof and Proving as a Triplet

Let us reflect on our theoretical perspective, conceptualizing proof as a triplet, to answer our research question. This perspective allows us to characterize proof-related activities, such as oral argumentation and written mathematical proofs, from a broader perspective than the existing frameworks. However, we do not intend to argue that these three aspects are necessary and sufficient for discussing the cultural specificities of proof and proving. Instead, researchers claim, based on the results of previous international comparative studies, that *the triplet allows us to explain that some of the three aspects are emphasized in a given institution more than other aspects, and some aspects are considered in a different way according to the institution*.

For instance, in a comparative study of French and German classrooms, Knipping (2004) identified the importance of writing in French classrooms, where each step in an argumentation chain should be made explicit. One may see in our analysis that the linguistic level of rigor required for proving in France is very different from that in Japan. Proving in a Japanese classroom is rather similar to that in a German classroom, which is called *intuitive-visual argumentation*. This example implies the relevance of our perspective that language is an aspect that strongly reflects cultural specificities, in addition to the function of proving mentioned above.

From the institutional perspective of ATD, students' relation to proof and proving could be different not only across the country, but also across grades and mathematical domains (e.g., algebra and geometry) within a country. While in this paper researchers consider as an institution the geometry lessons in a specific country, it is necessary to further analyze and compare empirical data from different institutions to corroborate our theoretical perspective and elaborate it to systematically investigate different proving activities by means of the triplet—structure, language, and function.

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TOULMIN ANALYSIS OF META-MATHEMATICAL ARGUMENTATION IN A JAPANESE GRADE 8 CLASSROOM

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In this research report we aim to analyse argumentation at two levels, using the so-called Toulmin model. We examine the structure of the mathematical argumentation, as well as the nature of the meta-mathematical argumentation justifying the validity of some proofs and the rejection of others in a Japanese grade 8 classroom. The results show that the analysis of a meta-mathematical argument allows us to gain a deeper insight into the proving process, although the role of the statements is more difficult to determine.

INTRODUCTION

Proof and proving are considered to be essential but challenging in the mathematics classroom. How argumentation may develop through proving processes in classroom interactions is one of the main research foci in recent publications (e.g., Mariotti et al., 2018; Stylianides et al., 2016). Argumentation analysis based on the work of Toulmin (1958) has been used extensively in mathematics education to investigate students' mathematical arguments in different ways. For instance, Pedemonte (2007) has analysed the relationship between argumentation and proof in students' working in pairs, while Krummheuer (2007) has adopted the model to analyse students' argumentation and participation in classroom processes. Although the Toulmin model was developed to be applicable to different rational arguments in different fields, including mathematical and non-mathematical argument, in mathematics education research it is less common to use it to investigate non-mathematical arguments. One exception is Potari and Psycharis (2018), who analysed pre-service mathematics teachers' argumentations while interpreting classroom incidents. They report that "different argumentation structures and types of warrants, backings and rebuttals [occur] in the … interpretations of students' mathematical activity." (p. 169). The lack of research on non-mathematical argumentation implies that a broader perspective is needed to take into account both mathematical and non-mathematical argumentation and how they can be related to proof and proving in classroom teaching and learning. In this research report we aim to analyse argumentation in a Japanese classroom at two levels.

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We examine the structure of the mathematical argumentation, and also the nature of the meta-mathematical argumentation justifying the validity of some proofs and the rejection of others. To attain this aim, we first consider how Toulmin model can be adopted to analyse the mathematical and meta-mathematical argumentation, and then reconstruct the classroom process in terms of the two-levels of argumentation.

THEORETICAL AND METHODOLOGICAL CONSIDERATIONS

Argumentation Analysis

We focus on argumentative structures which can be identified in collective processes in classroom interaction. For this purpose, the Toulmin model is adopted, following the methods of *argumentation analysis* described by Knipping and Reid (2015, 2019). At the heart of this method is a reduced Toulmin scheme describing arguments in terms of Claims/Conclusions, Data, Warrants, and Backings (see Figure 1). Briefly, an argument aims to establish a claim, based on specific data and general warrants (and backings). Toulmin's full scheme includes other elements (Qualifiers, Rebuttals) and Knipping and Reid include also Refutations in their analysis.

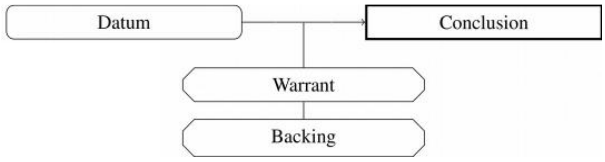


Figure 1: Toulmin model

Mathematical and Meta-mathematical Argumentation

Knipping and Reid (2015, 2019) analysed argumentation over time and reconstructed local (detailed) and the global (gross) argumentative structures. In this study, in order to reveal the nature of mathematical argumentation in-depth, we consider both mathematical argumentation and meta-mathematical argumentation. Our analysis shows that a mathematical argument can be supported (or rejected) by meta-mathematical argumentation which is talking about the argument. Since the meta-mathematical argumentation sometimes involves non-mathematical argumentation which represented by ordinary language, it is more complex and more difficult to analyse. The research question in this study is as follows: *How can the method of argumentation analysis be used to analyse both mathematical and meta-mathematical argumentation?*

Method of Analysis

Argumentation analysis (Knipping & Reid, 2015, 2019) consists of three stages for reconstructing arguments in classroom: first identifying 'episodes' of mathematical activity, then assigning roles (Conclusions, Data, Warrants, Backings) to statements, and grouping these into ‘steps’, assembling these steps into larger ‘streams’ in which a conclusion of one step is used as a datum (or occasionally a warrant) for the next step. These streams are in turn joined together into a ‘structure’ of the entire argumentation. Here we are interested in comparing the argumentation streams at two levels that occur in episodes of the lesson.

Data and Context

Our analysis is based on a transcript from a mathematics lesson in a mathematics class with 37 eighth-graders (age 13-14) at a junior high school in Japan. Before the lesson, the students learnt the terms ‘proof’ and ‘definition’, with several definitions of geometrical objects and fundamental assumptions (e.g., conditions for congruent triangles, SSS, SAS, ASA, and properties of parallel lines and angles) in Euclidean geometry. Based on these definitions and assumptions, they learned how to prove several statements related to triangles; in order to prove that two segments are equal in length, they learnt to focus on ‘finding’ a pair of triangles to which the two segments belong respectively as their sides (see Tsujiyama & Yui, 2018, for details). This experience is related to the main problem in the lesson (see below).

DATA ANALYSIS

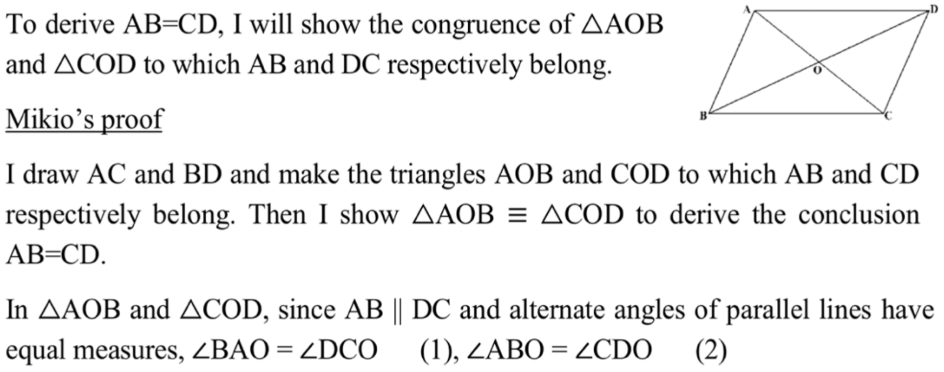
Identifying Episodes

The lesson can be divided into episodes, listed in Table 1. In Episode 1 the teacher asked the students about their prior knowledge about parallelograms; the students identified five properties of parallelograms. The teacher then (Episode 2) posed the problem: Prove that in a quadrilateral ABCD, if AB || DC and AD || BC, then AB = DC. After all the students had individually written down a plan and proof in their own way (Episode 3), the teacher asked two students to present their results. One showed △ABC ≡ △CDA (Episode 4). Miya then presented her proof, based on showing △ABD ≡ △CDB (Episode 5). The teacher then asked students who had done the first proof, why they had drawn the segment AC (Episode 6) and those who had taken Miya’s approach why they had constructed the segment BD (Episode 7). At the end of the lesson, the teacher discussed the faulty proof produced by a fictive student Mikio (Episode 9, see Figure 2).

Episode	Description	Transcript Lines
1	Recalling Prior Knowledge	Not included
2	Posing Problem	Not included
3	Individual Work	Not included
4	First Argument: △ABC ≡ △CDA	Not included
5	Miya's Argument	1–4
6	Why AC? Jiro's argument	4–9
7	Why BD? Etsu's argument	10–24
8	Omitted section	25
9	Mikio's 'proof'	26–72

Table 1: Episodes of the lesson

Here we analyse mathematical argumentations from Episodes 5 and 9, and meta-mathematical arguments from Episodes 6, 7 and 9. Although we have identified two types of meta-mathematical arguments (Episode 6 & 7) concerning the argument from Episode 5, we omit one of the meta-arguments due to limited space.



To derive AB=CD, I will show the congruence of △AOB and △COD to which AB and DC respectively belong.

Mikio's proof

I draw AC and BD and make the triangles AOB and COD to which AB and CD respectively belong. Then I show △AOB ≡ △COD to derive the conclusion AB=CD.

In △AOB and △COD, since AB || DC and alternate angles of parallel lines have equal measures, ∠BAO = ∠DCO (1), ∠ABO = ∠CDO (2)

Figure 2: Mikio's 'proof'

Miya's Mathematical Argument

Miya's argument is as follows:

2 Miya: I drew diagonal BD to show AB = CD, and proved that △ABD and △CDB are congruent. AB and BC are parallel from the assumption. And, since alternate interior angles of parallel lines are equal, ∠ABD and ∠CDB are equal, also ∠ADB and ∠CBD are equal. And, since they are common sides, BD = DB. Thus, ASA holds, and therefore △ABD and △CDB are congruent. Since corresponding sides of congruent figures are equal, well, I thought that AB = CD should be correct.

The argumentation stream for this argument is shown in Figure 3.

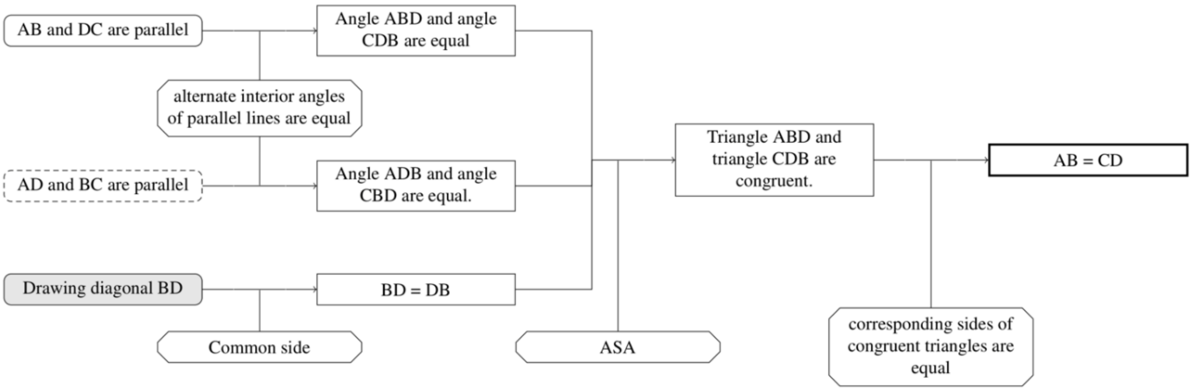

AB and DC are parallel
Angle ABD and angle CDB are equal
alternate interior angles of parallel lines are equal
AD and BC are parallel
Angle ADB and angle CBD are equal.
Triangle ABD and triangle CDB are congruent.
AB = CD
Drawing diagonal BD
BD = DB
Common side
ASA
corresponding sides of congruent triangles are equal

Figure 3: Miya’s argument

Miya’s argument is unusual only in that there are fewer implicit data or warrants, which are indicated by boxes with dashed lines, than often occurs in classrooms. The only implicit statement is AD || BC, which is not uttered but written on the blackboard. She asserts the existence of BD by drawing it into the diagram.

Why AC? Jiro’s Meta-Mathematical Argument

In Episode 6, Jiro offers the following argument to justify the construction of the diagonal AC.

4 T: Why was it necessary to draw AC? Jiro.

5 Jiro: Well, what we want to show now, we want to show that two sides are congruent.

6 T: Two sides are congruent?

7 Jiro: Two sides are equal, we want to show that two sides are equal in length. And, to show it, now I have a quadrilateral, but I do not know how to show congruency of quadrilaterals. I only have one quadrilateral and do not know how to show, so I tried to transform it to the one that I already knew. Then I drew the line [AC] and, well, made two triangles in that way, and I thought of showing the congruency [of two triangles]. I drew the line in this way.

The diagram for his argumentation is shown in Figure 4. It begins from the fact that they want to show that two sides are congruent. From this fact, via the implicit warrant that corresponding sides of congruent figures are congruent, Jiro implicitly concludes that he wants to show that two figures are congruent. The only figure he can see is a quadrilateral and he does not know how to show quadrilaterals are congruent, so he concludes he should show the congruency of two triangles. To produce the two triangles it was necessary to draw the segment AC.

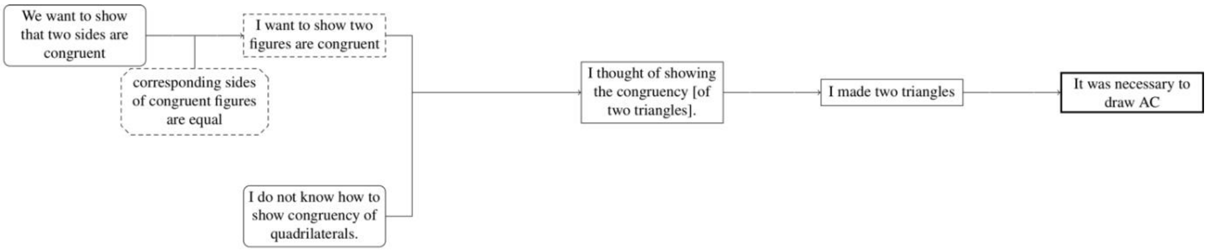

We want to show that two sides are congruent
I want to show two figures are congruent
corresponding sides of congruent figures are equal
I do not know how to show congruency of quadrilaterals.
I thought of showing the congruency [of two triangles].
I made two triangles
It was necessary to draw AC

Figure 4: Our analysis of Jiro's meta-mathematical argument

The analysis of a meta-mathematical argument is similar to the analysis of a mathematical argument, although the role of the statements is more difficult to determine. It is useful to begin from the conclusion, the necessity of drawing AC, and to find the data and warrants supporting it. This can be expressed as 'It was necessary to draw AC, because I wanted to make two triangles.' In turn, 'I wanted to make two triangles, because I wanted to show congruency of two triangles' and 'I wanted to show congruency of two triangles because I wanted to show congruency of two figures, and I do not know how to show the congruency of quadrilaterals.' More elements are implicit than in Miya's argument.

Mikio's Mathematical Argument

In Episode 9, the teacher showed Mikio's 'proof', which is the incomplete proof 'attempt' shown in Figure 2. Our analysis of it is shown in Figure 5.

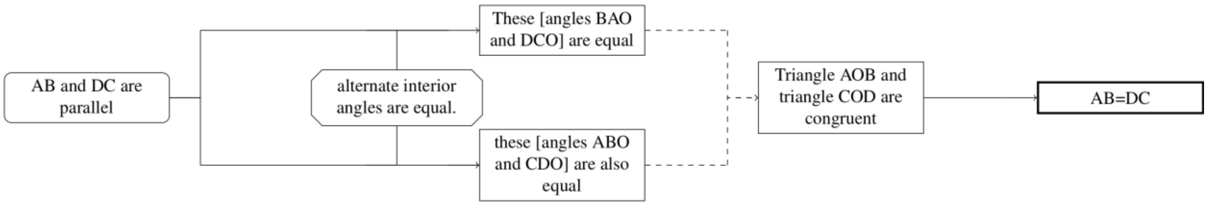

AB and DC are parallel
alternate interior angles are equal.
These [angles BAO and DCO] are equal
these [angles ABO and CDO] are also equal
Triangle AOB and triangle COD are congruent
AB=DC

Figure 5: Our analysis of the mathematical argumentation in Mikio's 'proof'

The diagram of Mikio's 'proof' reveals that is leaves two warrants implicit. One of these implict warrants conceals the flaw in this 'proof'. To assert that △AOB ≡ △COD he needs a congruent side so that he can use either AAS or ASA. But without asserting either the property he is trying to prove (opposite sides are congrent) or one of the other properties (that diagonals bisect each other) he cannot establish the congruent side.

The Meta-Mathematical Argument Concerning Mikio's 'Proof'

Related to the meta-mathematical argument concerning Mikio's proof, some parts of the transcripts are omitted here, since space is limited. Nevertheless, the transcript below shows how the teacher and students arrived at the reason why the given proof is impossible.

51	T:	You could not make it?
52	Ken:	Since there was no congruent side, well, no equal side.

53 T: Then, Mikio tries to prove, in this way. To create triangles that include the conclusion, AB and CD, he connects points A and C, B and D respectively. See? [These triangles] include the conclusion[i]. Fine. Now he makes [triangles] AOB and COD. And then, if he shows congruency [of △AOB and △COD], he can deduce the equality [of AB and CD] since they include [AB and CD]. So he tries to prove that. But here, he stopped at (1) (2). Why [did Mikio] get stuck? What do you think? Ken.

54 Ken: Well, under the assumption, uh, sides... We cannot show equality of [any pair of] sides, so we cannot prove this.

…

59 Ss […] We do not know properties of parallelogram.

60 T Yes. We cannot use other properties [of parallelogram].

61 Shin Then, uh, sides

62 T These are parallel. [with marking AB and DC in Mikio's diagram]

63 Shin Length of other sides, sides are not necessarily equal in length.

…

67 Mizu: Well, we can only use those above two [properties of parallelogram: (1) opposite sides are parallel, (2) opposite sides are equal in length] and cannot use the three below [properties: (3) the diagonals intersect at their midpoint, (4) opposite angles have equal measures and so on].

…

70 T: Mizu, you wanted to use this [referring to the property (3)], didn't you? You thought that you could succeed if you used this, didn't you? Then, bad teacher came and told you that you were not allowed to use this, so you were in trouble, didn't you? Then, this and this are like this [marking angles in Mikio's diagram], alternate interior angles are equal. So, it comes to this way. But how does it work? These [angles] are equal since they are alternate interior angles of parallel lines, these are also equal. So it is fine if we use AB = DC, isn't it.

71 Ss: The conclusion. / We cannot use the conclusion.

The meta-argument around Mikio's 'proof' has rather complex structure as shown in Figure 6. To justify the rejection of the proof, many statements involve negations and both warrants and implicit backings can be considered as meta-level reasoning or proving; such as 'a statement cannot be used for proving unless it is proven', 'circular reasoning', and 'it is necessary that a pair of sides at least is equal to use congruence conditions of triangles'. In Figure 6, the warrant 'other properties of parallelograms have not been established" and its backing are implicit, but it can be interpreted as such (Lines 59-50, 67, 70). The 'circular reasoning' as an implicit backing is also concerned with what the teacher and students uttered (Lines 70-71). Thus, it is interpreted that their discussions about Mikio's 'proof' are not only about how to prove the statement but also about what constitutes a proof.

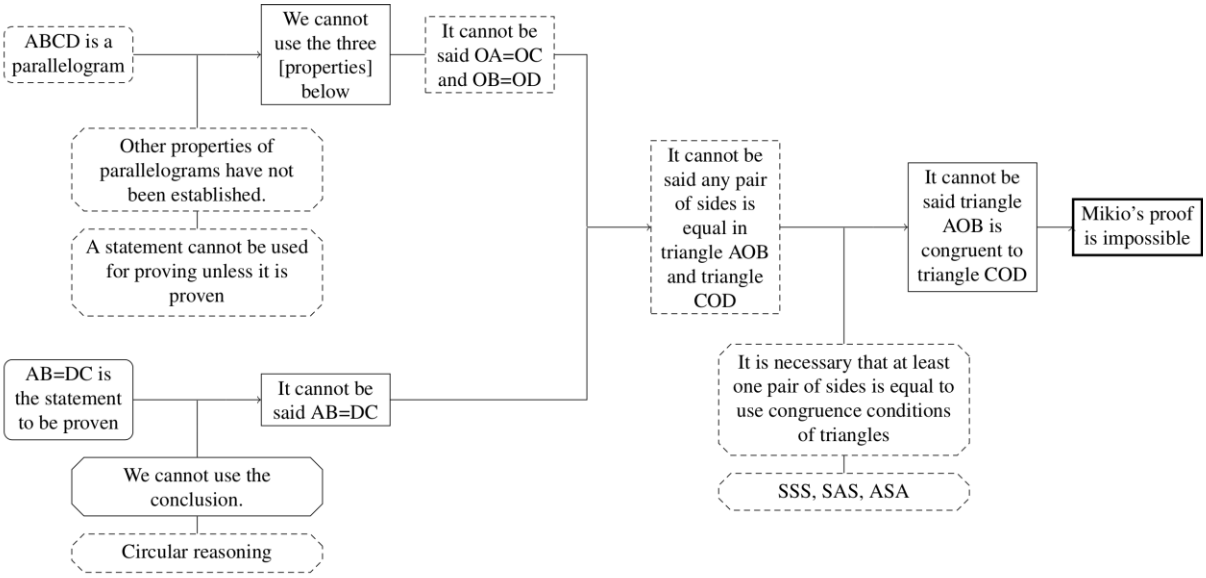

ABCD is a parallelogram
We cannot use the three [properties] below
It cannot be said OA=OC and OB=OD
Other properties of parallelograms have not been established.
A statement cannot be used for proving unless it is proven
It cannot be said any pair of sides is equal in triangle AOB and triangle COD
It cannot be said triangle AOB is congruent to triangle COD
Mikio's proof is impossible
AB=DC is the statement to be proven
It cannot be said AB=DC
We cannot use the conclusion.
Circular reasoning
It is necessary that at least one pair of sides is equal to use congruence conditions of triangles
SSS, SAS, ASA

Figure 6: Our analysis of the meta-mathematical argumentation around Mikio's 'proof'

DISCUSSIONS AND CONCLUSIONS

One of the characteristics of our paper is the use of Toulmin's model for the analysis of meta-mathematical argumentation, which allows us to gain a deeper insight into classroom processes involving proving activity. In this study, we considered the meta-mathematical argument as the justification of the validity and the rejection of proofs and analysed their structures. For example, Jiro's meta-mathematical arguments were for justifying the validify of Miya's proof. It seems that the teacher's question "Why was it necessary to draw AC?" (line 4) facilitated their discussions. Our analysis (Figure 4) showed that some statements include ordinary sentences, such as "we want to show…" and "I do not know…", rather than purely mathematical sentences. Another meta-mathematical argument was for reasoning about the rejection of Mikio's 'proof'. This meta-level discussion allowed them to reflect "what has (not) been proven" and "the conclusion is a statement to be proven (it cannot be used to prove)" and to understand what constitutes a proof in the class (Figure 6). Since our analysis is limited, further research is needed to investigate different structures and functions of meta-mathematical argument.

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