ACTIVITIES AND RESEARCH FOCI IN MATHEMATICAL MODELLING: A LITERATURE REVIEW

Stanislaw Schukajlow[1], Janina Krawitz[2]

[1]University of Münster, [2]University of Cologne

Mathematical modelling comprises various activities and focuses on diverse research foci. However, overviews of these aspects are largely missing. In a systematic literature review of 105 papers published between 2020 and 2024, we asked: (1) What modelling activities were addressed in research? (2) What research foci were explored, and what activities were analyzed within these foci? Analysis revealed that most studies examined the modelling process holistically. Among specific activities, research studies most frequently analyzed mathematizing, working mathematically, and validating. Modelling performance was the most frequently examined research focus, alongside affect, technology, and STEM. Creativity was analyzed holistically, whereas other foci (e.g., problem posing) were assessed through activities such as simplifying.

INTRODUCTION

An important goal of mathematics education is to prepare students for current and future life. This goal implies the ability to solve real-world problems with the help of mathematics. Problems in the real world often do not include all the information needed for a solution, thus providing the opportunity to develop multiple solutions and requiring demanding translation processes between reality and mathematics. Because modelling problems are highly relevant to students' lives, modelling is included in the curriculum in many countries all over the world. Given the growing interest in research on modelling, there is a strong need for modelling research overviews that can identify trends and help in the planning of future studies (Cevikbas et al., 2022; Schukajlow et al., 2023).

Recent literature reviews have addressed modelling competencies (Cevikbas et al., 2022), perspectives on modelling (Babb et al., 2023; Krawitz et al., 2025), or reflections on modelling as a research field (Geiger & Frejd, 2015). However, with the exception of one review (Schukajlow et al., 2023), these reviews did not focus on modelling activities and did not relate results to the overall research foci of the empirical contributions. To address this gap in the literature, we conducted a systematic literature review, which provides a focused synthesis of research on modelling activities and associated research foci.

MODELLING ACTIVITIES AND FOCI IN MODELLING RESEARCH

Numerous contributions in modelling have analyzed students' cognitive processes and the modelling activities they use to solve modelling problems. Modelling activities are

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M.S. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: Research Reports*, pp. 1307-1314. PME.

typically described in modelling cycles. Modelling activities comprise the transitions between states in modelling cycles (Blum & Leiss, 2007). The modelling process starts with understanding the problem and constructing a situation model. The situation model must be simplified to construct a real model before the real model can be mathematized and a mathematical model can be developed. After constructing a mathematical model, students work mathematically and get mathematical results, which must be interpreted in the real world and should be validated (i.e., examined for plausibility). In contrast to distinguishing between various modelling activities, some approaches underline the holistic nature of modelling processes. In the framework on modelling competencies, an approach to modelling is called *holistic* if it does not refer to distinct modelling activities, and it is called *analytical* if it does (Niss & Blum, 2020). An analysis of research on modelling competencies indicated a strong prevalence of the analytical approach (Cevikbas et al., 2022). As solution processes depend on the types of modelling problems, students' knowledge, and affective prerequisites (e.g., effort), modelling processes result in individual solution routes (Borromeo Ferri, 2018). Thus, the modelling activities that can be observed and analyzed in a specific contribution can vary between empirical studies.

A prior literature review and analysis of empirical findings from research on modelling indicated that each modelling activity could be demanding for students (Niss & Blum, 2020) and reported various research foci addressed in modelling research (e.g., technology, metacognition, mathematical knowledge, affect or problem posing). However, it is not clear which research foci predominate in the recent research on modelling.

The specific findings on research foci in relation to modelling activities highlight the following points (Schukajlow et al., 2023). Understanding: Studies have analyzed strategies for improving reading comprehension and understanding (e.g., by posing reading comprehension questions or by teaching the note-taking strategy). Simplifying: Simplifying activities have been analyzed concerning simplifying strategies (e.g., use of prior real-world knowledge), mathematical knowledge, and students' beliefs. Results indicate that simplifying strategies vary between students, that mathematical knowledge partly compensates for deficits in simplifying and vice versa (Ferrando & Albarracín, 2021), and that students' inappropriate beliefs about problems (e.g., all numerical information is given in the task) negatively affect simplifying activities. Mathematizing: Studies have analyzed real-world knowledge and mathematical knowledge. More specifically, the process of implemented anticipation of mathematics was suggested to be important for mathematizing (Stillman & Brown, 2014). Types of teaching (e.g., the use of technology or doing outdoor mathematics) have also been analyzed in relation to mathematizing. Working mathematically: Studies have focused mostly on students' mathematical knowledge, their use of cognitive and metacognitive strategies, and the role of technology. Interpreting: Interpreting (i.e., contextualizing of mathematical results) has rarely been addressed in isolation in the past but has

usually been analyzed as part of the modelling process. The complexity of mathematical procedures, simplifying, and more broadly, the classroom culture, were analyzed in relation to interpreting. Validating: Validating was found to interact with metacognition (e.g., planning and monitoring of solution processes) and with other activities. For example, while interpreting, Czocher (2018) observed three types of validation activities: (a) validation of mathematical procedures, (b) validation of alignment between mathematical procedures and assumptions used in the models, and (c) validating results against one's intuitions about the real world.

RESEARCH QUESTIONS

In the past, all modelling activities were analyzed analytically, i.e., regarding difficulties in solution processes. A holistic approach that does not specify one or several activities can also be expected to be represented in the research on the basis of prior literature reviews (Cevikbas et al., 2022; Niss & Blum, 2020). The research foci that were addressed in modelling are much more difficult to predict, as mathematical modelling can address very different goals, such as learning mathematics, applying mathematics in the real world, or fostering positive affect (e.g., motivation). What activities targeted what foci in this research is also an open question. The research questions in this study were:

- RQ1: What modelling activities were addressed in research from early childhood education to secondary school?
- RQ2: What research foci were analyzed, and what modelling activities were analyzed within these foci?

METHOD

Data collection and selection processes

To answer the research questions, we followed the PRISMA guidelines (Page et al., 2021) to conduct the systematic literature review. Data were retrieved from Web of Science (WoS), Scopus, and ERIC—databases that include a comprehensive collection of papers with high relevance for education research. To identify relevant studies, search strings were employed to include articles with "model*" in the title and "math*" and "model*" in the title or abstract. To ensure that the search results remained manageable, we restricted the search to journal articles published in the subject areas of social sciences or psychology, written in English, and published since January 2020. Furthermore, given the extensive body of research on modelling and the focus of our research questions, the review focused on learners from early childhood education to upper secondary school. The search was conducted on April 22, 2024.

The search initially returned 2,615 results from Scopus, 865 from WoS, and 565 from ERIC. Duplicate records across these databases were removed, reducing the total from 4,045 to 3,320 after eliminating 725 duplicates. The titles, abstracts, and keywords of the remaining records were screened, leading to the exclusion of 3,043 records based

on the criterion of a focus on mathematical modelling. This left 277 reports to be retrieved, of which ten full texts could not be accessed, resulting in 267 available reports. Additionally, studies from the ICTMA books were included due to ICTMA's significant role in mathematical modelling research. Two ICTMA books published since 2020, comprising 99 chapters in total, were identified at the time of the search.

The final assessment included 267 journal articles from the databases and 99 chapters from the ICTMA books. On the basis of the eligibility criteria, we excluded six journal articles and one ICTMA chapter (not focused on modelling), 182 journal articles and 62 ICTMA chapters (not focused on students from early childhood education to upper secondary school), and seven journal articles (not written in English). 108 papers—72 journal articles and 36 ICTMA chapters—were included in the literature review.

Data analysis: modelling activities and foci

We selected a journal article or a chapter from an ICTMA book as the unit of analysis and coded them for modelling activities and the main research foci they addressed.

Modelling activities. Modelling activities were coded with a deductive-inductive approach following Mayring's (2015) procedure for applying a content analysis. We coded the contributions for the activities: understanding, simplifying, mathematizing, working mathematically, interpreting, and validating (see overview in Table 1). Additionally, we coded an activity as holistic if the contributions described the modelling process without emphasizing specific modelling activities. If another activity than those listed in the coding book came up, we added it to the list of subcategories. Each contribution could be assigned to different categories. The first author performed the coding. For 15% of the contributions, the second author checked the assigned subcategories for accuracy. No disagreements appeared.

Research foci. The foci were coded inductively. The first author inductively identified subcategories. Each contribution could be assigned to multiple subcategories. The initially identified subcategories were then summarized into broader research foci.

FINDINGS

The analysis of 108 papers revealed that most of the contributions assessed modelling holistically (72 papers, 67%), followed by papers that applied mathematizing (17, 16%, see Figure 1), validating or working mathematically (15, 14%, each), interpreting or understanding (12, 9% each), and/or simplifying (10, 9%). Three activities: making assumptions (6, 6%), sensemaking (i.e., activities involving decisions about whether the results make sense in the real world; 2, 2%), and communicating (2, 2%) were identified inductively.

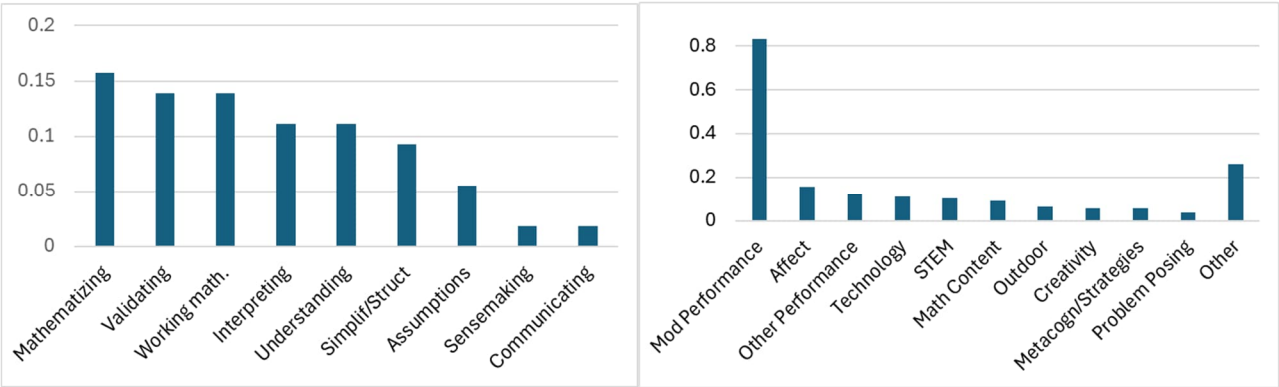
The analysis of the research foci revealed that the most frequent group of research foci referred to students' performance. Papers on performance included contributions on modelling performance (90, 83%) and/or other performance-related measures (13, 12%) such as grades, problem solving, or reading comprehension (see Figure 1). The

second group of research foci focused on the analysis of the non-cognitive outcomes. The most popular research focus in this group was affect (17, 16%) followed by creativity and/or strategies/metacognition (6, 6% each). The third group of research foci looked at instructional approaches and included research foci on technology (12, 11%), outdoor modelling (7, 6%), and problem posing (4, 4%). The fourth group of research foci targeted the science or mathematical content: STEM (11, 10%) and/or mathematical content (10, 9%), such as differential equations or fractions. More than one fourth of the papers (28, 26%) analyzed other research foci, such as group processes, representations, authenticity, international comparisons, and many others.

Table 1: Categories for coding.

Category	Subcategory	Description
Modelling activities	Understanding	If understanding was the focus
	Simplifying	If simplifying was the focus
	Mathematizing	If mathematizing was the focus
	Working mathematically	If working math was the focus
	Interpreting	If interpreting was the focus
	Validating	If validating was the focus
	Holistic	If the paper described the process holistically
	Inductive coding (presented in findings)	If another activity than those listed above was addressed
Research foci	Inductive coding (presented in findings)	If the contribution highlighted a specific focus

Figure 1: Frequencies of modelling activities and research foci assessed analytically. (Each paper could address several activities).



The analysis of modelling activities within the identified research foci revealed that most of the research foci addressed modelling by using holistic or analytical approaches. However, one exception was creativity research that did not analyze any specific modelling activities. In papers on creativity, researchers used a holistic approach (5, 85%) or did not analyze modelling activities (1, 15%). Similar results were found for the papers that focused on mathematical content, which used a holistic approach to assess modelling activities (9, 90%) and/or addressed only one activity: working mathematically (1, 10%). Interestingly, papers from the instructional approach group of research foci (outdoor modelling and instruction on problem posing) were more likely to assess specific activities than to use a holistic approach (holistic approach in outdoor mathematics: 2, 20% or in problem posing: 1, 11%).

DISCUSSION AND IMPLICATIONS FOR RESEARCH

Our analysis of students' modelling activities recently addressed in peer-review journal articles or ICTMA chapters revealed that in most contributions, researchers followed a holistic approach to modelling activities and did not focus on one or several specific modelling activities. This result differs from the findings by Cevikbas et al. (2022) who found that the analytical approach was predominant. One explanation for this difference is that we covered a broader area of modelling research, as we did not focus on modelling competence frameworks only. Another explanation may lie in differences in coding procedures. Whereas the study on competencies relied on references to a modelling framework in the theoretical background of the studies, we examined the coding procedures used in each study to decide how the activities were addressed.

Our analysis of specific activities indicated that the studies addressed all activities that have been suggested to be important for modelling, with a stronger focus on mathematizing. Simplifying was addressed the least. We argue that, in addition to addressing modelling holistically, a stronger focus on specific activities in the future is important, as such findings can contribute to a more differentiated picture of modelling research in both descriptive studies and intervention studies. For example, it is possible that teaching modelling did not have a positive effect on all activities equally, but only on some of them, it may have even promoted one activity but hindered another activity, or it may improve some activities but not affect the solution of holistic modelling tasks. Three specific research activities that we uncovered inductively included making assumptions, sensemaking, and communication. The attention given to making assumptions in these studies indicates that researchers have recognized its significance for mathematical modelling. Sensemaking in mathematics is one of the overarching goals of mathematical modelling (Niss & Blum, 2020), and our results demonstrate that it remains important for researchers in modelling. Communicating is a more general activity, which is not specific to mathematical modelling. Its appearance might be related to the popularity of the modelling cycle by Blum and Leiss (2007). This explanation should be tested in future literature review studies by looking at which activities were analyzed together in one contribution (for an example of a study that

analyzed validating in relation to other activities Czocher, 2018) and how the use of a specific modelling cycle is related to the analysis of modelling activities.

An inductive analysis of the research foci addressed in the articles showed that most research foci focused on students' performance, followed by non-cognitive outcomes and instructional approaches. Apart from the clear dominance of modelling performance as a central focus, a broad range of different research foci were analyzed in modelling research. Future research on mathematical modelling should further expand this thematic scope to reflect the diversity of research questions in mathematics education. The findings of this study highlight the research foci that were considered interesting and important in the modelling research community. For example, strategies and metacognition in modelling, which were introduced several decades ago, continue to be of interest in the field. Furthermore, research in modelling contributes to research foci that are analyzed beyond the modelling community, such as the role of technology, problem posing, or affect. Researchers interested in these overarching research foci would likely benefit from studies coming from research on modelling and might use the results to build conjectures about the productive use of technology or problem posing for learning mathematics.

The analysis of activities addressed within specific research foci indicates that some topics, such as mathematical creativity or learning of mathematical content, were addressed exclusively or for the most part holistically. However, most research foci were targeted both holistically and analytically (i.e., by the analysis of several modelling activities). These results highlight the diversity of research questions addressed within each research focus, with some requiring analytic assessments of modelling and others calling for more holistic approaches.

This study has important limitations. Future research should examine these findings by extending the time frame or expanding the sample by additionally including university students and teachers. Furthermore, using an in-depth analysis of each contribution could also contribute to overcoming the limitations of this study. Taking into account the potential dependence of the results on cultural traditions, authorships, or perspectives in mathematical modelling should also be considered in future literature review studies.

The novel contribution of this systematic literature review study is that it presents an overview of what activities and research foci were recently addressed in modelling research. This study contributes to research on modelling by shedding light on recent trends in modelling research and under-researched modelling activities and research foci. Furthermore, it contributes to general research in mathematics education by uncovering the research foci in which research on modelling contributes to broader topics such as technology, affect, or problem posing.

References

Babb, A. P. P., Acuña, F. P., Rocha, Y. A. O., & Rojas, A. S. (2023). Diversity of perspectives on mathematical modelling: A review of the international landscape. *Advancing and Consolidating Mathematical Modelling*, 43–57. https://doi.org/10.1007/978-3-031-27115-1_3

Blum, W., & Leiss, D. (2007). How do students and teachers deal with mathematical modelling problems? The example sugarloaf and the DISUM project. In C. Haines, P. L. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical Modelling (ICTMA 12): Education, Engineering and Economics* (pp. 222–231). Horwood. https://doi.org/10.1533/9780857099419.5.221

Borromeo Ferri, R. (2018). *Learning how to teach mathematical modeling in school and teacher education*. Springer. https://doi.org/10.1007/978-3-319-68072-9

Cevikbas, M., Kaiser, G., & Schukajlow, S. (2022). A systematic literature review of the current discussion on mathematical modelling competencies: state-of-the-art developments in conceptualizing, measuring, and fostering. *Educational Studies in Mathematics*, *109*(2), 205–236. https://doi.org/10.1007/s10649-021-10104-6

Czocher, J. A. (2018). How does validating activity contribute to the modeling process? *Educational Studies in Mathematics*, *99*, 137–159.

Ferrando, I., & Albarracín, L. (2021). Students from grade 2 to grade 10 solving a Fermi problem: Analysis of emerging models. *Mathematics Education Research Journal*, *33*(1), 61–78. https://doi.org/10.1007/s13394-019-00292-z

Geiger, V., & Frejd, P. (2015). A reflection on mathematical modelling and applications as a field of research: Theoretical orientation and diversity. In *Mathematical Modelling in Education Research and Practice* (pp. 161–171). Springer.

Krawitz, J., Schukajlow, S., Yang, X., & Geiger, V. (2025). A systematic review of international perspectives on mathematical modelling: Modelling goals and task characteristics. *ZDM – Mathematics Education*, *57*, 193–212.

Mayring, P. (2015). Qualitative content analysis: theoretical background and procedures. In A. Bikner-Ahsbahs, C. Knipping, & N. Presmeg (Eds.), *Approaches to qualitative research in mathematics education* (pp. 365–380). Springer.

Niss, M., & Blum, W. (2020). *The learning and teaching of mathematical modelling*. Routledge. https://doi.org/10.4324/9781315189314

Page, M. J., McKenzie, J. E., Bossuyt, P. M., Boutron, I., Hoffmann, T. C., Mulrow, C. D., Shamseer, L., Tetzlaff, J. M., Akl, E. A., & Brennan, S. E. (2021). The PRISMA 2020 statement: an updated guideline for reporting systematic reviews. *bmj*, *372*(71), 1–9.

Schukajlow, S., Kaiser, G., & Stillman, G. (2023). Modeling from a cognitive perspective: theoretical considerations and empirical contributions. *Mathematical Thinking and Learning*, *25*(3), 259–269. https://doi.org/10.1080/10986065.2021.2012631

Stillman, G., & Brown, J. P. (2014). Evidence of implemented anticipation in mathematising by beginning modellers. *Mathematics Education Research Journal*, *26*(4), 763–789.