TEACHERS' RESPONSES TO EARLY NUMBER INSTRUCTIONAL EXPLANATIONS

Hamsa Venkat[1,2] and Angel Mukuka[1]

[1]CASTeL, Dublin City University, [2]University of the Witwatersrand

In this paper, we share the process of developing an analytical framework for considering data comprised of teachers' responses to an instructional explanation video offering help to a child who is stuck on how to solve: _ - 23 = 14. In the paper, we overview the relatively small literature base on instructional explanations and illustrate some of the ways in which the available (almost exclusively 'global north') frameworks failed to capture salient aspects of a South African dataset. This led us into a grounded, but literature-informed, development of an analytical framework that better represented the underlying data. In the body of the paper, we detail this analytical framework and the axial coding that was used to develop indicators linked to a hierarchy based on 'nature/extent of reference to mathematics'.

INTRODUCTION

Teachers' understandings of early primary mathematics content and pedagogy have received increasing interest in recent years in sub-Saharan Africa. In South Africa, this interest accompanies evidence of learning lags linked to core early number concepts (Spaull et al., 2022). The evidence on teachers' knowledge of Foundation Phase number ideas is mixed. Some evidence points to teachers with difficulties in their own understandings of early number ideas (Porteus, 2023). However, other evidence points to teachers largely being able to solve Foundation Phase number problems for themselves, but unable to produce coherent instructional explanations that assemble talk, representations and rationales in ways that would support children's independent working with similar problems (Bowie, Venkat & Groenewald, 2025).

In this paper, we explore teachers' understandings of an instructional explanation linked to a missing minuend problem: _ – 23 = 14. The explanation, presented in a short video, involved a 'trial and error' approach to solving the problem. In the video the teacher stated that if children didn't know what to do to get the missing number, they could try subtracting the given numbers (23 – 14 was written down with 9 as the answer). 9 was then inserted into the given equation and rejected on the basis that 9 – 23 did not produce 14. 23 + 14 was then tried, producing 37 as the answer. This answer was checked in the given equation and accepted because 37 – 23 did equal 14.

This task was drawn from a baseline assessment in a professional development (PD) project linked to an after-school NGO-led Maths Clubs programme in South Africa. It asked teachers to state whether they liked the instructional explanation in the video, and to give reasons for their answer.

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M.S. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: Research Reports*, pp. 1531-1538. PME.

LITERATURE REVIEW

Leinhardt's (1993) writing on instructional explanations remains widely cited. In this work, she describes instructional explanations in terms of what they achieve:

> 'Instructional explanations clarify concepts, procedures, events, ideas and classes of problems in order to help students understand, learn and use information in flexible ways.' (p.47)

She notes that the goal of instructional explanations is to: 'move students into a position in which they can enter the disciplinary dialogues of the field' (p.47) by 'describing and modelling in the discipline of interest' (p.48). She also notes that instructional explanations, in contrast to 'disciplinary explanations' (explanations anchored in 'a core of conventions within the discipline'), frequently involve redundancy/repetition alongside completeness, because they must take children's understandings into account.

Güneş et al. (2024) note that the body of writing on instructional explanations is small and likely linked to the emphasis in the mathematics education research field to student-centred working. A recurring finding in the available writing is that instructional explanations focus more on the sequence of steps to be carried out in problem-solving than on the reasoning that underpins the sequence. Kinach (2002) notes that pre-service teachers often show little awareness of connections, rationales for steps, or the warrants that establish ideas within a discipline. Güneş et al's (2024) literature overview notes that teachers often add context or illustrations to support explanations but have difficulty distinguishing the quality of explanations, exhibiting 'a low sensitivity to omissions and vagueness' (p.402).

Leinhardt's point about instructional explanations needing to take account of both the discipline and children's understandings is complicated further by compounding aspects in the South African evidence base: gaps in teachers' content and pedagogic content knowledge; problems with coherence in instruction; explaining ideas in English which is not a first language for most teachers or learners.

The evidence of predominantly teacher-led instruction in sub-Saharan Africa makes it important to support the development of good quality instructional explanations that take in the more extensive reference to the discipline outlined in Kinach's (2002) analysis. However, the caveats mentioned above also make a typological search for Kinach's, or others', category descriptors in our data difficult.

METHODOLOGY

The baseline data used to develop the conceptual frameworks consisted of the written responses to the focal video clip of 73 teachers from four South African provinces during an initial assessment/professional development day at the start of 2025. The 1-minute video clip was played to the group. Each teacher had a sheet on which to record their thoughts on the video, with reasons for their response. Most teachers included one or two sentences within these responses.

We initially worked with Kinach's descriptors but found that, even at the 'sequence of steps' or 'content' level – which Kinach describes as the norm for pre-service teachers – we were not seeing clear correlations. For example, several responses indicated either liking the video explanation because it was: 'Easy to understand. Easy method to grasp' [Teacher C6] or not liking it because it was: 'Too much explanation for this age group, confusing learners' [Teacher A16]. Some responses also suggested misinterpretations of the trial-and-error approach, e.g., stating that 9 – 23 should not be included, or that the video offered 'two methods' for solving the problem.

This led to a more grounded approach to the data, informed by the literature base noting the predominance of communicating the steps and difficulties with spotting omissions. Mindful that we could see vagueness and omissions even in relation to problem-solving steps in explanations, we devised a hierarchy based on 'nature and extent of reference to mathematics' – as presented in the video clip for studying teachers' responses. Coherent working at the higher levels involved a vantage point on the logic of the trial-and-error approach: that an operation will be tried out with the result checked for fit, resulting in either the acceptance of the answer and route, or the elimination of this route in the event of a lack of fit. Making sense of the explanation in the Video required awareness of this vantage point on trial and error as a valid mathematical approach. Across our dataset this awareness was expressed in a range of ways: erroneous; partial; hollow; relatively rote; more rational terms. These emergent indicators were collated into hierarchic levels that were developed through distilling similarities across the responses placed within each category – axial coding, in Corbin and Strauss' (2008) terms.

Working in this way with the data led us into the development of a framework that encompassed the breadth of phenomena within our dataset, taking forward what we were able to see with existing concepts and models in the literature.

OUTCOME: FRAMEWORK FOR ASSESSING SOUTH AFRICAN TEACHERS' EVALUATIONS OF INSTRUCTIONAL EXPLANATIONS

The framework levels and key indicators and descriptors within levels that we have currently developed are outlined in Table 1 and discussed thereafter.

A key feature that this framework illuminates is of a 'base' level that is different from the base level presented in Kinach's (2002) model, where a rote procedural emphasis on stating the steps to enact to solve the problem predominates amongst the pre-service teachers in her study. This relaying of a sequence of steps sits at Level 2 in our framework, with teacher evaluations of explanations indicating either a comprehensive noticing of the steps enacted in the video or offering a comprehensive alternative sequence of steps for problem-solving. However, our data indicated a substantial number of evaluation comments that did not comment comprehensively on the trial-and-error approach to choosing the operation to be enacted in the video or offer a comprehensive alternative problem-solving procedure. Instead, in the various sub-categories that we ended up with in Level 1, we saw: (1a) misreadings or erroneous

interpretations of the offered explanation – for example, evaluations that suggested that the explanation offered ‘two methods’ for problem-solving – addition and subtraction, or stating that the explanation made use of the ‘inverse method; (1b) evaluations that made no direct reference to the video, but offered partial alternative approaches; (1c) what we came to describe as ‘hollow’ evaluation comments – comments with limited, if any, specific mathematical referents; (1d) partial reference to the steps detailed in the video clip.

It is at Level 3 that we see a transcending of what Skemp (1976) described as ‘rules without reasons’ (p.20). At this level, responses provide some communication of the logic, or rationales, for steps. These were offered either for the approach detailed in the video clip, or for an alternative approach. We have noted in Table 1 that at this Level, evaluations communicated – for example, some awareness of why the missing number had to be larger than the subtrahend value.

Table 1: Levels and indicators of nature/extent of reference to mathematics

Level 1a	**Level 1b**	**Level 1c**	**Level 1d**
Misreading/Erroneous Interpretations that incorrectly describe or interpret the mathematical or pedagogical approach demonstrated. Indicators: • Naming a method/strategy as being used that did not match the trial-and-error approach in Video 1 (e.g., inverse operation was used; two methods were shown for solving the problem) • Seeing the initial subtraction (23 – 14) as confusing for learners rather than necessary to establish that subtraction will not work here • Unclear or ambiguous responses that may suggest a misunderstanding.	**No direct reference to video** – comment states partial elements that may reflect the teacher's own knowledge about this problem, without direct reference to how this knowledge links to the video explanation, or reference to some alternative route to solving the problem: Indicators: • Correct alternative partial strategy/method named/ suggested, but no unpacking. E.g., 'it will be simple if we work by 10 counts', use PPW diagram, etc.	**Hollow** – Explanation that mentions 'steps' being presented generically rather than mathematically, or focuses on learner ease, familiarity, enjoyment, or general praise or dislike of explanation without any specific mathematical referent. Indicators: • Generic reference to 'steps' to take with no specific mathematical referent • Focus on ease, fun, clarity, for self or learner, with no mathematical detail (e.g. 'Learners can easily understand this", "this is easy to follow", "it is fun and simple", "this is straightforward, even struggling learners can understand this one", "use what you know," etc. • Doesn't like it because it is; "confusing", "not clear", "not simple", "too long", etc.	**Partial attending**– Noting of, or claim about, some of the steps or aspects of working shown in the video. Responses in this category have a mathematical referent but attend only to some steps or aspects rather than to all of them Indicators: • Refers to "trial-and-error" without stating how it is applied • Work with given numbers/Use what you have • 'Before is bigger in subtraction' • 'Will help learners to calculate correctly … be able to use addition and subtraction.

Level 2a	**Level 2b**
Comprehensive noticing of the steps, but no reference to how the steps offer a rationale for the calculation that is carried out. Indicators: • Provides a sequence of steps or states what was done (and how it was done) • References a trial-and-error method and how it was applied, but no rationale is offered	**Comprehensive solution steps of an alternative route** to solving the problem – no explicit reference to trial-and-error. Indicators: • States what the learner needs to do with no reference to video 1 explanation (add the given numbers', add 14 and 23, etc)

Level 3a	**Level 3b**
Responses reflect a vantage point on trial-and-error reasoning to justify why addition works, or why video 1 explanation should be rejected. Indicators: • Responses that frame the explanation as logical, trial-and-error-based, or relational, e.g., recognition of the number sentence structure (e.g., "The missing number must be larger since 23 is being subtracted from it—so 23 and 14 must be added to get the missing number.", "trial and error showed that adding the numbers gives the missing total, so addition works.")	**Vantage point on alternative approach.** No explicit reference to video 1 explanation

DISCUSSION

For Kinach (2002) and others who have noted instrumental emphases in pedagogy, Level 3 noticing and/or enactments of instructional explanations is the key goal with a range of expansion categories noted within this level. Our dataset, in contrast, suggests that Level 2 is not always in place as a basis from which to work in professional development programmes. Additionally, our ambitions for Level 3 are currently more modest than Kinach's expansive focus on concepts, problem-solving processes and epistemic warrants in the discipline. Based on our dataset, we are simply looking for some expression of awareness of why missing minuend values have to be larger than the subtrahend value, or some overview vantage point on the overall procedure.

Our sense is that our broader methodology of offering students different explanations to consider and compare (the broader task included three further explanations for this task) is useful for getting insights into aspects of mathematical working that teachers show awareness of and value. In the broader video task, we noted with interest that the video clip telling learners to simply add the two given numbers of 23 and 14 was the video that had the highest number of positive responses as a 'good explanation'. This result was also evident in the written pretest data, where the same group of teachers was asked to explain the concept to learners before viewing video clips of different instructional explanations. Our analysis of these written responses (Mukuka & Venkat, 2026) showed that more than half relied solely on symbolic representations (such as column or horizontal addition of 23 and 14) without any accompanying explanations or justifications. In contexts of low attainment, where gaps in teachers' mathematical knowledge have tended to be highlighted, such insights are useful for directing our attention in more nuanced ways at the challenges that need to be addressed. Our analysis of teachers' evaluations of this and the other explanations points to some of these nuances: first, attention to precision in talking about mathematical working; and second, contrasting examples of more rote and more rational explanations and discussing what makes the latter more useful, rather than simply presuming that explanations with rationales for the choices of steps are self-evidently better.

Our current application of the levels and indicators to a second video explanation for the same task suggests that the framework is more broadly applicable. Further, recently collected data from the same teacher cohort on the explanations they would give a child stuck on the same missing minuend problem presented in this paper suggest substantial moves into Level 2 of our model, with few instances of errors in the explanations offered. This move, while partial from the perspective of strong mathematical knowledge for teaching, still points to a move forward from the balance of data seen in Level 1 in the baseline dataset from which the framework presented in this paper was derived.

Acknowledgements

This work stems from research funded by Taighde Éireann – Research Ireland under Grant number COALESCE/2024/3903.

References

Bowie, L., Venkat, H., & Groenewald, C. (2025). Teachers' understandings of early number models: Data from an online pilot study. In Proceedings of the 33rd Annual Conference of SAARMSTE, 52-57.

Corbin, J. & Strauss, A. (2008). *Basics of qualitative research: Techniques and procedures for developing grounded theory (3rd ed.).* Thousand Oaks. SAGE. https://doi.org/10.4135/9781452230153

Güneş, C., Kercher, A., & Zaskis, R. (2024). Teachers' noticing mathematics within imprecise instructional explanations. *Canadian Journal of Science, Mathematics and Technology Education, 24*, 400-417. https://doi.org/10.1007/s42330-025-00355-3

Kinach, B.M. (2002). A cognitive strategy for developing pedagogical content knowledge in the secondary mathematics methods course: Toward a model of effective practice. *Teaching and Teacher Education, 18*, 51-71. https://doi.org/10.1016/S0742-051X(01)00050-6

Leinhardt, G. (1993). Weaving instructional explanations in history. *British Journal of Educational Psychology, 63*, 46-74.

Mukuka, A., & Venkat, H. (In press, 2026). Insights from primary school teachers' instructional explanations in early grade mathematics. *Proceedings of the 34th Annual Conference of the Southern African Association for Research in Mathematics, Science and Technology Education.*

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching, 77*, 20-26.

Spaull, N., Pampallis, I., Ardington, C., Sapire, I., & Isaac, P. (2022) Not adding it up: Grade 1 mathematics outcomes in the Eastern Cape and Limpopo. In H. Venkat & N. Roberts. Early Grade Mathematics in South Africa. Oxford University Press.