HOW TO DEVELOP AT-RISK FIRST GRADERS' PATTERNING SKILLS: EVALUATING A SUPPORT PROGRAM BEYOND ACCURACY THROUGH EYE TRACKING

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Patterning skills in first grade are a robust predictor of later mathematics achievement, underscoring the need to foster them. This study examines not only whether a support program for first-graders at risk of developing mathematical difficulties (RMD) reduces errors on repeating-pattern tasks, but also what eye tracking can reveal about pattern-recognition processes. Eighteen RMD first graders were randomly assigned to an RMD support group that received one-to-one instruction on repeating patterns and an RMD control group without additional support. Pre–post test measures showed significant reductions in error rates for both groups. However, only the RMD support group showed gaze shifts towards relational processes. These findings highlight the value of combining process with performance measures when evaluating interventions.

INTRODUCTION

Intervention studies in early mathematics learning have repeatedly demonstrated that targeted instruction can improve children's performance. These studies often document improvements through product-oriented measures such as accuracy on task-specific assessments. While such outcome measures are indispensable for evaluating whether an intervention "works," they provide only a limited view of how children's thinking changes, especially in domains where underlying processes—rather than merely the correctness of answers—are central.

One such domain is early patterning. Repeating patterns (e.g., AABBAABB) are widely regarded as a foundation for early algebraic thinking (Clements & Sarama, 2020) and are internationally recognized as central to early mathematics curricula. Early proficiency with repeating patterns predicts later mathematical success (Lüken et al., 2014; Rittle-Johnson et al., 2019), yet many first-graders—particularly those at risk of developing mathematical difficulties (RMD)—struggle with patterning, often relying on superficial or inefficient processes (Lüken & Sauzet, 2021; Rittle-Johnson et al., 2015). Existing patterning interventions document substantial gains in performance (e.g., Kidd et al., 2013; Wijns et al., 2021), but they rarely combine performance measures with process-oriented data.

In this study, we address this gap by using eye tracking to capture how RMD first-graders visually process repeating patterns before and after a support program, alongside their error rates. Eye tracking allows us to infer processes that remain

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invisible in accuracy data alone. We investigate *to what extent a support program on repeating patterns (a) reduces error rates and (b) shifts the distribution of processes in first-grade RMD students relative to RMD peers without additional support*. Methodologically, we ask: *Compared to performance measures, what does eye tracking reveal after support, and which aspects remain invisible?*

THEORETICAL BACKGROUND

Patterning in Early Mathematics Learning

Patterning is an early algebraic activity in which children identify a predictable regularity, verify its structure, and use it, for example, to continue a pattern. Repeating patterns consist of a finite unit of repeat (e.g., AB) that is iterated without change (e.g., ABABAB). Understanding repeating patterns entails recognizing similarities across representations and flexibly identifying the unit of repeat as an early introduction to algebraic thinking, and preschoolers who can reproduce, extend, and explain more complex patterns before school entry show more advanced number-concept growth later on (Lüken et al., 2014). While preschoolers can usually copy simple and more complex repeating patterns, extending them is of intermediate difficulty, and success rates decline as patterns become more complex (Lüken & Sauzet, 2021; Rittle-Johnson et al., 2015). Such features shape how children approach patterns, identify, and validate the unit of repeat, which are *pattern-recognition processes*.

Identifying Processes in Repeating-Pattern Tasks (using eye tracking)

Children's work with repeating patterns is a sequence of processes: hypothesizing a candidate unit of repeat, verifying it against adjacent elements, and extending the pattern once validated. Regarding students' cognitive processes, we can identify two types of thinking: recursive and relational (Junker et al., 2024). In recursive thinking, the pattern is extended element by element, focusing on local appearances ("after red comes blue, then red again"), whereas in relational thinking, the unit of repeat is understood as a composite whole that is applied flexibly. Across different approaches—observations, interviews, or drawings (Lüken & Sauzet, 2021; Papic et al., 2011)—typologies converge on a spectrum between recursive, elementwise and relational, unit-based processes. Lüken and Sauzet's (2021) five increasingly sophisticated processes can be situated from no reference to pattern, via elementwise and recursive chanting, to an explicit "view of unit of repeat." Papic et al. (2011) similarly document a developmental shift from random or direct-comparison responses and simple alternation toward explicit use and transfer of the unit of repeat. Junker et al. (2024) showed that both low- and high-number-sense first-graders frequently used recursive strategies; however, only children with high number sense consistently shifted to relational strategies, whereas students with weaker number sense employed a wider range of recursive variants and often failed to generalize the unit of repeat.

Recent work has begun to use eye tracking to derive pattern-recognition processes in repeating-pattern tasks (Baumanns et al., 2024), as it allows a detailed insight into

cognitive processes (Schindler et al., 2025). Baumanns et al. (2024) derived four pattern-recognition processes from such gaze-overlaid videos (see Figure 1): (1) looking at one unit of repeat, (2) looking at one unit of repeat and validating/applying it, (3) looking at each element, and (4) unsystematic jumping over the pattern. These processes can be situated along a continuum from no pattern reference (4) → element-by-element recursive processes (3) → unit-based relational processes (2). Based on gaze alone, process (1) can reflect attention to singular characteristics (recursive) or a view of unit of repeat (relational). Using eye tracking with RMD students is particularly valuable because they are often unable to self-report their solution processes in a way that yields valid data for research.

Interventions and Support Programs on Patterning

A growing body of intervention studies shows that explicit teaching of repeating patterns can accelerate young children's mathematical development. In Grade 1 and even before formal schooling, small-group instruction and one-to-one tutoring that systematically incorporate copying, extending, repairing, and discussing patterns have enabled struggling, low-achieving, and low-income children to outperform untreated peers on pattern tasks, develop flexible use of the unit of repeat, and achieve higher overall mathematics and later arithmetic scores (Hendricks et al., 2006; Kidd et al., 2013; Starkey et al., 2004). Beyond documenting performance gains, Papic et al. (2011) provided evidence of process change from random or direct comparison and simple alternation toward identification and transfer of the unit of repeat, accompanied by advances in spatial structuring. Across interventions, certain design features recur: a graded progression from simple to more complex units, frequent talk about the unit of repeat, and the use of multiple representations to highlight structural similarities. What remains underexplored—especially for RMD students—is how children's pattern-recognition processes change under instruction. Prior studies have rarely tracked real-time processes, leaving open whether performance gains stem from careful element-by-element matching or from a genuine shift to unit-based reasoning.

METHODS

Study Design

The study followed a pre–post control-group design with first-graders at risk of developing mathematical difficulties (RMD). In the sixth week of Grade 1, all participating children completed a standardized mathematics test (MBK 0; Krajewski, 2018) and an eye-tracking diagnostic on repeating patterns. Children scoring at least one standard deviation below the age norm on the MBK 0 were classified as RMD and formed the target cohort. Twenty RMD children were randomly assigned to either an RMD support group or an RMD control group (10 per group). The two groups did not differ significantly on the MBK 0. Between pre- and post-test, attrition in the control group reduced the final sample to 18 children (support: $n = 10$; control: $n = 8$). During the six-week intervention period, the RMD support group received additional

instruction; the control group followed regular classroom instruction. After data collection, the school provided comparable pattern activities to the control group.

Tasks and Procedures

Children completed 21 repeating-pattern tasks at pre- and post-test (7 pattern structures × 3 representational formats). We used a range of simple and more complex units (e.g., AB, AAB, AABC, ABAC) in three representational modes (colors, numbers, shapes). Items were in identical order at both time points. Testing took place individually in a separate room at the school. Each trial displayed a repeating pattern ending with a white blob. Children responded orally by naming the color, number, or shape that they believed continued the pattern. The experimenter recorded each response and advanced to the next item. Incorrect responses were not corrected. Other tasks were interleaved but are not analyzed here. Eye movements were recorded with a Tobii Pro Spectrum eye tracker (120 Hz) with stimuli presented on the integrated 23.8-inch monitor. Children were seated approximately 60–65 cm from the screen. Each session began with a 5-point calibration and 4-point validation (mean validation accuracy: 0.59°).

Information on the support program

Children in the RMD support group participated in six weekly one-to-one sessions (20 min. each) conducted by the author in a separate room. The sessions centered on hands-on activities that progressively deepened children's understanding of repeating patterns and the unit of repeat. Across sessions, children duplicated, created, extended, and repaired repeating patterns of increasing complexity and translated them across multiple representations (colors, numbers, shapes, body percussion). The unit of repeat was made explicit (e.g., using a *repeating-pattern-stick*) and repeatedly invoked when describing, extending, or repairing patterns. Throughout all sessions, the instructor consistently prompted explicit language about patterns and the unit of repeat.

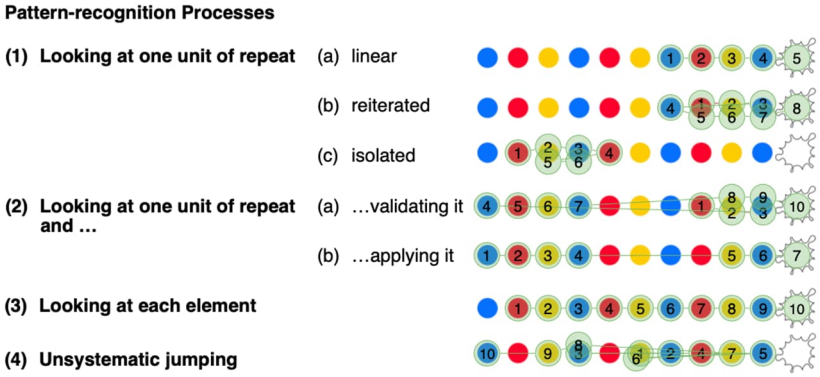
Qualitative coding of pattern-recognition processes and quantitative analyses

Gaze-overlaid videos were exported from Tobii Pro Lab and analyzed to identify children's pattern-recognition processes. We used a deductive content analysis approach based on the category system developed in Baumanns et al. (2024) that distinguishes four mutually exclusive processes (see Figure 1): (1) looking at one unit of repeat, (2) looking at one unit of repeat and validating/applying it, (3) looking at each element, and (4) unsystematic jumping. Across pre- and post-test, 1,554 task videos were available. Trials coded as process (4) were cognitively indeterminate and, therefore, excluded from quantitative analyses. Trials with insufficient gaze data were coded as data loss. A second rater independently coded 35% of the dataset with an almost perfect interrater agreement of $\kappa = 0.86$.

For shifts in processes, we computed 3 × 3 transition matrices (pre-test process × post-test process) for both groups separately. Changes in the distribution were tested with McNemar–Bowker tests. Error rates were computed as the proportion of incorrect

responses per child and time point (non-responses counted as incorrect). Pre–post changes were analyzed using paired Wilcoxon signed-rank tests.

Figure 1: Pattern-recognition processes coded in this study



RESULTS

Shifts in Pattern-Recognition Performance

For both groups, error rates on the repeating-pattern tasks decreased substantially from pre- to post-test. In the RMD support group, the average error rate dropped from 51.2% at pre-test to 19.0% at post-test, a significant reduction with a large effect ($W = 54$, $p = .004$, $r = 0.96$). The RMD control group showed a similar pattern: error rates decreased from 44.3% to 13.1%, again with a significant and large effect ($W = 33$, $p = .021$, $r = 0.83$). Thus, based solely on error rates, both groups exhibited marked improvements in performance over time.

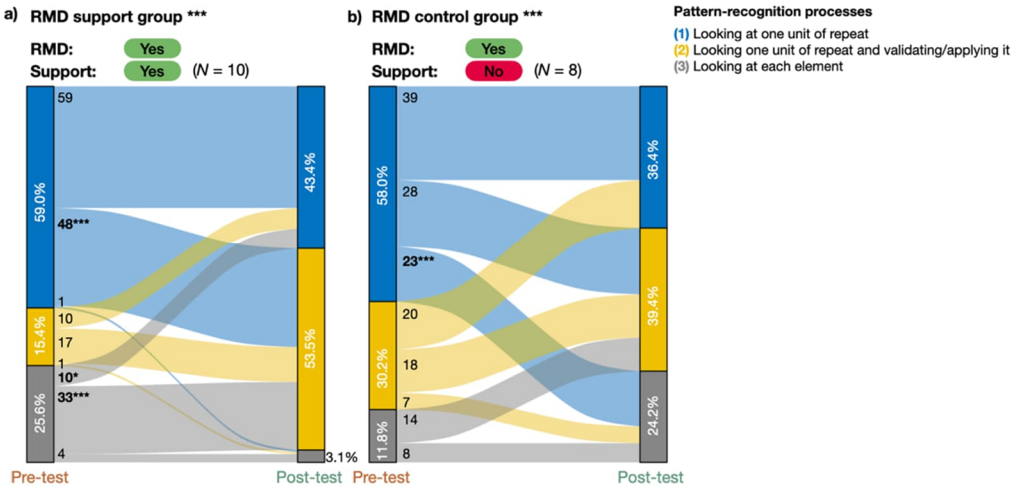
Shifts in Pattern-Recognition Processes

Eye-tracking data reveal qualitatively different changes in both groups' pattern-recognition processes (see Figure 2). For the RMD support group, item-level transitions from pre- to post-test indicated a highly significant redistribution of processes with a large effect ($\chi^2(3) = 62.38$, $p < .001$, $g = 0.38$). Item solutions that had been coded as process (3) ("looking at each element", recursive) at pre-test were substantially more likely to be coded as process (2) ("looking at one unit of repeat and validating/applying it", relational) at post-test than vice versa ($p < .001$, $g = 0.47$). Likewise, solutions initially coded as process (1) ("looking at one unit of repeat" without validation) moved significantly more often into process (2, relational) than out of it ($p < .001$, $g = 0.33$). Thus, post-test solutions in the support group shifted toward unit-based validation, and away from elementwise checking or unvalidated unit focus.

In the RMD control group, pattern-recognition processes also changed significantly over time, but with a different profile and a smaller overall effect ($\chi^2(3) = 26.67$, $p < .001$, $g = 0.21$). Here, post-hoc analyses showed that item-level solutions were more likely to move from process (1) to process (3, recursive) than in the reverse direction ($p < .001$, $g = 0.50$). In other words, without targeted support, RMD students

tended to shift toward a recursive approach. Taken together, eye tracking reveals that similar improvements in accuracy across the two groups are underpinned by different process changes: a move towards relational, unit-based validation in the support group versus intensified elementwise checking in the control group.

Figure 2: Pre/post-test processes of RMD support (a) and RMD control group (b)



What Eye Tracking Can and Cannot See

Although eye tracking can reveal intervention-related shifts in processes, it is neither a stand-alone replacement for other data sources nor a methodologically neutral approach. In the following, we juxtapose verbal data from support sessions to illustrate both the added value and the blind spots of eye tracking. The following transcript from an early support session illustrates the limitations of relying on *verbal* explanations to reconstruct first graders' pattern-recognition processes:

1	I	Close your eyes. I will take away one colour. Which colour is missing?
2	Anju	Yellow!
3	I	How do you know that? How do you know?
4	Anju	(*softly spoken, not understandable*)
5	I	How do you know that yellow is missing? It could also have been red (*places a red counter in the gap*). Why doesn't that fit? Doesn't it fit?
6	Anju	(*shakes head*)
7	I	Why not?
8	Anju	Because yellow has to be here.
9	I	Why?
10	A	Because … [*8 s pause*]

Even in this one-to-one context, the child is unable to articulate their process, so we cannot reconstruct whether they are attending to the repeating unit, comparing local elements, or drawing on a more global relational structure. In other words, the verbal

data confirm *that* the child successfully repaired the pattern, but they do not make the process by which the child arrived at this solution accessible to process analysis.

In an activity called "Flash pattern" (the repeating pattern is visible for 2–3 seconds and the child must name the next element), the student Kafei first gives a wrong answer, then settles on the correct answer "Purple!", and justifies this choice by marking the unit of repeat with a stick and explicitly naming it as the "unit of repeat." When the interviewer challenges his answer by suggesting that green could also continue the pattern and asks him to explain using the unit of repeat, Kafei contrasts the correct configuration "green, orange, purple" with the hypothetical, incorrect completion "green, orange, green," pointing to the corresponding positions in the pattern. Kafei offers a relational justification by invoking the unit of repeat. His explanation shows emergent relational language and the ability to warrant his choice. These qualitative shifts in how he justifies his solution are not captured by gaze data. Thus, eye tracking can only represent one dimension of the support program's benefits.

DISCUSSION

This study investigated how a support program on repeating patterns affects first-graders at risk of developing mathematical difficulties, focusing on both performance and eye-tracking–based pattern-recognition processes. Two main conclusions emerge: First, both the RMD support and RMD control group showed significant reductions in error rates. If we had evaluated the support program solely on product measures, we would have concluded that the program had no distinct added value. The eye-tracking analyses revealed that only the RMD support group showed shifts of processes away from recursive, elementwise checking towards relational, unit-based processes. For at-risk students, this shift may be educationally meaningful, as it provides access to algebraically valuable ways of structuring patterns rather than knowing "how to get the next element right." In contrast, RMD control students moved from a presumably cursory focus on one unit to recursive "looking at each element". Thus, similar gains in accuracy in the two RMD groups were underpinned by different process changes.

Methodologically, the study illustrates the value of combining performance and process data when evaluating interventions. At the same time, the interview excerpts highlight that gaze data do not capture developments in students' language and argumentation, which are another central dimension of learning. Understanding how interventions work seems to require triangulating multiple data sources.

Several limitations qualify these conclusions. The sample was small and drawn from a single school, limiting statistical power and generalizability. The support program was delivered by a researcher under one-to-one conditions rather than by classroom teachers, so its impact in classroom settings remains to be tested. Also, we do not know how durable the observed process shifts are over time. From an instructional perspective, the present results suggest that brief, structured support can move at-risk children away from effortful element-by-element checking toward relational, unit-

based pattern recognition. The rationale for such intervention is thus not only to raise performance, but to open up more advanced ways of thinking about repeating patterns—ways that are likely to be foundational for later algebraic thinking.

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