INTERDISCIPLINARITY IN MATHEMATICS EDUCATION: A DIALOGUE AT THE BORDERS BETWEEN DISCIPLINES

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This paper proposes a theoretical framework for understanding interdisciplinarity in mathematics education as a comparison of rationalities. To this aim we refer to Habermas's construct of rationality and conceptualise disciplinary borders as semiotic spaces of negotiation, according to the lens offered by De Luca Picione and Valsiner. The framework is operationalised by analysing a case study, based on the Fagnano's problem, illustrating how students navigate mathematical and physical rationalities. The findings show how communicative acts mediate epistemological negotiations at the border between the disciplines, supporting the co-construction of meaning and a synthesised interdisciplinary understanding.

INTRODUCTION AND RATIONALE

In mathematics education, interdisciplinary activities are often seen as vehicles that naturally encourage the application of mathematical concepts across domains and cultivate students' dispositions to generalise mathematical knowledge. The claim that mathematical topics form networks of connections within and across disciplines and extend into learners' lived experiences is consistent with this framing. In line with this view, we adopt Klein's definition of interdisciplinarity as "a means of solving problems and answering questions that cannot be satisfactorily addressed using single methods or approaches" (Klein, 1990, p. 196). This perspective provides an operational, problem-driven criterion for discipline integration while remaining method-neutral and fully compatible with inquiry-oriented designs. This entails orchestrating mathematical activity in dialogue with other domains, deploying mathematics as an instrument within the sciences and importing everyday knowledge into modelling, so that learners engage with meaningful problems and sustain inquiry-based trajectories (Capone, 2022). The need of developing robust research on integrated interdisciplinary teaching that builds deep connections between mathematics and other disciplines has been strongly emphasized in recent literature (e.g. Goos et al., 2023). Research on boundaries and boundary-crossing has also shown how interactions across socio-cultural differences can become resources for learning. Within this tradition, boundary objects (Star, 1989) can facilitate interactions, and boundary-crossing designs may scaffold flexible thinking as students engage with distinct epistemic practices (e.g. Bakker & Akkerman, 2014) thereby fostering collaborative problem-solving for complex tasks. Yet, while influential, these approaches rarely illuminate how interdisciplinary meanings are actually unfolded in classroom

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interaction. In our view, we lack theoretical tools that make visible how communicative acts mediate epistemological negotiations at the border between disciplines. We contend that progress requires a focus on the dialogical processes, on the negotiations through which disciplinary epistemologies meet, collide, and align. To fill this gap in the literature, we propose a framework that extends the notion of disciplinary borders as semiotic spaces of negotiation (De Luca Picione & Valsiner, 2017), incorporating Habermas's (1981) construct of rationality to account for the role of communicative acts in shaping interdisciplinary meaning-making. We attempt to operationalise this framework, by addressing the following research question: How does applying this framework to a case study involving Mathematics and Physics reveal mechanisms of interdisciplinary meaning-making in the negotiation of disciplinary rationalities?

To illustrate the analytical potential of the framework for interpreting how meanings are co-constructed at the border between disciplines, we draw on an interdisciplinary teaching experiment centred on Fagnano's problem and its application in billiard physics (Adesso et al., 2025). This context offered a fertile opportunity to observe how high school students navigated tensions between mathematical (proof-based) and physical (empirical) rationalities, and how communicative acts mediated the negotiation of these perspectives. The case study does not aim to produce generalizable findings but serves as an illustrative context for demonstrating how the proposed framework can be operationalised in empirical research.

THEORETICAL FRAMEWORK

To offer an analytical lens for interpreting how meanings are co-constructed through dialogue at the border between disciplines (Capone & Faggiano, 2024), we combine two complementary conceptual tools. First, we adopt from Semiotics the metaphor of the border as a semiotic cross-cultural space (De Luca Picione & Valsiner, 2017), conceptualizing disciplinary interaction as a space of negotiation rather than a mere interface between separate domains. From this perspective, borders regulate processes of differentiation, comparison, and meaning transformation, functioning as liminal spaces where cultural and disciplinary meanings can be reorganized. In mathematics education, this allows us to interpret interdisciplinarity as a dynamic process in which mathematical and non-mathematical epistemologies interact, generating dialogical tensions that can become engines for learning. Second, from Pragmatics, we integrate Habermas's (1981) construct of rationality to analyse how communicative acts support interdisciplinary meaning-making. Following Boero (2006), rationality is understood not as a property of knowledge products but as a characteristic of intentional problem-solving behaviours shaped by socio-cultural contexts. Habermas distinguishes three interrelated components of rationality: epistemic rationality (the justification of knowledge claims), teleological rationality (the orientation of action toward goals through chosen means), and communicative rationality (the orientation toward mutual understanding through discourse). In our proposal, these components are considered as analytical categories for tracing how students negotiate and validate disciplinary perspectives during interdisciplinary dialogue. Applying these components allows us

to delineate, for instance, key differences between mathematical and physical rationalities. Epistemically, Mathematics relies on deductive proof and formal consistency, whereas Physics grounds validity in empirical evidence and experimental control. Teleologically, Mathematics aims at abstraction and generalization, while Physics seeks to explain and predict physical phenomena. Communicatively, Mathematics privileges symbolic precision and proof-based argumentation, whereas Physics combines mathematical formalism with descriptive, experimental, and representational forms of communication.

The following case study illustrates how this framework can be applied to analyse classroom interactions and to reveal mechanisms of interdisciplinary meaning-making.

METHODS

We explored our research question through an interdisciplinary teaching intervention with two nineth-grade and four tenth-grade classes. The activity was designed around five principles: sustaining work within a defined disciplinary frame; inviting cross-disciplinary connections; cultivating language that gradually supports conceptualization (in the sense of Vergnaud, 2013) and argumentation; valuing alternative viewpoints; and recognizing multiple solution strategies. Co-designed by researchers and teachers, the activity examines Fagnano's problem (the minimal-perimeter inscribed triangle in an acute triangle) and leads students to see how Snell's optical law in a triangular billiard coheres with the relevant geometric properties.

Description of the designed activity

The activity was designed to make disciplinary contrasts visible and to foster communicative acts through which students negotiate meanings and recognize the complementarity of mathematical and physical perspectives. It begins with a geometric analysis of Fagnano's problem: historical sources situate the task, and a guided exploration of Fejér's proof in GeoGebra foregrounds invariants, logical deduction, and the collaborative construction of the minimal-perimeter inscribed triangle (the "orthic trajectory"), refining language and criteria of proof. The work then shifts to billiard dynamics: students build a triangular table to test the orthic trajectory, reflect on friction as a physical constraint and consider its implications for idealization, and finally use a "light billiard" to verify the trajectory, connecting geometric optics with the physics of billiards.

Data collection and analysis

We adopted a qualitative, interpretative approach to data collection and analysis. Data was collected across three sessions and included video and audio recordings of group work, students' written productions, and researchers' field notes. The analysis was not aimed at empirical generalization but at illustrating how the framework operates in practice. The focus was on how communicative acts mediated meaning-making at the border between disciplines. Relevant episodes were identified and selected excerpts were transcribed and translated into English; all data were anonymized through

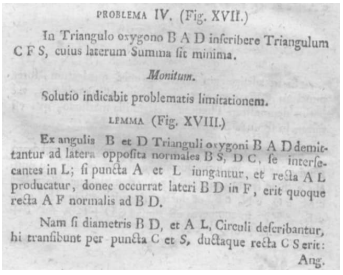
alphanumeric coding. Episodes were interpreted through the lenses of the semiotic border and the components of rationality proposed by Habermas (epistemic, teleological, and communicative), to trace how students negotiated mathematical and physical rationalities while working on Fagnano's problem.

FINDINGS

In this section we present selected episodes and highlight how the suggested framework reveals mechanisms of interdisciplinary meaning-making in the negotiation of disciplinary rationalities.

According to the design of the activity, during the first phase, students were introduced to Fagnano's problem through the historical sources (Fig. 1a).

Fig. 1: a.) Fagnano's problem in "Problemata quaedam ad methodum maximorum et minimorum spectantia" and b.) reproduction in GeoGebra



PROBLEMA IV. (Fig. XVII.)

In Triangulo oxygono B A D inſcribere Triangulum C F S, cuius laterum Summa ſit minima.

Monitum.

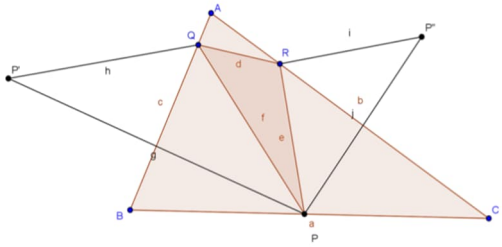
Solutio indicabit problematis limitationem.

LEMMA (Fig. XVIII.)

Ex angulis B et D Trianguli oxygoni B A D demittantur ad latera oppoſita normales B S, D C, ſe interſecantes in L; ſi puncta A et L iungantur, et reſta A L producatur, donec occurrat lateri B D in F, erit quoque recta A F normalis ad B D.

Nam ſi diametris B D, et A L, Circuli deſcribantur, hi tranſibunt per puncta C et S, ductaque recta C S erit: Ang.

a.



b.

Notably, students were not presented with the proof, but they were guided by the teacher in constructing the triangle of minimum perimeter inscribed in the acute triangle ABC. The text that follows is an excerpt from the discussion in one of the classes while a construction like the one in Fig. 1b was displayed on the shared screen.

Teacher: Under what conditions would you expect triangle PQR to have a minimum perimeter?

Student_1: When the distance between P' and P'' is minimal, because this distance coincides with the perimeter.

Teacher: Let us add something to this consideration...

Student_2: When points P' and P'' are aligned.

Teacher: You are still missing something.

Student_3: When Q and R are aligned with the points P' and P''.

The process of constructing Fejér's proof exemplifies mathematical epistemic rationality. Students justified their claims based on established geometric principles, such as axial symmetry (PQ ≅ P'Q and PR ≅ P''R). Validity was established through logical deduction within the geometric system. When prompted about the minimum perimeter condition, students initially focused on distance minimization ("*when the distance between P' and P'' is minimal*"). Still, they refined this through discussion to

include the alignment of points (“*When Q and R are aligned with the points P’ and P’’*”), demonstrating collaborative construction of meaning grounded in geometric logic. The goal was explicitly mathematical: to prove the existence and nature of the minimal inscribed triangle.

Divided in groups, students were then requested to understand how to minimise the length of P’AP’’. GeoGebra was not just a visualization tool but an environment for precise construction and formal exploration guided by geometric principles. In the final collective discussion, students were guided by the teacher to complete the geometric proof based on their observations. The text that follows is an excerpt of the discussion within one of the groups.

Student_4: Moving P along BC we see that the lengths of AP’, AP and AP’’ change and the length of P’P’’ also changes, but we can also say that the length of P’P’’ is minimum when the length of the other three is minimum.

Student_5: Yes, right, P’P’’ is minimal when AP is minimal....

Student_6: Good point. And what is the minimum length of an AP?

Student_5: That is when AP is perpendicular to the side BC.

Student_4: Right, namely, when AP is the altitude relative to BC of triangle ABC.

Student_6: So, the inscribed triangle of minimum perimeter is the one that is obtained by taking as P the foot of the altitude conducted from A to BC...

Student_4: ...and as Q and R the intersections of P’P’’ with AB and AC

Student_5: Let’s check this with GeoGebra.

Student_6: Is something still missing?

Student_4: Of course, who can tell us that Q and R are also the feet of the other two altitudes of the triangle?

These students used the dragging in GeoGebra with a clear teleological purpose: to observe invariant properties (amplitude of the angle P’AP’’) and test hypotheses (minimizing AP). This shows an awareness of using the tool to pursue a mathematical investigation. The utterance “*...who can tell us that Q and R are also the feet of the other two altitudes?*” reveals an emerging meta-mathematical awareness of proof sufficiency and rigor, illustrating teleological rationality understood as goal-directed reasoning toward a fully established proof.

At the outset, the connection between minimizing P’P’’ and the position of P remained implicit. Guided exploration, most notably the focus on minimizing AP, rendered this connection, together with its link to the altitude, as explicit mathematical knowledge. In parallel, student questions brought to the fore the need for completeness in proof (verifying Q and R). Interpreted thus, the mathematical phase illustrates how the framework accounts for meaning-making at the border: epistemic and teleological rationalities shaped students’ actions toward solution and proof, whereas

communicative rationality mediated their negotiation of what counts as valid and rigorous.

In the second phase, students built a triangular billiard table using poor materials and drew the orthic trajectory on it with markers. The aim was to use a ball and verify that if it is thrown from one of the vertices of the orthic trajectory, it goes through its sides and comes back to the starting point (Fig. 2a).

Fig. 2: Construction of a.) the triangular billiard and b.) the "light billiard"

a. b.

To do so, as the excerpt below shows, they have used the knowledge built in the first phase of the activity.

Student_5: Ok… with these strips, we have our triangular billiard table. Now we have to use it to verify Fagnano's problem.

Student_6: And what does verifying the problem mean?

Student_7: Um... see if a ball bounces over the edge and back to where it came from?

Student_6: Yes, but if you try this way it doesn't... It doesn't always happen.

Student_7: In fact, we have to figure out when the ball goes back to where it came from.

Student_5: I see… we have to draw the orthic triangle and roll a marble with a stick.

Student_8: Yes! Right! That's why last time we saw that if you take the altitudes and join the feet, you have the minimum triangle.

Student_6: Yes, but let's see if it's true that by following that triangle, the ball goes back to where it started.

Since friction affected the ball's motion, confirming that the orthic triangle realizes a closed, minimal-perimeter trajectory proved challenging. Therefore students were suggested to construct a "light billiard" with mirrors on the sides. In this optical analogue, Fermat's principle applies: a light ray connecting two points selects the path that minimizes travel time (Fig. 2b).

By explicitly positing a mathematical construct to guide physical action ("*we have to draw the orthic triangle and roll a marble with a stick*"), students display a teleological orientation grounded in epistemic rationality. The dialogue exemplifies communicative rationality at the border, where the negotiation of "verifying the

problem" translates abstraction into procedure ("*see if a ball bounces... and back*") and aligns prior knowledge (minimum triangle = orthic triangle) with the task at hand ("*That's why last time we saw...*"), thereby bridging the disciplines. The failed empirical attempt rendered the frictionless idealization of the mathematical model explicit at the border, clarifying its scope and limits. Through teacher guidance and the experimental design, the law of reflection and Fermat's principle were made explicit as epistemic warrants for the "light billiard" analogy, aligning teleological aims (toward a complete demonstration) with communicative rationality in the negotiation of validity across domains. Crucially, the border is not seen as "to be crossed", but the locus where students confront epistemic and teleological contrasts and adapt their strategies, where the framing acquires meaning and where semiotic transformations occur.

During the whole-class discussion groups reported their work and the teacher oriented the exchange toward how Fagnano's problem had been approached from mathematics and physics and what this revealed about their relationship. One contribution captured this interdependence: "*Without doing the construction to find the orthic triangle with GeoGebra, we would not have understood where and how to direct the laser, because if you don't direct it well, the ray continues to reflect, and the triangle is not formed.*" This underscores how mathematical construction functioned as an epistemic scaffold for experimental action, making the border between disciplines visible and productive rather than contrasting.

DISCUSSION AND CONCLUSIONS

Building on a theoretical framework that conceptualises interdisciplinarity in mathematics education as a comparison of rationalities, and that treats disciplinary borders as semiotic spaces of negotiation (De Luca Picione & Valsiner) in dialogue with Habermas's construct of rationality, we operationalised the framework through a case study centred on Fagnano's problem and its physical application. Applied to this context, the framework illuminates the mechanisms through which communicative acts mediate epistemological negotiation at the border between mathematics and physics.

In the first phase, epistemic rationality supported the construction and refinement of knowledge (e.g., establishing the orthic triangle and proof sufficiency), while teleological rationality organised purposeful progression toward a logically coherent and communicable argument, foregrounding issues of validity and rigor. In the second phase physical epistemic rationality sought empirical verification; the friction-induced mismatch with the idealised geometric model rendered idealization and physical constraints explicit, prompting a teleological shift in means. The adoption of the "light billiard" invoked the law of reflection and Fermat's principle as epistemic warrants, enabling purposeful adaptation when initial methods failed and bridging mathematical construction with experimental practice through negotiation at the border.

In relation to the research question the analysis shows that epistemic, teleological, and communicative rationalities interact productively at the border, where the framing

acquires meaning and semiotic transformations occur. The border need not be crossed; it should be recognised as the locus where disciplinary perspectives meet, collide, and align, and where communicative acts sustain the co-construction of an integrated, interdisciplinary understanding.

Finally, consistent with our stated objective, the case functions as an illustrative context rather than a source of generalisable claims. Its value lies in demonstrating the analytical potential of the proposed framework, making visible how communicative acts mediate epistemological negotiations at the border and how purposeful strategy adaptations (teleological rationality) coordinate with knowledge-building (epistemic rationality) across domains. Future work may extend this operationalisation to diverse settings and grade levels, examining how variations in task design and facilitation further shape negotiation, validity, and rigor at the border.

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