LEARNING BEGINNING ALGEBRA IN AN EXCEL ENVIRONMENT AS A COMPLEX DYNAMIC SYSTEM

Michal Tabach[1], Einat Heyd-Metzuyanim[2], Tamar Aviram[1]

[1]Tel Aviv University, School of Education, [2]Technion - Israel Institute of Technology

This study employs an integration of commognition and complex dynamic systems (CDS) theory to investigate how beginning algebra students learn in an Excel environment. Through a case study of two low-achieving seventh-grade students, we trace how noncanonical and canonical mathematical narratives emerge, stabilize, and shift during spreadsheet-mediated activity. Findings show that Excel's feedback loops reinforced students' prevailing meta-rules—whether correct or not. A phase shift occurred only when the interviewer explicitly targeted the underlying non-canonical meta-rule, enabling reorganization toward a canonical discourse. We claim that the CDS lens offers terms for describing difficult-to-capture processes of stabilization and perturbation in students' learning with technological tools.

INTRODUCTION

Research indicates that technology-enhanced mathematics learning positively influences students' achievement (e.g., Higgins et al., 2019), and extensive work has examined how technological environments shape mathematical activity. Spreadsheet tools such as Excel, in particular, have been shown to foster exploration, experimentation, and student agency by enabling learners to generate, test, and revise mathematical expressions (Tabach & Friedlander, 2008).

Existing process-oriented frameworks have been less effective in explaining why certain technology-mediated forms of mathematical activity stabilize over time, including cases where students' discourse remains noncanonical despite active engagement and rich feedback. Discourse-analytic approaches such as Commognition (Sfard, 2008) offer powerful tools for examining learning processes in micro-detail, but they provide limited means for modeling the persistence, resistance to change, and reorganization of learning over time. The theory of Complex Dynamic Systems, has proven useful for studying such emergent phenomena in technology-enhanced learning (Shvarz et al., 2021). This study therefore adopts a complexity-discursive perspective to investigate how early algebra students learn with Excel, illustrating how canonical and noncanonical mathematical discourses emerge, stabilize, and transform in technologically mediated environments.

THEORETICAL BACKGROUND

Commognition conceptualizes mathematics learning as a change in discourse (Sfard, 2008). Mathematical discourse is defined by its characteristic words, visual mediators,

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routines, and narratives. Narratives describe objects, relations, or processes and are subject to endorsement or rejection within a discourse community. Routines are recurrent patterns of action expressed as task–procedure pairs that specify both the task type and the procedure deemed appropriate (Lavie et al., 2019). Learning is therefore examined through shifts in how these discursive components are employed and coordinated.

A central distinction in commognition is between *object-level* and *meta-level* learning (Sfard, 2008). Object-level learning entails expanding or refining narratives and routines within an existing discourse. Meta-level learning, by contrast, involves changes in the *meta-rules* that regulate word use, determine the applicability of routines, and govern the endorsement of mathematical narratives. These meta-rules are typically implicit and acquired through participation rather than explicit instruction (Sfard, 2008).

As long as meta-rules remain intact, learners may question results, reject procedures, or try alternatives while still operating within the same interpretive framework (Chan & Sfard, 2020). Consequently, changes in procedures or outcomes do not necessarily signal shifts in the rules governing the interpretation of mathematical expressions, and discourse may remain stable despite local inconsistencies or unsuccessful actions.

Within commognition, canonicity is defined discursively rather than by "objective" correctness: a narrative is canonical when it is endorsed by the relevant discourse community (Sfard, 2008). Thus, narratives may be internally coherent and locally accepted while remaining noncanonical relative to the task or broader mathematical norms. Although commognition explains the coherence and persistence of such noncanonical narratives, it offers limited tools for modeling the mechanisms through which they stabilize or destabilize over time. To address this gap, we draw on complex dynamic systems theory.

Complex dynamic systems theory

Complex dynamic systems (CDS) consist of multiple interacting components whose local interactions produce emergent global behavior over time (Hilpert & Marchand, 2018). These systems evolve without centralized control and are characterized by nonlinearity, whereby small changes can yield disproportionate or unpredictable effects. Instead of progressing linearly, CDS develop through continual component interactions, generating periods of relative stability alongside moments of qualitative change.

A key feature of CDS is the presence of *feedback loops*—recursive interactions through which the system's current state shapes subsequent activity. These loops may be *stabilizing*, reinforcing existing patterns, or *destabilizing*, weakening them and increasing variability (Byrne, 1998; Hilpert & Marchand, 2018). As feedback accumulates, systems tend to organize around *attractors*, or preferred patterns of activity toward which they repeatedly gravitate.

In educational settings, and particularly from a discursive perspective, an attractor may appear as a recurring configuration of narratives, routines, and meta-rules (Heyd-Metzuyanim, 2025). When such a configuration persists, the system is said to be in a *phase*—a temporally extended manifestation of an attractor. A system or phase is considered *stable* when stabilizing feedback loops dominate, enabling activity to converge and persist; it is considered *unstable* when destabilizing loops increase variability and heighten the likelihood of organizational change.

Changes in a system's organization may arise through *perturbations* - differences, internal or external, that alter patterns of interaction (Byrne, 1998; Hilpert & Marchand, 2018). Whether an event functions as a perturbation is system-relative: the same prompt, question, or representation may be ignored, absorbed, or amplified depending on the system's current organization and stability. Perturbations do not produce change in a linear fashion; rather, through their interaction with existing feedback loops, they may contribute to destabilization and, under certain conditions, to a *phase shift* - a transition from one attractor-based phase to another.

The dynamics of a CDS are also shaped by *control parameters*, relatively stable features that constrain the range of possible system behaviors without being directly altered through moment-to-moment interaction (Byrne, 1998; Hilpert & Marchand, 2018). In discursive systems, meta-rules can serve as such parameters by delimiting which routines, task interpretations, and narratives are available at a given time. In this study, the system under analysis is constituted not by the students, interviewer, or technological tools individually, but by the *discursive configurations* they co-produce. Within this system, technological artifacts such as Excel generate and display narratives, thereby participating in feedback loops taken up by both students and interviewer. The meta-rules governing interpretation thus function as key control parameters shaping the system's trajectory.

Research question

1. What were the phases of the Excel-mediated discussion?
2. How were the phases sustained, perturbed and shifted?

METHODS

This study is part of a larger project investigating how spreadsheet-mediated tasks can support beginning-algebra, low-achieving students in shifting their mathematical discourse toward canonical forms. The project involved five pairs of seventh-grade students identified by their teacher as low achieving. Each pair participated in ten one-hour sessions with the third author in an out-of-school setting, meeting once every two weeks. During these sessions, students engaged in specially designed spreadsheet activities aligned with the seventh-grade mathematics curriculum. The researcher intervened only when students were unable to continue. All sessions were video-recorded, one camera capturing the screen and another capturing the students and computer, and transcribed verbatim.

In this case study, we focus on Ido and Uri (pseudonyms) as they work on the second task in the first meeting. The task involves calculating discounted prices for bottles of water. The scenario states: "The street shop is having a sale. For every bottle of water you buy, you pay only two-thirds of its original price. Add a column with the new discounted price to the table." Prior to this, the students had created an Excel table with two columns: column A listing the number of bottles and column C listing the original prices (6 NIS per bottle).

	A	C
1	number of bottles	price in the street shop
2	1	6
3	2	12
4	3	18
5	4	24
6	5	30

We first segmented the transcript into episodes and analyzed the data to identify feedback loops, attending to phases and potential phase shifts. The analytic process is illustrated in the findings section.

FINDINGS

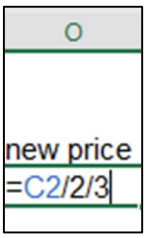
In addressing RQ1, we identified three episodes, corresponding to phase one, a phase shift, and phase two, within the 12 minutes during which Ido and Uri worked on the task. In phase one (8 minutes), a noncanonical procedure was endorsed: "to calculate two-thirds of six you do 6/2/3," yielding the narrative "the price is 1 NIS." This narrative was governed by a meta-rule linking *of* with division. Because 6/2/3 = 1 is mathematically valid, the sub-steps of dividing 6 by 2 to obtain 3 and then dividing 3 by 3 to obtain 1 became bonded.

A change occurred in the next 2.5 minutes (second episode), when the narrative and procedure derived from the meta-rule in episode one were questioned. Specifically, whether "a bottle at the new price really costs 1 NIS." This doubt triggered a back-propagation of uncertainty, leading the students to question the procedure 6/2/3.

In the third and final episode (1.5 minutes), a different, canonical meta-rule, associating *of* with multiplication, was explicitly recalled and applied, leading to the endorsement of a canonical procedure and narrative: "two-thirds of six is 6×2/3 = 4".

Having identified the phases, we now turn to RQ2 to examine how these phases were sustained and shifted through the students' interactions with Excel and the interviewer.

Phase 1- following a non-canonical meta-rule

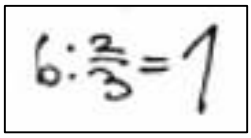


Ido immediately enters the Excel expression =C2/2/3. Pressing *Enter* produces the output "*1*," which surprises him; he asks, "*Six divided by two-thirds, how much is that*?" and then rejects the computer's feedback, saying, "*No, that doesn't make sense, I think*." Uri, however, accepts it, responding, "*Yes, it does*." These turns mark the beginning of the dynamics that will eventually lead the students to endorse the narrative (and associated procedure), that a bottle costs 1 NIS. At this stage, however, the system remains unstable. Ido continues to resist the narrative: when he drags the expression downward and Excel generates a column of 1, 2, 3, and so on, he immediately deletes it, saying, "*No, it's not*".

Having rejected the original formula he entered, Ido searches for an alternative procedure. He mentally calculates two-thirds of six, stating: "*A third of six is two, two-thirds is four, two-thirds of six is four*." We interpret this as calculation-based feedback, which at this moment could have served as a destabilizing loop countering the narrative that 2/3 of 6 is 1. However, as we later show, this destabilizing feedback was insufficient to perturb the system.

After calculating two-thirds of six mentally, Ido reframes the task: "*We need to get here* [in Excel] *to a situation where there's four… six divided by two-thirds. How do you write that?*" This indicates that he now endorses 4 as the value that should appear in the Excel cell and represent the price of one bottle. His sub-task becomes finding an Excel formula that will produce 4. Crucially, however, he still describes the underlying procedure as "six divided by two-thirds," showing that he continues to follow a meta-rule in which *of* implies division. We argue that this meta-rule, shared by Uri as well, functions as a control parameter, constraining the system's dynamics.

The implications of the divide-meta-rule are evident in Ido's repeated, unsuccessful attempts to construct an Excel formula that will yield "4," including =C2/2/3, =2/3/C2, and =C2/0.66. Each of these attempts generates destabilizing Excel-based feedback relative to Ido's evolving narrative, which comprises two sub-narratives: first, that the original formula (=6/2/3) is incorrect, and second, that the cell should display "4".



As Uri observes Ido's repeated failures to generate a satisfactory formula, he attempts a similar approach and manually computes 6/2/3. His calculation leads, canonically, to the result "1." We interpret this as a calculational feedback loop that stabilizes the narrative "2/3 of 6 is 1," along with the broader set of associated narratives and procedures (the Excel formula =6/2/3, the claim that one bottle costs 1 NIS, the correctness of the column 1, 2, 3, …, etc.). All of these align with the meta-rule that "2/3 of" should be translated as division.

The destabilization of Ido's narrative, combined with the stabilizing feedback loops provided by Excel and by Uri's handwritten calculations, leads to a rapid acceptance of the "divide" narrative ("*Oh really? So we were right?*"). As we show in the next section, this narrative exhibits many characteristics of an attractor.

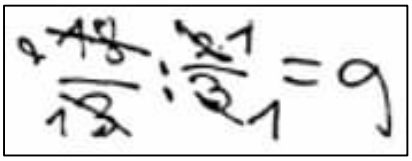
Phase shift- questioning the non-canonical meta-rule

Several indicators suggest that the "divide" narrative and its associated procedures have entered an attractor state. First, both students express satisfaction, Uri saying "*yeah, yeah, yeah*" and Ido responding "*Ah, right, yes*." A second indicator is the interviewer's repeated, unsuccessful attempts to perturb this narrative. Her initial, gentle prompt ("*Yeah, is that good? So what are you saying, actually, that what?*") elicits confident repetition from both students that the prices for one and two bottles are 1 and 2. Her next attempt, reciting the task ("*For each bottle you pay two-thirds of its price, so…*"), is interpreted narrowly: Uri treats it as a technical question about

where to write the answer (“*Let’s write here…*”), while Ido interprets it as a request to locate the value in Excel (“*This* [the Excel list], *this is the two-thirds of its price*”). These interpretations highlight the strength of the attractor and the stability of the governing meta-rule: neither student perceives the interviewer as even hinting that their procedure might be problematic.

Only with the interviewer’s third move (“*Really*?”), delivered in a tone difficult to interpret as agreement, does Ido begin to treat her stance as disagreement rather than a request for clarification. He responds, “*Umm, umm, no, we were wrong*” marking the onset of an authority-led destabilizing feedback loop targeting the narrative “2/3 of 6 is 1.” This destabilization is not immediately shared by Uri, who first asks “*why*?” and then attempts to re-establish the original narrative by repeating his written calculation. When this calculation again yields “1,” Ido briefly returns to the initial narrative (“*right, yeah*”). Thus, although the third prompt is taken up as a challenge, the students respond by reinforcing the same operation and procedure; the repeated calculation of 6/2/3 continues to function as a stabilizing loop for the narrative “2/3 of 6 is 1”.

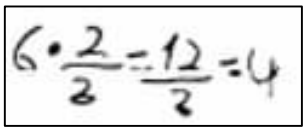
The interviewer’s fourth move, “*so two-thirds of six is one?*”, continues the challenge by restating the claim in a way that highlights its implausibility. This authority-led feedback appears to intensify Ido’s destabilization; he laughs and initiates a new calculation. However, this calculation still follows the “of is division” meta-rule and yields 9. The result generates a strong destabilizing feedback loop, producing confusion and frustration (“*I have no idea anymore*”) rather than prompting a reassessment of the underlying procedure.



Only after the preceding challenges have destabilized the students’ confidence without offering an alternative path does the interviewer’s next question function as a sufficient perturbation, triggering a phase shift. This time, her question targets the meta-rule itself, how they decide which procedure to use: “*If you need to do two-thirds of six, what exercise would you do?*” Unlike the earlier prompts, the students interpret this as requiring a different kind of response. Both Uri (“*Uhhhh…*”) and Ido (“*wait, wait…*”) hesitate, signaling a breakdown in their previously stable way of proceeding.

Phase 2- Endorsing a canonical meta-rule

Following this hesitation, the interaction shifts. The interviewer repeats her question, “*What do you think, what’s the exercise?*”, prompting Uri to tentatively propose a multiplication procedure (“*I think six times two-thirds… no…*”). Her follow-up “*And then what do you get?*”, functions as an authoritative feedback loop, signaling that this procedure is canonical and helping stabilize the fragile new multiply meta-rule. Uri responds by calculating on paper, and Ido joins him, explicitly invoking the meta-rule (“*I remember we learned that*”). Ido’s recall serves as a stabilizing feedback loop for the emerging narrative that *of*



should be computed through multiplication. When their calculation yields "4," he quickly affirms it ("*nice*"). Ido and Uri then enter the new procedure into Excel without hesitation, and the resulting output of "4" provides an additional stabilizing feedback loop.

The accumulation of authoritative, mathematical, and computerized feedback loops supporting attractor two—together with the reinforcement of the narrative "4," first articulated by Ido in phase one—creates a cluster of stabilizing influences that rapidly orient the system toward the canonical procedure and narrative. In contrast to the multiple trials, negations, and exploratory pathways characteristic of phase one, phase two is marked by swift convergence: 2/3 of six is now expressed as =C2*2/3, yielding 4 for one bottle and, when dragged, producing the column 4, 8, 12.

DISCUSSION

In this study, we examined how learning unfolds in an Excel environment, focusing on how both canonical and noncanonical mathematical discourses become stabilized and transformed. Excel's feedback loop acted as a stabilizing force regardless of the meta-rule in play—reinforcing "of as division" in phase one and "of as multiplication" in phase two. Even when Ido rejected the initial feedback, the technology offered no indication of error, a pattern also noted by Tabach and Friedlander (2008).

Our findings show that expert recognition of a noncanonical conclusion is not, by itself, sufficient to destabilize a stable discursive attractor. In our case, although the interviewer repeatedly questioned the students' conclusions—an action that typically signals to students that something is amiss—these early prompts did not function as effective perturbations because they did not explicitly target the meta-rule governing the interpretation of *of*. As long as this meta-rule remained implicit and intact, challenges to results or task interpretations were absorbed within the existing discursive configuration. Destabilization occurred only when the interviewer's prompt directly addressed the procedure-selection rule ("*What exercise would you do?*"), thereby making the meta-rule available for reconsideration.

From a complexity-discursive perspective, this helps explain the persistence of the noncanonical phase: the meta-rule operated as a control parameter, constraining the space of possible actions and interpretations. A shift toward canonical discourse required a reorganization of this control parameter, a process that commognition characterizes as meta-level learning. Prior research suggests that such meta-level learning is notoriously difficult to initiate, particularly in peer interactions (Ben-Dor & Heyd-Metzuyanim, 2025).

Adding a complex dynamic systems lens to commognition clarifies how discursive constraints are maintained and challenged over time. While commognition explains how meta-rules shape and limit mathematical interactions, complexity theory offers tools for modeling how these constraints stabilize activity and why challenges to noncanonical conclusions may be repeatedly absorbed without producing change.

From this perspective, what commognition terms “meta-level learning” can be understood as a phase shift, a non-linear reorganization of activity that emerges only under particular conditions of destabilization. This lens also allows us to conceptualize Excel not as a corrective authority but as an active participant in stabilizing dynamics, reinforcing existing discursive configurations regardless of their canonical status.

From a practical standpoint, our study suggests that Excel cannot function as an “expert” in its own right. Although it provides immediate and consistent feedback, this feedback stabilizes whichever discourse is currently active. Supporting movement toward canonical mathematical discourse, especially across meta-level divides, requires the presence of a human expert who can intervene in ways that render new meta-rules accessible without undermining students’ agency.

References

Ben-Dor, N., & Heyd-Metzuyanim, E. (2025). Unifying dialogic learning and mathematics learning: A discursive lens for the study of dialogic mathematics peer learning. *Journal of the Learning Sciences*, *34*(1), 1-34.

Byrne, D. (1998). *Complexity theory and the social sciences: An introduction*. Routledge.

Chan, M. C. E., & Sfard, A. (2020). On learning that could have happened: The same tale in two cities. *The Journal of Mathematical Behavior, 60,* Article 100815.

Heyd-Metzuyanim, E. (2025). Failure to teach/learn mathematics: A complexity-discursive perspective. *Educational Studies in Mathematics*, *119*(3), 515–533.

Higgins, K., Huscroft-D’Angelo, J., & Crawford, L. (2019). Effects of technology in mathematics on achievement, motivation, and attitude: A meta-analysis. *Journal of Educational Computing Research*, *57*(2), 283-319.

Hilpert, J. C., & Marchand, G. C. (2018). Complex systems research in educational psychology: Aligning theory and method. *Educational Psychologist*, *53*(3), 185–202.

Lavie, I., Steiner, A., & Sfard, A. (2019). Routines we live by: From ritual to exploration. *Educational Studies in mathematics*, *101*(2), 153-176.

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing*. Cambridge university press.

Shvarts, A., Bos, R., Doorman, M., & Drijvers, P. (2024). Reifying actions into artifacts: process–object duality from an embodied perspective on mathematics learning. *Educational Studies in Mathematics*, *117*(2), 193-214.

Tabach, M., Friedlander, A. (2008). Understanding equivalence of symbolic expressions in a spreadsheet-based environment. *International Journal of Computers in Mathematics Learning*, *13*, 27–46.