WHAT IS 3+50%? VARIOUS STUDENTS' MEANING ABOUT %

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This Research Report presents preliminary findings on a study about students' ways of making-sense of %. Grade-8 students' strategies for solving one arithmetic task is looked into: the 3+50% task. The initial analyses of students' strategies for '3+50%' represent a first step in better understanding varieties of conceptions of % that students engage with, as well as the (implicit or explicit) roles they give to the %-referent.

INTRODUCTION

Although % is recognized as one of the most useful mathematical concepts in everyday life, it is also frequently "oversimplified in instruction […where] the richness of the topic has often been left unexplored, while heavy instructional emphasis has been placed on percent procedures" (Parker & Leinhard, 1995, p. 429). As such, studies have been reporting, and for a long-time, on challenges students of all ages face with %, especially concerning their overreliance on formulas and techniques at the cost of understanding (see Erdem et al., 2023). These challenges do not only concern students, and other studies have focused on e.g. consumers' misunderstanding of how % combine (Chen & Rao, 2007); on workers' incorrect use of operations with % (Hahn, 1999); on specialists' misuses of % in various reports (Packard & Boardman, 1988); and the list could go on. Although made more than 30 years ago, Lembke and Reys (1994) assertion that "percent is one of the most difficult topics for students and adults to conceptualize and to develop facility in using" (p. 237) seems to continue to be apt and current. As such, beyond these above calculational challenges, what meanings do students give to %? In what ways do they engage with % tasks? These questions are core to our research objectives. This RR presents preliminary findings about students' meanings given to % when solving a task concerned with arithmetical operations.

THEORETICAL STEPS TOWARD CONCEPTUALIZING %

As presented at CERME (Proulx, 2025), the literature dedicated to % defines it along numerous conceptions: ratio, fraction, decimal number, operator, index, etc. Taken as a whole, this puts forth a large variety about what % are and can be; something to subsequently reinvest for analyzing students' sense-making and engagement with %.

% as ratio: relations. The ratio conception is probably the most natural one, with its historical origins in taxation practices (Parker & Leinhardt, 1995). Literally, an assertion like 22% is to be read "22-per-cent", pointing to a comparison between two quantities (here, 22 and 100 or cent). Presented as a "specific sort of rate" (Champlain et al., 1996) or a "special ratio involving parts per hundred" (Glatzer, 1984), % is read as "for every 100": 22% is 22-for-each-100. Heuvel-Panhuizen (1994) phrase % as a

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M.S. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: Research Reports*, pp. 1187-1194. PME.

ratio of so-many-out-of-so-many, hence so-many-out-of-hundred, leading to transform a situation in relation to 100. For 215 kids in a group of 860 persons, this requires asking "what if it were for 100 persons?", leading to 25% of the group being kids. Establishing % implies finding equivalent ratios through proportions, where e.g. 215 is to 860 as 25 is to 100, represented symbolically by $\frac{215}{860} = \frac{25}{100}$. That it be a ratio with 100, a special relation to 100, or other comparisons with 100, the ratio conception emphasizes the relation between two quantities (one being 100).

% as fraction: partitioning. If the ratio conception is more 'natural', the fraction conception may be the most encountered one, found in vocabularies and symbolisms linked to fractions for expressing % (e.g. fraction on, fraction of, numerator, denominator, a/100). Here, transformations to 100 at the denominator are grounded in equivalent fractions. Another way is to see % as fractions part-of-a-whole, where a whole is partitioned in 100 pieces and the numerator represents the number of pieces taken: 28% means that 28 pieces were taken from a whole cut into 100 pieces, hence 28/100. This conception points to % as a partitioning, a part of something.

% as decimal number: numerical representations. The decimal number conception leads to convert % into a manipulable form for operating. Converting 35% in decimal form gives 35÷100=0.35, leading to the equality 35%=35/100=0.35. Transformed as such, the % can be treated as a number, e.g. by placing it on the number line: 35% as 0.35 is between 0.3 and 0.4. This conception focuses on the numerical representation of %, its number dimension, and the possibilities it offers for operating with it.

% as operator: calculating. This conception focuses on seeing the % sign as a request for undertaking calculations. The % symbol e.g. in 27% leads to multiply 27 by 1/100 or 0.01, or divide it by 100: it becomes 27×1/100, 27×0.01 or 27÷100. This operator conception is mostly focused on calculations, on how to do them and their results.

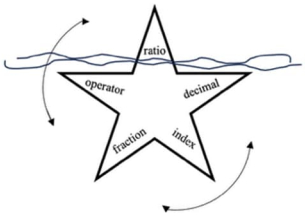
% as index: indicating. Another number conception is one where % is conceived as an index, linked to a referent (Berry et al., 1999). The % represents here an indication of the part to be taken from that referent. As index, the % does not have a predetermined value without that referent, and it is not possible to operate without it: e.g. 4% is not a number *per se* on the number line, nor is it necessarily equivalent to 4/100 or 0.04, because 4% can take many values, from 12 (4% of 300), 3217 (4% of 80425), etc. The focus is here on % as a relative object, through its connection with its referent-number.

Complementarity and flexibility between various conceptions of %

Although presented separately, these conceptions of % are however strongly entwined. One finds tons of equalities of the sort "20%=20/100=1/5=0.2", with mentions of the necessary index-link of these % to their referent. Beyond numerical conversions, this points toward the complementarity existing between conceptions, which allows to address % in flexible ways. Each conception is not isolated and is opened to a transition toward other conceptions: a connectedness enabling the passage from number, operator, fraction, index, etc., to any other one, depending on what is at stake. Inspired

by Janvier's (1983) star metaphor to conceptualize complementary modes of representation of functions, conceptions of % can also be envisaged as each representing points of an immersed star (Figure 1). When one conception of % is put forward, the other conceptions are there, connected to it, waiting and wanting to be made salient if need be. This %-star aims to illustrate an array of possibilities, through the complementarity between conceptions when working and interpreting %.

Figure 1: The %-star with its various complementary and interconnected conceptions



The absolute|relative interplay with %

Schoolwork focusing on conversions between %, fractions and decimal numbers has received ample critiques, particularly since it frequently ends up becoming a mechanical endeavor rid of meaning (see e.g. at PME: Streefland & Heuvel-Panhuizen, 1992). Throughout these critiques, an argument is often raised that % is not a topic reducible to technical matters, and that % do not exists in the absolute, that is, without the consideration of referents. This situation is complex and filled with nuances, because % may be "empty" without its referent, to reuse Gauvrit (2014) expression, but it is nonetheless hardly interpretable without undergoing some conversions toward fractions, ratios, decimal numbers and operations to establish its value. For example, finding 31% of 728 can imply, first, to numerically convert 31% in something malleable for operating (e.g. multiplying by 0.31 or 31/100). But, in this conversion, 31% is not equivalent to 0.31 or 31/100 in the absolute, but is 0.31 *of something* or a relation of 31 parts for each 100. Once converted, this 0.31 or 31/100 can be multiplied by 728 to get the value looked for. Working with % can engage in an interaction between numerical conversions of % and the relativity of the % to its referent; a back-and-forth interplay between both absolute and relative dimensions when managing %.

METHODOLOGICAL DIMENSIONS

To investigate ways in which students give meaning to %, eight 75-minutes sessions were held in four different Grade-8 classrooms (26-27 students, 13-14 years old). This study is part of a wider research programme in which the methodology used is referred to as "mental mathematics" (see PME report in Proulx, 2019). The research sessions were conducted by the PI, following a structure similar to that described by Douady (1994): (1) The PI offers a task orally or on the board; (2) The students have about 15 seconds to solve the task mentally, without access to paper-and-pencil or any other material aid; (3) The students are invited to describe their answer in detail; (4) The PI take notes on the board or students come to the front board to explain their solution; (5) The PI invites other students who may have solved differently (and step (3) repeats).

The tasks given in the sessions were chosen to cover a large array of issues, inspired by Barrata et al.'s (2010) classification, implying arithmetic operations, comparisons and estimations about %. Additionally, task selection was guided by the fact that mental mathematics environments impose specific constraints of time on students, and thus tasks have to be directly accessible, with simple numbers and context (which in turn could lead students to avoid plunging head-first in long-calculations). This RR offers preliminary results by reporting on the arithmetic task "What is 3+50%?". This task, frequently found in textbooks, was chosen for two main reasons. First, both 3 and 50% are accessible, which could potentially avoid students' errors of conversions and calculations. Second, it is deliberately ambiguous, where the referent of the % is not explicit. Both reasons could augment accessibility to the meanings students give to %.

Strategies engaged in all classrooms were similar, if not identical, so no distinction is made in the analysis. Students' strategies (as verbal explanations and front board notes) generated the data for the study. Data is analyzed through the components of the frame presented above, that is, the %-star and the absolute|relative interplay. Following Douady (1994), the goal is not to report on all students' strategies, nor to discuss the long-term outcomes of this work on them. Aligned with the research objectives, the analyses focus on the meanings students gave to % through engaging in tasks.

PRELIMINARY ANALYSES OF STRATEGIES – WHAT IS 3+50%?

A large variety of strategies were offered by students, which in turn led to an additional variety of answers. Through these, students engaged in what can be called "Readings" of the task. These offer initial illustrations of meanings students gave to %.

Reading #1 – 3 + 50% calls for conversions: In this Reading, students transform elements of the task, be it the 50%, the 3, or both, in order to give their answers.

> One student mentioned that 50% is 50/100, which "if you take the zeros out" is equal to 5/10, so is 5 tenths or 0.5. The answer is thus 3 + 0.5 = 3.5.
>
> One student expressed that 3 is 3 units, which is 300% plus 50%, that gives 350%.
>
> One student said that 50% is half of 100, it is one half. And 3 is 3 units, giving 3 and a half.

The % are converted to numbers: 50% is made equal to 0.5, 3 is converted to 300%. Different types of answers were given with these conversions, from 3.5, 350%, to 3 ½. Direct equivalences are made between %, fractions and decimals, where students fluidly navigate from one to the next to find an adequate representation, and then produce an answer. As well, no referent is explicitly mentioned, making the % appears to be worked on in an absolute sense: 50% *is* 0.5; 50%=50/100; 3 *is* 300%, etc.

Reading #2 – 3+50% calls for linking the % to its referent-number: In this Reading, students determine a value of 50%, by linking it to 3 as its referent. The 50% does not have a precise value itself, but needs to be connected to its referent.

> One student asserted that 3 plus "3 divided by 2", which is 50% *of 3* amounts to 4.5.
>
> One student explained that 50% is 50% *of* 3, so 3 plus half of 3 gave 3 + 1.5 = 4.5.

> One student said that 3 is 3 units, hence 300%. Adding 300% to 50% *of 300%* gives 450%.

In these, the % has no value *per se*, and is relative to its referent. In other words, 50% is relative to the number 3 taken from the question. There is also a fluidity between 50%, half, and 1.5, where these are linked through the relativity and attachment of 50% to 3 (or 300%). The % is here conceived as an index-number.

Reading #3 – 3+50% calls for linking the % to a unit: Here, the % is again connected to a referent, which is 3. However, it is not the number 3, but mainly 3 taken as the complete or whole-unit in itself. The 50% is thus linked to one unit of 3.

> One student said that 3 is 3/3, and so it is 100%. Adding 100% and 50% gives 150%.

> One student explained that "3 is one" and that "3 is equal to a unit". Converting 50% in fraction gives ½, and so the answer is 1 and ½.

The answers 150% and 1 ½ insist on the relative value of %. These are not numerical values like 1.5, 4.5 or 3.5, but indicators of what '3+50%' represents. As the unit, the 3 is taken as a thing, an entity, and not a numerical value. While in Reading #2 it is used to calculate the value of 50%, here the 3 acts as the unit of reference, representing 100%, but not for making definitive calculations. One could surmise that if the task had been '8 + 50%', 150% and 1 ½ would still hold as answers. The conception of % is of an index-number; and, to some extent, the answers 150% and 1½ are too.

Reading #4: 3+50% calls for clarification: In this Reading, students react to the task itself, asserting that information is missing, and that a precise answer cannot be given.

> One student uttered "you do not say what you are talking about, it could be 50% of 8, 50% of I-don't-know", adding that an answer cannot be given since "the sentence is not finished, because you can say 3+50%, but 50% of what? 50% of bananas, 50% of the number".

> One student explained that the answer would be obtained by adding 3 and "a number divided by 2", which was expressed as (?/2). Hence, the answer is unknown.

Here, the % is asserted relative, calling for knowing explicitly its referent (for then being able to produce an answer). Lack of information prevents from doing so. The 3 is not conceived as a plausible option for that referent since, as one student said: "You cannot use 50% of 3, because it does not say so". There is a feeling of dismay in reaction to the task, and even some frustration: "I don't know what the number is"; "you do not say what we are talking about". The % is conceived as an index, related to a referent 'to be known', hence the process does not yet lead to a definitive answer.

Reading #5: 3+50% calls for calculations: In this Reading, students explain operating, converting, and establishing equivalences to get an answer.

> One student explained that 50% can be obtained by 50 ÷ 100, because it is 50 over 100, so it is 50/100, which is also 0.5. Thus, 3 + 0.5 = 3.5. And, "to put it back in %, I did times 100", leading to 3.5 × 100 = 350, and so to 350/100, which is also 350%.

> One student explained doing equivalences and "took the % away, and did 3/1 + 50/100". Proportions were established to find "3/1 is how much for 100?" by asking "1 times

> something equals 100, so it is 300", and "doing the same thing" with "50/100 is how much for 100?" gave 50. One "here has 300 and here 50, which equals to 350" with $\frac{300+50}{100}$.

When questioned, students appeared in doubt as to their definitive answer (is it 350/100? 350? 350%? 3.5?). It might be 'all of it', where the calculations, techniques, values, and equivalences *as a whole* appear to act as their answer to '3+50%'. Their operations do end at one point, but could have ended before: actions are undertaken to "take the % away", leading to equivalences and additional operations, which are retransformed, giving other answers, etc., in a continual sequence. Focusing on the operator conception, these answers showcase fluidity and ease of calculations with %.

Reading #6: 3+50% calls for thinking of exams and grades: In this Reading, students position the task in a context of examination related to grades obtained at school.

> One student expressed that the answer is 80%. To get it, "I told myself that 50% is a little like '5 over 10', so 5/10, and then 3/10", which added gives 8/10, "so 80%".
>
> One student said 53%, because "it's logical, for example if you had 50% in an exam and you tell your teacher that 3 points are missing, he will add 3 points and do 3 + 50".

The answers 80% and 53%, while unusual, are at the same time well-justified in their exam context (one might say that this mostly represents students' experience with %). The 50% denotes a grade, where the 3 is brought into the 50%: it becomes 3 points or 3% leading to 53%, or a '3 out of 10' to match the 5/10. The % are here operated on as numbers (while assuming a same referent, something addressed below).

DISCUSSION & FINAL REMARKS ON STUDENTS' MEANING ABOUT %

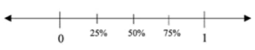
On issues of complementarity. Although preliminary and only about one single task, student appear to make uses of different conceptions of %, and in a variety of ways and ease. As for complementarity between different conceptions of %, it can be said to have been maintained implicit in students' ways of solving. There obviously were moments where students explicitly mentioned a sequence of equivalences between %, decimal numbers and fractions (e.g. with 50% being a half made equal to 0.5, 5/10 and 50/100, or with 3 being 3/1, 300% or 3 for 1 as ratio). This said, most strategies focused on one unique conception, without explicitly navigating to the next. This might open ways for the future design of tasks that can trigger complementarities between conceptions, for investigating students' engagement with these.

About the role of the %-referent. These preliminary analyses also point to reflections on students' ways of addressing the role of the %-referent. Each Reading does not address issues of %-referent in the same manner, some making it explicit whereas other keeping it implicit (or even absent). As such, entwined with their %-conceptions, students' ways of handling (or not) the referent appeared instrumental in the answer they offered and engaged with. In Reading #1, when students convert % directly in numbers (with 50%=0.5 or 3=300%), the % is conceived in the absolute, as a direct numerical equivalence where no explicit consideration of the referent is made. Being

usual in schoolwork, these conversions *might* assume an implicit referent of 1 or '1 unit', leading e.g. to 50% *of 1* and then 0.5 *of 1*. The same for Reading #5 about calculations, where the operations conducted did not make apparent links with a %-referent. Reading #6, with 53% and 80% as answers, also seems to keep issues of referent implicit, treating the 3 and 50 as numbers that can be added directly (what Parker & Leinhard, 1995, call "ignore % sign"). When questioned, however, some of these students mentioned that the % sign *is* present, but simply lacking from the 3 and that it was the PI "laziness" that led to write it as such: making thus their answer a 3%+50%=53%. As Gauvrit (2014) explains, this equality is true in rare cases where all % have the same referent. Here, again, the implicit nature of the referent comes to the fore, this time through an assumption that all % share the same referent and so can be operated on directly. On the opposite, Readings #2, #3 and #4 make explicit the presence of the %-referent. In Reading #2, this referent is taken as the value 3, in Reading #3 it is taken as a unit of 3, and in Reading #4 it is 'needed, but unknown'. The impact of these various engagements with the %-referent is salient in these Readings, raising in turn considerations about the absolute|relative interplay.

Beyond the absolute|relative interplay. The issue of implicit or explicit referent brings forth a triplet of perspectives on %, going beyond the above absolute|relative interplay. At one extreme, Readings #1, #5 and #6 consider the % in the absolute, as a number or something to operate on as a number. At another extreme, in Readings #3 and #4, the % is presented as fully-relative. For Reading #4, the % is 'maintained' relative, asserting that until its referent is made explicit, it cannot be known. Similarly, for Reading #3, when students say that 3 "is its 100%", it is not about 3 as a value, but about 'one' unit of 3; hence the answers 1 ½ and 150% are relative (i.e. index) with no definitive values. And, in between, lies Reading #2, navigating both extremes of absolute and fully-relative views of %, where the 50% is read as 50% *of* 3. Reading #2 appears midway, since the %-referent is relative but *this relativity is acted on*, with a referent pinned-down to '3'. This midway perspective offers a temporary state, one viable for the task at hand. When 3+50% is read as '3+50% of 3', the value 1.5 is not universal for all cases of 50%; it is for now, for this precise task. It also makes possible different operations, since % become linked to the same referent (e.g. 25%<50% and 50%+25%=75%, for the same %-referent). These perspectives (Table 1) go beyond the absolute|relative interplay, and offer a rich panorama of students' ways of handling %.

Table 1: Three perspectives for working with %

Absolute perspective on %	**Midway perspective**	**Relative perspective on %**
Implicit referent (or not mentioned)	Referent fixed temporarily	Variable referent
% as number, its value is obtained by converting it (for operating)	Value of % linked to a fixed unit	% as index-number, its value is relative to a referent (to be known)
50% = 50/100 = 0.5 50% > 25% 50% + 25% = 75% 	50% of is 50% of is 50% of is 50% > 25% and 50% + 25% = 75%	The added dotted square in is both an addition of 50% or 33.$\overline{3}$% of the new whole. John spent 50% of his money. Mary spent 25%. Who spent more?
Reading #1, #5, #6	**Reading #2**	**Reading #3, #4**

All in all, although preliminary and only about one task, students' engagements with 3+50% brought forth their various uses of %-conceptions, leading to considerations of their complementarity. And, the Readings of the 3+50% task gave rise to an additional variety of ways in which students gave meaning to the role of the %-referent, enlarging considerations of the absolute and relative takes on % and its output value. These varieties raise enthusiasm for continuing further research, for deepening understandings about ways in which students give meaning to % and engage with them.

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