MATHEMATIZING IN A PLAY-ORIENTED PRESCHOOL CONTEXT

Hanna Palmér[1], Jessica Elofsson[2], Maria Walla[3], Camilla Björklund[4]

[1]Linnaeus University, [2]Linköping University, [3]Dalarna University
[4]University of Gothenburg

This study examines how preschool teachers design mathematics teaching that enables young children to mathematize, drawing on Freudenthal's notion of mathematics as genuinely needed within an activity. Within a design-based research project in Swedish preschools (children aged 1–3), 68 video-documented activities were analysed to identify instances of mathematizing. Three approaches were identified: drawing on scripted activities, drawing on known counting chants, and drawing on an experience familiar to the children. Across these, mathematizing occurred when teachers framed and sustained mathematical necessity within play-oriented situations. The study provides empirical insight into how mathematizing can be realized in early mathematics education.

INTRODUCTION

There is a broad consensus that early mathematics education is of great importance. Studies consistently link young children's mathematical abilities to later school achievement, not only in mathematics but also in other school subjects (e.g., Aunio & Niemivirta, 2010; Siegler & Braithwaite, 2017). As a contribution to the field of early mathematics education, this study focuses on *mathematizing*, a notion introduced by the Dutch mathematician Hans Freudenthal (Freudenthal, 1978). In brief, mathematizing refers to the process of applying mathematical thinking and skills to solve problems in situations where mathematics is genuinely needed to accomplish a task. Central to Freudenthal's work is the idea that mathematics should be experienced as meaningful and purposeful for the learner, rather than as an application of pre-given procedures distant from lived experience. The context of the study is an educational design research project (Bakker, 2018) focusing on mathematics education in Swedish preschools (financially supported by the Swedish Institute for Educational Research, grant no. 2022-00010).

In Sweden, preschool is available to all children aged 1–5 years and is guided by a national curriculum that clearly states that, based on a play-oriented approach, preschool should include teaching and opportunities for children to explore different knowledge areas, including mathematics (National Agency for Education, 2025). Within this play-oriented approach, a tension has been identified between the openness of play and the goal-oriented nature of teaching (Björklund & Palmér, 2024). While mathematics frequently appears in preschool activities, it does not always become

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meaningful or necessary for the children, and it risks being reduced to tasks without a clear purpose. In contrast, *mathematizing* emphasizing mathematics being genuinely needed to solve a task.

In this study, we address this tension by exploring how mathematizing may be realized in early mathematics education. Rather than evaluating or judging how mathematics *should* be taught in preschool, the aim is to examine how mathematizing can be realized. In the study, we specifically ask: How do preschool teachers design their teaching when the intention is to enable children to mathematize? As Freudenthal's work was not conducted in a preschool environment, this study contributes empirically grounded knowledge about how mathematizing can be planned for in early mathematics education.

MATHEMATIZING

Research in mathematics education indicates that young children learn mathematics most effectively when it is embedded in meaningful problems where mathematical ideas serve a clear purpose (Clements et al., 2017). This aligns closely with Freudenthal's view of mathematics as a human activity, in which children should actively organize and structure situations rather than merely reproduce predefined procedures (Gravemeijer & Terwel, 2000). According to Freudenthal (1978, 1981), the starting point for his work on mathematizing was the prevailing practice in school mathematics at the time, which often began in the sophisticated knowledge and strategies of experts. Freudenthal argued that this approach frequently led to learning objectives that made sense from the experts' perspective but not necessarily from that of students. He also argued that mathematics is always both form and content and thus should never be taught in isolation; instead, teaching should focus on the interplay between them. For example, while children may learn to carry out mathematical procedures (e.g., count objects using one-to-one correspondence) and memorize facts (e.g., a triangle is a polygon with three sides), these skills and facts are of limited value if students do not understand their purpose or underlying meaning.

Freudenthal emphasized that mathematics should be taught so that the knowledge becomes useful and meaningful for the learner, which is why all mathematics teaching should be based on the learner's world and experiences (Freudenthal, 1978, 1981; Gravemeijer, 2004). Instead of decomposing ready-made expert knowledge, students should elaborate, refine, and adjust their current ways of knowing (Gravemeijer, 2004). Freudenthal distinguished between horizontal and vertical mathematization, both necessary for developing mathematical competence. Horizontal mathematization refers to the use of mathematics as it appears in the "real world" based on the learner's life and experiences, while vertical mathematization is the process where symbols are shaped, reshaped, and manipulated to make a problem solvable by reducing the "noise" of the "real world" However, in both cases, the subject matter that is to be mathematized should be experientially real for the students. In this study, the focus is primarily on horizontal mathematization.

Translating Freudenthal's ideas into a preschool context poses challenges. In a school context, he clarified that "the real world" refers to contexts where mathematics is genuinely needed to solve problems. Further, "real world" should be understood based on the English verb *to imagine*, which in Dutch is translated into *zich realiseren*, a term that emphasizes the act of making something real in one's mind. Accordingly, the contexts used in mathematical problems do not have to stem from the real world in a literal sense (Van den Heuvel-Panhuizen, 2020). Thus, what constitutes the "real world" may vary across individuals; for instance, mathematical objects are real for mathematicians differently than they are for young children (Freudenthal, 1981). Recent studies on mathematizing in preschool contexts have shown that the "real world" for children can include imagined scenarios, and that both meaningful tasks and the nature of teacher questioning are crucial for supporting mathematization (Palmér & Björklund, 2023; Björklund & Palmér, 2024). In a preschool study, Papadakis et al. (2017) showed that mathematizing in line with meaningful and real-world contexts enhanced mathematics learning. The start was in horizontal mathematizing, where children were to interpret everyday situations, use informal strategies, and develop their own representations, with teachers guiding them toward vertical mathematizing and more structured mathematical understanding. Another study focusing on horizontal mathematizing, but with slightly older children, showed how students' learning of basic addition was strengthened when they engaged in meaningful situations where teacher guidance supported a shift toward vertical mathematizing and more formal mathematical understanding (Leung & Hung, 2018). In preschool, Freudenthal's approach has also been enacted through guided picture-book reading, which research has shown to support emerging mathematical understanding by embedding, for example, early number, pattern, and spatial awareness within meaningful narrative contexts (Van den Heuvel-Panhuizen, 2015).

CONTEXT OF THE STUDY

The empirical material analysed in this paper originates from a study on preschool mathematics. First, mathematics education giving the youngest preschoolers (1- to 3-year-olds) the best conditions in which to learn mathematics was developed and tried out in authentic Swedish preschool settings (see for example Björklund & Palmér, 2022; Palmér & Björklund, 2023, 2024). Based on these positive results, guiding material was developed in collaboration between researchers and preschool teachers. This guiding material was structured into five parts, each corresponding to one design principle whereof the first was mathematizing. In line with Freudenthal, this design principle referred to mathematics being genuinely needed to accomplish a task and, therefore, being meaningful. The five parts of the guiding material were structured in the same way, one text to read (approximately three pages), one video illustrating the content of the text in authentic preschool teaching settings, questions to discuss with colleagues, scripted activities to be carried out with children and video-documented, and, finally, one activity to be planned, carried out, and video documented by the preschool teachers.

METHOD

For this study, an analysis was conducted on video documentations of activities planned and carried out by preschool teachers when implementing the above-described guiding material. All Swedish ethical guidelines for research regarding information, consent, and confidentiality were followed throughout the study (approved ethical review 2023-01344-01).

In total, 68 video-documented self-planned and carried out activities from 15 preschools were analysed. The analysis was conducted in several steps. First, all video recordings were viewed in full to identify activities in which mathematizing was realized. In this analysis, mathematizing was considered to be realized if mathematics became genuinely needed in the activity. Second, activities in which mathematizing was identified were analysed in order to identify similarities and recurring patterns in how preschool teachers conducted the teaching. Through iterative comparison and discussion among the researchers, three descriptive categories describing different ways in which preschool teachers enabled opportunities for children to mathematize were developed. These categories will be presented in the results section and illustrated with empirical examples. No quantification is provided, as the purpose of the analysis was to explore and characterize qualitatively different ways of enabling mathematizing rather than to focus on their frequency. This approach aligns with established qualitative analysis practices that emphasize the richness and meaning of observed patterns rather than statistical generalization (e.g., Patton, 2015). As the study is embedded in an educational design research project (Bakker, 2018), the focus is on teachers' ways of enabling mathematizing rather than on evaluating individual children's learning outcomes. Finally, it is important to note that the analysis in this study focused exclusively on activities in which mathematizing was identified; the study does not examine when or why mathematizing occurs in preschool in general, but rather how it can be realized in a play-oriented context.

RESULTS

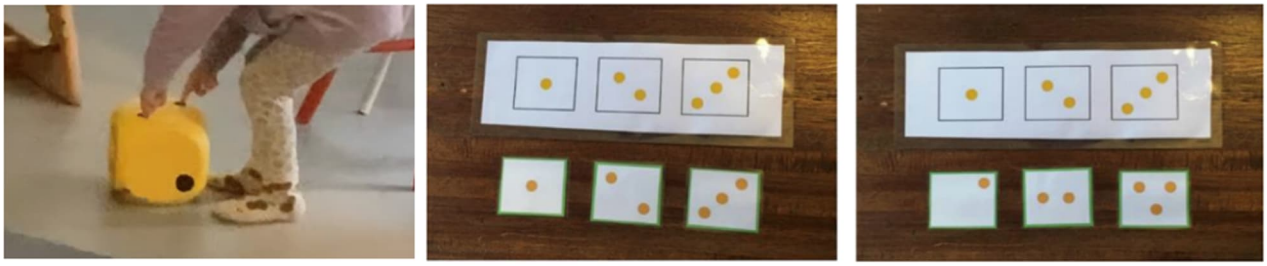
In this section, the three categories of how preschool teachers enabled opportunities for children to mathematize are presented with illustrative examples. The three categories are 1) drawing on a scripted activity in the guiding material, 2) drawing on a known counting chant and, 3) drawing on an experience familiar to the children.

Drawing on a scripted activity from the guiding material

The first category shows how preschool teachers enable mathematizing by drawing on a scripted activity from the guiding material and making minor adaptations. All scripted activities are planned in line with mathematizing, but it is not self-evident that this is kept when the teachers make adaptations. One example of an adaptation in which mathematizing is kept is the game "hop-hop", which is presented in the script and video in the guiding material. The game is based on a song about jumping. In the guiding material, the activity is framed so that, before each verse, the preschool teacher and the

children agree on “how many to jump on”. If the children choose to “jump on three”, the teacher explores with them how this can be done (e.g., jump on two feet and one hand or on two hands and one foot). When exploring with the children, the teacher is to use number words and finger representations, helping the children to identify different ways of jumping on three. The activity becomes an example of mathematizing as the children need to explore the numbers to perform the jumping correctly in the next verse. One example where mathematizing is kept when drawing on this scripted activity is a teacher adding a large die (1–3 dots) to the game (Figure 1a). This die is used to decide how many to jump on. Before each verse, one child is asked to roll the die (approximately 16 × 16 cm) and then the teacher and children first explore the number of dots, then how to jump, and then in the next verse they jump on the decided number. Thus, mathematizing is kept in the adaptation as the children need to explore the numbers to perform the jumping correctly in the next verse.

Figure 1a: The die added to the game. Figure 1b: The lottery game.



Another example in this category is the lottery game. In the scripted activity, the lottery game consists of a game plan and playing cards (Figure 1b). Initially, the dots on the cards are positioned identically, which means that the children do not need to distinguish cardinality to match the playing cards. This is to introduce the children to the rule that one playing card should be matched to one square on the game plan. When the children understand the rules, the playing cards are changed to cards whose dots are no longer in the same pattern as on the game plan. Now the children need to distinguish the number of dots to match the playing cards, thus introducing cardinality as a genuinely needed aspect (mathematizing) of the game. One example where mathematization is kept when drawing on this scripted activity is when teachers replace the yellow dots with objects chosen by the children, for example, buns instead of yellow dots on the game plan and playing cards. Apart from this adaptation, the activity is conducted in the same way (thus still mathematizing).

Drawing on a known counting chant

The second category of how preschool teachers enable mathematizing is their drawing on a known counting chant. An example is the counting chant “Five Little Monkeys Jumping on the Bed”. In this chant, five monkeys jump on the bed and one at a time fall off the bed until none remain. While reciting the rhyme “five/four/three … little monkeys jumping on the bed”, the teacher and children jump with their fingers in their

hands (Figure 2). Thus, knowing the number of fingers to jump with, corresponding to the number of monkeys they sing about, is essential to the activity (mathematizing). To support the children, the teacher uses both verbal counting words and one-to-one finger counting as well as matching the children's fingers to her own to ensure the correct quantity. Variations within this category concern whether teachers use materials for the chants, for example, figures of monkeys for the "Five Little Monkeys Jumping on the Bed", or if they rely solely on words and gestures.

Figure 2: The teacher jumps with the fingers on her hand.

Drawing on an experience familiar to the children

The third category shows preschool teachers enabling mathematizing by drawing on an experience familiar to the children. One example is when a teacher bases her teaching on a children's television programme, *Babblarna*. *Babblarna* is a well-known Swedish children's television programme that focuses on early language development through playful interaction with colourful characters, songs, and simple narratives. The teacher is aware of the children's strong interest in this programme, and that they know characters' names and the songs. The teacher then plans and conducts a situation in which the characters from *Babblarna* are having a snack, requiring counting to ensure that there are enough plates and other items for them. She builds the story so that the number of items is genuinely needed to be able to help *Babblarna* sort out their snack problem (mathematizing). To support the children, the teacher uses verbal counting words and one-to-one finger counting as well as finger representations to represent numbers.

DISCUSSION

In this study, the focus is on how preschool teachers design teaching when the intention is to enable children to mathematize. By empirically illustrating how mathematics becomes genuinely needed within play-oriented situations, in line with Freudenthal's notion of mathematizing, this study provides insight into how mathematizing can be realized in early mathematics education. The results illustrate how mathematizing within a play-oriented context among 1- to 3-year-old children can be realized through three approaches: drawing on scripted activities, drawing on known counting chants, and drawing on an experience familiar to the children. Although the results are descriptive rather than quantified, examples from the category drawing on scripted activities constituted most of the data. Thus, drawing on scripted activities from the guiding material seems to support teachers in sustaining mathematical necessity while making minor adaptations. However, there is a risk that inappropriate adaptations could

make the mathematical content less visible, highlighting the need for clear guiding material regarding the intended design and implementation of activities (Palmér & Björklund, 2023).

The study also contributes to understanding what may constitute the "real world" in early mathematics education. In the present study, horizontal mathematizing was realized in settings of games, counting chants, and shared cultural references familiar to the children. While these settings may not correspond to traditional interpretations of real-world contexts, they were meaningful and real for the children involved, thus in line with "real world" within the notion of mathematizing (Van den Heuvel-Panhuizen, 2020). These findings support previous research suggesting that imagined, playful, and narrative-based situations can provide legitimate contexts for horizontal mathematizing in preschool (Palmér & Björklund, 2023; Van den Heuvel-Panhuizen, 2015). Adaptations of familiar activities do not automatically result in mathematizing, and mathematical aspects may be weakened or lost if necessity is not carefully maintained. Across the three approaches, mathematizing is realized as children need to use numbers for the activity to progress. In line with Björklund and Palmér (2024), Leung and Hung (2018), and Papadakis et al. (2017), mathematizing depends on how teachers sustain mathematical necessity through their interactions within this framing.

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