COMPARING DIGITAL AND TEXTBOOK INTERVENTIONS FOR EQUATION SOLVING: THE EFFECTIVENESS OF COMBINING MODELING AND TRANSFORMATIONAL APPROACHES IN GRADE 6

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This study compares four classroom-based interventions designed to support Grade 6 students' understanding of equation solving. The first intervention employed a digital environment that emphasized working with non-letter-symbolic and letter-symbolic equations via a balance-scale model. The second intervention employed a digital environment focused exclusively on transformational activities with numerical equalities and letter-symbolic-equations. The third intervention combined these two approaches, while the fourth relied solely on the mathematics textbook. Using a pre-test–post-test design, learning outcomes were analyzed. Results indicated that the combined intervention was the most effective, indicating that coordinating modeling and transformational activities can support algebraic thinking.

INTRODUCTION

Research on early algebra has expanded substantially since the 1990s, moving beyond questions of feasibility toward a richer understanding of early algebraic thinking (Ellis & Özgür, 2024). Despite these advances, there remains a need for studies that explicitly describe instructional interventions that support different dimensions of early algebraic thinking, particularly in authentic classroom settings (Kierna, 2022). Within this broader context, solving equations is one of the most conceptually demanding aspects of students' algebraic development (Kieran, 2022). It involves producing a chain of equivalent equations, in which each equation is logically deduced from the previous one by applying transformations to both known and unknown quantities (Radford, 2014). These transformations rely on fundamental properties of equality, such as the preservation of equivalence under addition, subtraction, multiplication, and division, and require students to treat unknowns analytically, as if they were known quantities. In this sense, equation solving exemplifies the *analytic dimension* of algebraic thinking, in which reasoning is grounded in deductive transformations (Radford, 2014).

Prior research has shown that many students initially struggle to reason about transformations that preserve equality when they first encounter letter-symbolic equations. Students often rely on inverse operations or trial-and-error strategies rather than analytic thinking (Xie & Cai, 2022; Pitta-Pantazi et al., 2024). While these challenges are well documented, less is known about how specific classroom-based instructional interventions can effectively support students in overcoming them

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(Kieran, 2022). In response to these concerns, the present study examines how different classroom-based instructional interventions can support Grade 6 students in shifting toward analytic operations on the unknown in equation solving. By situating the study in authentic classroom contexts and employing empirical assessment, this research aims to provide detailed evidence on instructional features that are pivotal in supporting students at a critical juncture in their development of algebraic thinking.

LITERATURE REVIEW

A growing body of research has examined how instruction can support students' understanding of equation solving prior to the formal introduction of letter-symbolic algebra. Studies focusing on numerical equalities have shown that, with targeted instructional support, students can learn to transform both sides of a numerical equality in equivalent ways, thereby anticipating key features of formal algebraic manipulation (Kieran & Martínez-Hernández, 2022). Other studies focusing on non-letter-symbolic equations have demonstrated that the use of concrete referents, such as boxes and marbles or envelopes and stamps, allows students to model the relationships between known and unknown quantities using equations and to work analytically to determine the unknown quantities (Lenz, 2022; Radford, 2022). Research engaging students with the balance-scale model, further showed that it can foster understanding of algebraic syntax (Otten et al., 2020). By highlighting that both sides must remain in balance, it encourages strategies that preserve equivalence, allowing actions on the balance, such as adding or removing equal amounts, to be coordinated with corresponding equation transformations (Vlassis, 2022). In addition, studies of digital tools and interactive environments suggested that these technologies can support equation solving by enabling students to coordinate and translate across representations, including model-based and symbolic forms (Kapon et al., 2019; Naftaliev & Yerushalmy, 2022).

Collectively, these studies suggest that sense-making across contexts and representational fluency can support early forms of analytic algebraic thinking. However, relatively few intervention studies have examined how instruction supports upper elementary students' transition from non-letter-symbolic to letter-symbolic equations (Kieran, 2022), and much of the existing research relies on small-scale teaching experiments, leaving questions about effectiveness in typical classroom contexts (Ellis & Özgür, 2024).

THEORETICAL BACKGROUND

Considering the findings reported in the previous sections, this study contends that, to better conceptualize the elements needed to design instructional interventions that support students' transition from non-letter-symbolic to letter-symbolic equations, two foundational yet enduring theoretical ideas remain essential: algebraic activity as modeling (Kaput, 2008; Kieran, 2004) and algebraic activity as transformational (Kieran, 2004). Modeling, as an algebraic activity, can take the form of using algebraic language—such as that used to express an equation—to explicitly describe a situation

involving known and unknown quantities (Kaput, 2008). Transformational algebraic activity involves "rule-based" procedures, such as transforming an equation while preserving its relational meaning (Kieran, 2004).

In the case of equation solving, a balance-scale activity can encompass both modeling and transformational features. Initially, the balance scale supports students in developing a conceptual understanding of the relationship between known and unknown quantities, as well as the operations needed to determine the unknown, without requiring the interpretation of letter-symbolic representations (Vlassis, 2002). This relationship can subsequently be expressed more compactly using letter-symbolic equations. However, at a certain point in instruction, the equation must become detached from the model and be treated as an algebraic object in its own right. When the symbols in the equation no longer directly refer back to the balance-scale context, equation solving depends primarily on transforming the equation according to algebraic syntax (Radford, 2022).

AIM OF THE STUDY AND RESEARCH QUESTIONS

Building on the theoretical perspective that both modeling and transformational algebraic activities seem essential for developing the analytic dimension of algebraic thinking (Radford, 2014), this study explores how these activities can be effectively integrated into instruction for Grade 6 students' learning of equation solving. To this end, we designed four classroom-based interventions. The guiding research question is: *How do four classroom-based interventions affect Grade 6 students' ability to solve equations using equivalence-preserving transformations?*

METHOD

Participants

The sample comprised 69 Grade 6 students drawn from two public primary schools in Cyprus. Students were taught in their regular classrooms by trained teachers who had prior training in the instructional interventions to ensure consistent implementation across classrooms. Participation was based on availability and school consent, consistent with a convenience sampling strategy.

Instructional interventions

All interventions addressed the same mathematical content (equality, the additive and multiplicative properties of equality, variable and equation solving), but differed in the instructional environment used. They had the same duration (eight 40-minute lessons) and were implemented over a two-week period during regular mathematics lessons.

Intervention 1 used in a digital environment that emphasized interactive balance-scale representations. Instruction allowed students to explore how balance is maintained when the same operation is applied to both sides of the scale. Variables were gradually introduced, initially represented as shapes with unknown weights and later evolving into letters. Letter-symbolic equations were then used to model the balance-scale

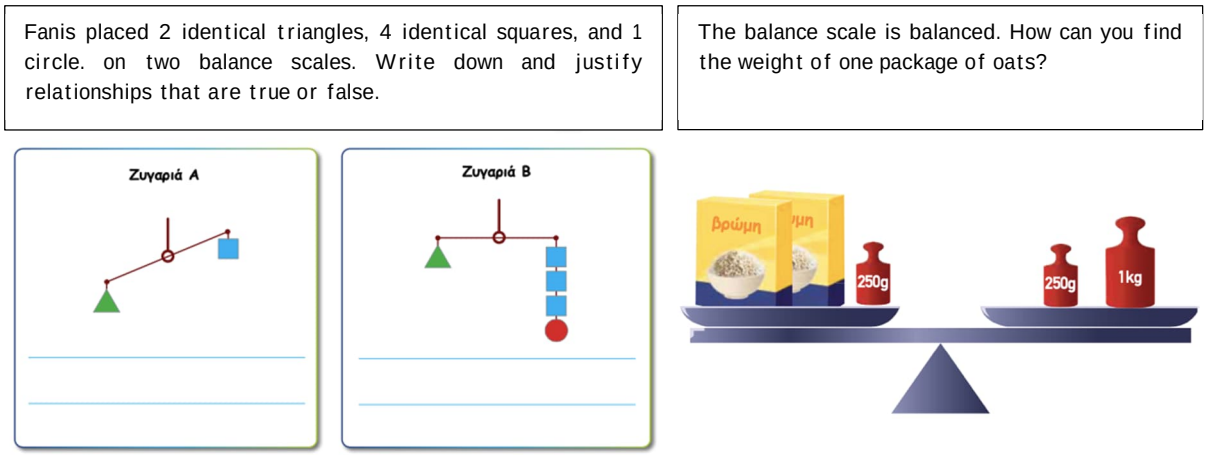
situations. The dynamic, visual nature of the environment supported both modeling and transformational activity. Teachers could monitor student progress through a dashboard and provide feedback in real time.

Intervention 2 used a digital environment that foregrounded transformational algebraic activity by treating numerical equalities and letter-symbolic equations as objects to be directly manipulated. Students engaged in transforming equations while the environment provided continuous, rule-based feedback that prevented invalid moves and constrained students' actions to equivalence-preserving transformations. By making the structure and logic of valid algebraic transformations explicit, this design emphasized equation solving as a transformational process rather than a modeling or rote procedural task. The environment supported conceptual discussion and aimed to reduce reliance on memorized procedures. A teacher dashboard was available in this case as well.

Intervention 3 combined the two digital environments to leverage their complementary strengths. The modeling-focused environment supported students' initial conceptual understanding of equality and equation solving through the balance-scale model, while the transformational-focused environment subsequently provided opportunities to work with equalities and equations once they were detached from the model.

Intervention 4 served as a control condition, relying exclusively on the official mathematics textbook (Figure 1). Students engaged with the same content as in Intervention 3, but worked solely in a paper-and-pencil format following the standard curriculum sequence. No digital tools or interactive representations were used, providing a baseline for comparison with the technology-enhanced intervention.

Figure 1: The Mathematics Textbook Environment in Intervention 4



Test

Learning outcomes were assessed using pre- and post-tests administered before and after the intervention period. The post-test included all pre-test items (common items) that assessed prior knowledge (e.g., translating verbal expressions into algebraic form, simplifying algebraic expressions and content that had not yet been formally taught,

(e.g., solving letter-symbolic equations). In addition, the post-test included an additional set of items specifically targeting the modeling of verbal expressions and word problems with equations, as well as equation solving. This distinction ensured that the assessment captured both students' understanding of basic concepts and procedures and learning gains directly attributable to the interventions.

Table 1: Examples of test items

Test	Types of items	Example
Pre-test / Post-test	Translation of verbal expressions into algebraic expressions	Subtract 5 from k. Multiply l by 4 and add 8.
Pre-test / Post-test	Simplification of algebraic expressions	Simplify the expression $6x+8+4x-3$
Pre-test / Post-test	Translation of word problems into algebraic expressions	Christina is b years old. Paris is 4 years younger than Christina. Write an algebraic expression representing Paris's age.
Pre-test / Post-test	Equation solving	Find the value of r, if $5r+8=23$
Post-test	Translation of verbal expressions into equations	Twice the sum of m and 3 is equal to 28.

Analyses

Prior to conducting the main analyses, data were screened for missing values and outliers. The final sample included 69 participants distributed across four instructional groups: Group 1 (n = 20), Group 2 (n = 14), Group 3 (n = 19), and Group 4 (n = 16). To address the research question concerning the effectiveness of the four interventions, two separate analyses of covariance (ANCOVAs) were conducted. The first ANCOVA examined performance on common items appearing in both the pre-test and post-test, while the second examined performance on additional items. In both analyses, pre-test scores were included as a covariate to control for baseline differences.

For the ANCOVA analyzing the common items, Levene's test indicated that the assumption of equal error variances across groups was violated, $F(3, 65) = 3.62$, $p = .018$. However, because ANCOVA is generally robust to moderate violations of this assumption, the analysis was retained. Moreover, a Bonferroni correction was applied, adjusting the significance threshold to account for multiple comparisons. For the ANCOVA analyzing the additional items, the assumption of homogeneity of variance was met, $F(3, 65) = 0.83$, $p = .481$. In addition, the homogeneity of regression slopes was assessed by including the interaction between pre-test scores and instructional group. This interaction was not significant for the additional items, $F(3, 61) = 2.03$, p

= .119, indicating that the relationship between pre-test and post-test scores was consistent across groups.

RESULTS

Descriptive statistics for post-test performance on the common and additional items for each instructional group are reported in Table 2. The ANCOVA for the common items, showed a significant and large effect of instructional group on post-test scores, $F(3, 64) = 7.10$, $p < .001$, partial $\eta^2 = .25$. The pre-test covariate was also a strong and significant predictor of post-test performance, $F(1, 64) = 172.50$, $p <.001$, partial $\eta^2 = .73$. The ANCOVA for the additional items revealed a statistically significant, small-to-medium effect of instructional group, $F(3, 64) = 2.90$, $p = .042$, partial $\eta^2 = .12$. Pre-test performance again significantly predicted post-test scores on the additional items, $F(1, 64) = 45.13$, $p < .001$, partial $\eta^2 = .41$.

Table 2: Descriptive statistics for Post-Test performance by instructional group

		Common items		Additional items	
Group	*n*	*M*	*SD*	*M*	*SD*
1	20	15.80	1.71	4.13	0.58
2	14	14.21	3.74	3.71	1.22
3	19	15.53	2.38	4.16	1.17
4	16	14.75	4.40	4.19	1.11
Total	69	15.16	3.10	4.07	1.02

Table 3 presents pairwise comparisons of the adjusted post-test means. The significance levels for these comparisons were adjusted using the Bonferroni method, allowing us to determine which instructional groups differed significantly from each other on the common and the additional items. For the common items, the results indicated that Group 3 significantly outperformed all other groups: Group 3 versus Group 2 (mean difference = 2.64, $p < .001$), Group 3 versus Group 1 (mean difference = 1.53, $p = .037$), and Group 3 versus Group 4 (mean difference = 1.54, $p = .045$). For the additional items, the results indicated that Group 3 and Group 4 obtained similarly high scores, both outperforming Group 1 and Group 2. However, only one statistically significant difference was identified: Group 3 scored significantly higher than Group 2 (mean difference = 0.77, $p = .050$).

Table 3: Pairwise Comparisons of Adjusted Means with Bonferroni Correction

	Common Items			Additional Items		
Comparison	Mean Diff	95% CI	*p*	Mean Diff	95% CI	*p*
Group 1 vs. Group 2	1.11	[-0.45, 2.66]	.340	0.29	[-0.46, 1.05]	1.000
Group 1 vs. Group 3	-1.53	[-3.00, -0.06]	.037	-0.48	[-1.20, 0.23]	.428
Group 1 vs. Group 4	0.00	[-1.50, 1.51]	1.000	-0.32	[-1.06, 0.41]	1.000
Group 2 vs. Group 3	-2.64	[-4.23, -1.05]	<.001	-0.77	[-1.55, 0.00]	.050
Group 2 vs. Group 4	-1.10	[-2.73, 0.53]	.422	-0.61	[-1.41, 0.18]	.234

Group 3 vs. Group 4	1.54	[0.02, 3.05]	.045	0.16	[-0.58, 0.90]	1.000

DISCUSSION

This study investigated the effects of four instructional interventions on Grade 6 students' early algebraic thinking, focusing on equation solving. The interventions differed in how they engaged students with modeling and transformational algebraic activities: Intervention 1 supported modeling activity by allowing students to reason about relationships between known and unknown quantities represented via the balance-scale model; Intervention 2 focused exclusively on symbolic manipulations, promoting transformational activity through rule-based operations; Intervention 3 integrated both approaches to provide complementary opportunities for modeling and transformation; and Intervention 4 followed traditional textbook instruction.

Pre-test and post-test included common items measuring prior knowledge as well as additional items targeting newly taught knowledge. On the common items, Group 3 (combined/digital) significantly outperformed all other groups, suggesting that the combination of modeling with transformational activities in a digital environment helped students consolidate prior knowledge. Group 1 (modeling/digital) and Group 4 (combined/textbook) performed similarly, while Group 2 (transformational/digital) lagged behind, indicating that focusing solely on transformational activity is less effective for reinforcing prior knowledge.

For the additional items assessing newly taught modeling problems and algebraic manipulation, Group 3 (combined/digital) outperformed Group 2 (transformational/digital), highlighting the importance of combining modeling with transformational practice, particularly the opportunity to work with transformations detached from the model. Interestingly, Group 4 (combined/textbook) performed similarly to Group 3, despite lacking interactive digital tools, suggesting that it may be the combination of itself, rather than interactivity per se, that supported learning strategies of algebraic strategies.

These findings underscore the value of integrating modeling and transformational activities. Balance-scale tasks supported relational reasoning about equality (Lenz, 2022), while symbolic manipulations facilitated engagement with algebraic rules and equivalence-preserving transformations (Radford, 2022). By combining these approaches, a pathway is created for students to transition from concrete to more abstract, analytic algebraic thinking, leveraging complementary affordances: concrete and visual representations for modeling equivalence, and symbolic notation for performing transformational operations.

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