UNPACKING META-DIDACTIC PRAXEOLOGIES IN TEACHER PREPARATION

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This study examines how meta-didactic praxeologies are structured in mathematics teacher education to prepare pre-service teachers for teaching with instructional technology. Using the Anthropological Theory of the Didactic, a qualitative case study was conducted with two mathematics teacher educators at a Ghanaian university, drawing on course plans and interviews. The analysis focused on the types of tasks, techniques, technologies, and theories at the meta-didactic level. The findings reveal contrasting praxeologies oriented toward design-based conceptual exploration and remedial–diagnostic stabilisation. Representation and decomposition dominated as the meta-didactic technique, while approximation was limited, and justificatory discourse remained largely tacit.

INTRODUCTION

Instructional technology has become a central feature of contemporary mathematics classrooms, with digital tools such as dynamic geometry environments, virtual manipulatives, coding platforms, and video-based assessment technologies increasingly used to support conceptual understanding and student engagement (Quarder et al., 2025; Yildiz and Arpaci, 2024). I will refer to instructional technology as 'IT' hereof. In response, mathematics teacher education programmes have expanded their focus on technology integration through dedicated courses, blended learning designs, and resource-oriented pedagogical approaches to strengthen pre-service teachers' (PSTs) technological and pedagogical capacities (Greene et al., 2023). Simultaneously, practice-based teacher education (PBTE) has gained prominence as a framework for preparing PSTs through direct engagement with teaching practice. PBTE seeks to support the development of adaptive and context-sensitive teaching expertise by foregrounding activities such as observing, decomposing, rehearsing, and reflecting on instruction (Grossman et al., 2009; Lampert et al., 2013; Kavanagh et al., 2020). Within mathematics education, PBTE has been widely positioned as a promising approach for supporting PSTs' learning to integrate IT in pedagogically meaningful ways.

Despite these developments, research consistently shows that PSTs' preparation for IT integration remains uneven and fragile. Numerous studies have reported persistent gaps between program-level aspirations and classroom enactment, with PSTs often encountering digital tools without developing the capacity to use them coherently within student-centred, conceptually oriented mathematics instruction (Eshetu et al.,

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2022; Mensah et al., 2024). These recurring difficulties suggest that the challenge is not merely one of access to tools or technical competence but also of pedagogy. A critical yet underexplored dimension of this problem is the teaching practices of mathematics teacher educators (MTEs). While PBTE highlights the importance of carefully designed learning experiences, existing research has paid limited empirical attention to how MTEs organise, justify, and enact IT integration in their teaching. Consequently, IT risks being treated as an add-on or procedural requirement rather than an integral component of pedagogically principled practice (Kavanagh et al., 2020; Wilson & Yonas, 2024). This limitation is especially pronounced in resource-constrained contexts, where infrastructural limitations, institutional norms, and policy demands strongly condition the forms of technology integration that are feasible and legitimate (Nyaaba et al., 2024). However, empirical studies examining how MTEs navigate these conditions in their teaching practices remain scarce.

This study addresses this gap by examining how MTEs design and enact teaching practices that prepare PSTs to teach mathematics using IT. Focusing on MTEs' instructional decisions and pedagogical reasoning, this study investigates how learning experiences related to IT are organised, justified, and negotiated within specific institutional and material conditions. By foregrounding MTEs' teaching practice, this study aims to contribute empirically grounded insights into how IT integration is pedagogically constructed within mathematics teacher education.

THEORETICAL CONSIDERATIONS AND RESEARCH QUESTION

This study is situated within the Anthropological Theory of the Didactic (ATD) (Chevallard, 1992) and draws on the constructs of meta-didactic praxeologies (see Arzarello et al., 2014). Meta-didactic praxeology refers to the structured ways in which teacher educators, institutions, or programmes organise, justify, and theorise practices for teaching future teachers how to teach (Arzarello et al., 2014). I will refer to meta-didactic praxeology as 'MDP' hereof. Within the ATD, teaching is understood as a praxeological activity organised by institutional norms, epistemic commitments, and legitimising discourses. When teaching becomes the object of teaching, as in mathematics teacher education, these praxeological structures operate at a meta-didactic level. Meta-didactic transposition foregrounds how MTEs interpret, adapt, and institutionalise external didactic inputs, such as instructional technologies, within teacher education. Rather than assuming direct implementation, this perspective emphasises the negotiated reconfiguration of pedagogical purposes, practices, and values as IT is integrated into institutional settings.

MDP provides an analytical lens for examining how MTEs organise and legitimise their pedagogical work. In the ATD, praxeologies consist of types of tasks (t), techniques (τ) for solving the type of task, technologies (θ) that rationalise those techniques, and theories (Θ) that confer epistemological legitimacy. Applied at the meta-didactic level, this structure enables an analysis of how learning experiences for teaching with IT are designed, justified, and constrained. This lens is particularly suited

to examining IT integration in resource-constrained contexts, where institutional conditions delimit the feasible and legitimate forms of pedagogy. To operationalise the analysis, the study draws on pedagogies of practice—representation, decomposition, and approximation (Grossman et al., 2009) as meta-didactic techniques embedded within broader praxeological configurations. Representation involves showing teaching practice so that PSTs can see what professional practice looks like; decomposition involves breaking complex teaching practice into smaller, learnable components; and approximation provides opportunities for novices to enact teaching practice in settings that are close to, but safer than, real classrooms (Grossman et al., 2009). Guided by the ATD, the study addresses the following research question: *How are the meta-didactic praxeologies of mathematics teacher educators structured in preparing pre-service teachers to teach mathematics using instructional technology*?

METHODS

This study employed a qualitative case study design to examine how MTEs structure their MDP for IT integration. The research was conducted at a public Ghanaian university offering a Bachelor of Education in Mathematics, where policy-driven digital reform expectations intersect with infrastructural limitations and rigid curriculum structures. IT–focused mathematics education courses, in which MTEs exercised curricular discretion, served as primary analytical sites. Two MTEs, *Benjamin* and *Abo (pseudonyms)*, participated in this study. Each taught a distinct, IT–oriented course (EMA 335 and EMA 325), held a doctoral degree in mathematics education, and had over 8 years of experience in teacher education. Data were drawn from two sources: course plans and semi-structured interviews with the teachers. Course plans were analysed as institutional texts articulating pedagogical intentions and positioning IT within formal teacher preparation, while interviews provided insights into pedagogical reasoning, perceived constraints, and justificatory discourses. Data analysis was guided by the MDP framework of the ATD, focusing on the types of tasks (t), techniques (τ), technologies (θ), and theories (Θ). Pedagogies of practice—representation, decomposition, and approximation were used as analytical indicators. Coding was both deductive and interpretive, and was refined iteratively through analytic memo writing. The interview data were treated as situated accounts, with attention to emphasis and omission. Trustworthiness was ensured through triangulation, member checking, and reflexive engagement. The researcher's positionality was treated as an interpretive resource and was supported by reflexive memo-writing.

RESULTS

Types of Tasks (t)

Across both courses, IT was integrated into *t*, which was directly tied to mathematical work. The nature of these *t* differed in terms of how technology was positioned in relation to teaching and learning. In EMA 335, Benjamin designed *t* that required PSTs

to construct technology-mediated instruction. Rather than adopting existing digital resources, PSTs were expected to design instructional uses of technology that enhance mathematical clarity. As Benjamin explained, *"I want them [PSTs] to think about how technology can make the concept clearer, not just how to use the software."* This *t* orientation foregrounded pedagogical purpose over technical proficiency, positioning IT as a means of shaping mathematical meaning. These *t* were explicitly collaborative. PSTs were expected to negotiate instructional decisions collectively by pooling and refining ideas through discussion. Benjamin noted, *"They bring their [PSTs] ideas together, and then as a group they decide what will work best for the lesson."* The *t* completion, therefore, depended on shared pedagogical reasoning rather than individual tool use, situating IT within collective instructional design.

In contrast, *the t* in EMA 325 was structured around Abo's assessment of PSTs' limited readiness for technology-mediated instruction. The *t* design prioritises foundational mathematical understanding before pedagogical application. Abo stated, *"Before we can talk about teaching with technology, they [PSTs] must first understand the mathematics properly."* IT primarily functions as a support for disciplinary consolidation rather than as a design medium. Abo further described how *t* used digital tools to surface and correct misconceptions through structured activity: *"When they [PSTs] work with the spreadsheet, the mistakes show clearly, and then we can correct them step by step."* Here, IT was embedded in *t*, emphasising error visibility, sequencing, and correction, framing it as a stabilising resource for mathematical reasoning.

Techniques (τ)

Across both courses, representation and decomposition of practice dominated as the meta-didactic techniques for making IT teachable. The approximation of practice appeared only in limited and indirect forms. In EMA 335, Benjamin relied on representation by modelling complete instructional sequences in which digital tools were embedded within the lesson flow. These demonstrations positioned PSTs as learners observing enacted practices rather than as teachers enacting instructional moves. Benjamin explained, *"I demonstrate the lesson first, so they [PSTs] can see how the technology fits into the teaching."* Thus, representation functioned as a primary τ for making instructional decisions visible while maintaining control over enactment. Benjamin also employed an explicit decomposition of instructional practice, analytically separating lesson components. He described this as, *"We look at the lesson in parts… the digital material, what students do with it, and how they [PSTs] use it."* Decomposed practice supported PSTs' analytical understanding of instructional structure but did not extend to rehearsed enactment.

In EMA 325, Abo similarly relied on representation, although with a stronger procedural emphasis. Teaching with IT was modelled through step-by-step demonstrations that focused on correct execution. As Abo stated, *"I show them[PSTs] the steps first, then they try the same thing on their own."* Decomposition occurred

primarily through task sequencing: *"I move from one operation to the next, so they [PSTs] understand each part before going on."* This supports functional fluency without unpacking teaching decisions.

The approximation of practice appeared only in partial forms. In EMA 335, PSTs discussed and refined instructional designs collectively: *"They [PSTs] explain their design to the class, and we talk about what could be improved."* In EMA 325, PSTs demonstrated digital procedures individually: *"Each student has to explain what they are doing when they [PSTs] present their work."* These activities involved reasoning about teaching but did not require PSTs to rehearse the instructional interaction. Overall, representation and decomposition were foregrounded, whereas approximation remained structurally constrained.

Technologies (θ)

Across both courses, IT was justified through pedagogical rationales linked to mathematical meaning-making, rather than efficiency or novelty. In EMA 335, Benjamin justified the use of IT as an epistemic resource for conceptual exploration. Dynamic mathematics software was described in the literature as a medium for visualising and interrogating mathematical relationships. He explained, *"With the animation, students can see how the relationship changes instead of just hearing the explanation."* Technology gained pedagogical value only when coherently embedded: *"If the technology is not connected to the task and the discussion, then it doesn't really help."*

In EMA 325, Abo articulated a technological rationale shaped by concerns regarding accuracy and learner readiness. IT was justified as a means of stabilising mathematical understanding and revealing errors: *"When they [PSTs] use Excel, the answers don't lie."* Abo also emphasised accessibility: *"These are tools they [PSTs] already have access to, so we can focus on the mathematics."* Technology was legitimised through feasibility and diagnostic clarity rather than exploratory affordances.

Theories (Θ)

MTE does not explicitly reference formal learning theories. Instead, their pedagogical reasoning reflects tacit epistemological orientations. In EMA 335, Benjamin's reasoning reflected an implicit view of learning as emerging from exploration and collective sense-making. He stated, *"I want them [PSTs] to observe what is happening and then draw their own conclusions."* Dialogue and reflection were central: *"After they share what they have seen, we discuss it as a class."* IT was legitimised as a catalyst for noticing and conjecturing.

In contrast, Abo's reasoning reflected a tacit orientation toward learning as a gradual acquisition of accuracy and procedural reliability. He explained, *"If the foundation is not correct, everything else will collapse later."* Teaching was framed as guided correction: *"My role is to guide them step by step until they get it right."* IT was legitimised as a support for controlled progression and error correction.

DISCUSSION

In addressing how MDP are structured in PSTs to teach mathematics with IT, this study shows that such praxeologies are locally configured through the interaction of pedagogical intentions, epistemic commitments, and contextual constraints. Rather than reflecting the direct implementation of policy ideals or generic technology frameworks, the practices examined reveal negotiated configurations of tasks, techniques, justificatory discourse, and tacit theories.

At the task level, IT was embedded within mathematically purposeful activity but oriented toward different pedagogical ends. Benjamin's design-oriented tasks position technology as an epistemic resource for conceptual exploration and instructional coherence. By integrating digital artefacts with worksheets and lesson planning, his approach aligns with didactic engineering traditions that emphasise technology's role in structuring enquiry and mathematical meaning-making (Trouche et al., 2019). In contrast, Abo framed remedial–diagnostic tasks as a stabilising tool for addressing misconceptions and ensuring procedural accuracy. This pragmatic orientation reflects adjustments to PSTs' limited readiness, consistent with research showing that technology integration in teacher education is often reshaped by learner constraints rather than by curricular aspirations (Caniglia & Meadows, 2018; Eshetu et al., 2022).

The techniques used to enact these tasks further clarified the structure of the MDP. Both MTEs relied primarily on representation and decomposition of practice, consistent with practice-based teacher education approaches that emphasise making teaching visible and analysable (Grossman et al. 2009). However, the approximation of practice remained limited and indirect. While PSTs engaged in reasoning about instructional design or in explaining digital procedures, opportunities to rehearse technology-mediated teaching moves or to respond to student thinking were scarce. This result suggests a structurally thinned enactment of approximation, shaped by concerns about feasibility and readiness—tensions also identified in PBTE research (Kavanagh et al., 2020).

At the level of justificatory discourse and tacit theories, IT was legitimised through distinct epistemic orientations. Benjamin framed digital tools as epistemic mediators for exploration and collective meaning-making, aligning with the view of technology as a cognitive and representational resource in mathematics education (Hoyles & Lagrange, 2010). In contrast, Abo justified technology on the grounds of diagnostic precision and accessibility, privileging correctness and control (Drijvers, 2015). These differences support the claim that technology integration is shaped more by beliefs about teaching and learning than by technical competence (Ertmer & Ottenbreit-Leftwich, 2010). Notably, these epistemic commitments have remained largely tacit. As Chevallard (2006) cautions, the absence of explicit logos in teacher education may limit reflective alignment between practice and theory when teaching becomes the object of instruction.

Overall, the findings suggest that MDP for IT integration are structured through adaptive and context-sensitive configurations rather than uniform best-practice models. While these configurations demonstrate pedagogical agency under constraints, they also point to the need for structured spaces in mathematics teacher education where justificatory discourses and epistemic assumptions about IT are made explicit and grounded in theory.

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