UPPER ELEMENTARY STUDENTS' PERFORMANCE AND REASONING ON FUNCTION-GRAPH TASKS

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This study examines Grade 5 and 6 students' performance on and reasoning about function-graph tasks. Drawing on a synthesis of frameworks, we designed tasks addressing five conceptual categories. Data from 271 students were analyzed using mixed methods. Results show stronger performance on coordination of quantities tasks, particularly with quantitative values, and substantial challenges with tasks requiring functional inferences. Performance on covariational and graph-as-object tasks was more variable. Findings include a framework for analyzing upper elementary students' reasoning of function graphs, with insight on how task categories foreground distinct reasoning.

INTRODUCTION

Functional thinking is a core dimension of algebraic thinking across grade levels (Pinto & Cañadas, 2021; Stephens et al., 2017). It encompasses multiple conceptions of functions (Doorman et al., 2013), including functions as input–output assignments, dynamic processes of covariation, correspondence relations emphasizing input and output pairings, and mathematical objects (Pittalis et al., 2020). Developing these conceptions requires coordinating representations, including equations, tables, diagrams, and graphs (Kieran, 2022). Among these representations, graphs play a central role in functional thinking (Radford, 2025; Tasova & Moore, 2025), as they uniquely support all major conceptions of functions, enabling coordinating changes in input and output quantities, interpreting direction and magnitude of change, and translating between representations (Carlson et al., 2002). Despite these affordances, little is known about how students—particularly in upper elementary grades—interpret and reason with function graphs. Addressing this gap, the present study examines how Grade 5 and 6 students perform on and reason about function-graph tasks.

THEORETICAL CONSIDERATIONS

To investigate performance related to function graphs, we draw on complementary theoretical perspectives. Central to our approach is the framework proposed by Strachota et al. (2024), originally developed with Grades K–2 students and grounded in Sfard's (1991) object–process theory**.** This framework conceptualizes reasoning about function graphs as dynamic and context-dependent, with multiple forms of reasoning potentially coexisting**.** It distinguishes qualitatively different ways of engaging with graphs, ranging from attention to individual graph components and

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coordination of quantities represented by points to generalized functional inferences and reasoning with the graph as a mathematical object.

To further interpret students' performance, we align this framework with research on graphical reasoning that distinguishes between local, quantitative approaches—such as reading exact values, choosing scales, or plotting points—and global, qualitative approaches, which emphasize interpreting overall trends, direction of change, and relative relationships (Duijzer et al., 2019). In addition, we draw on research on functional thinking that characterizes students' reasoning as covariation. Carlson and colleagues' (2002) define this reasoning as the mental coordination of the direction and magnitude of change in two quantities. This nuanced characterization of functional thinking allows us to differentiate reasoning about how quantities vary together from reasoning that attends only to successive values. Together, these perspectives provide a coherent theoretical lens for developing and analyzing function-graph tasks with upper elementary students, a key goal of this research.

METHODS

Participants and Background Knowledge

The sample comprised 271 students (130 Grade 5 & 141 Grade 6 students), selected through convenience sampling from public primary schools in Cyprus. Data were collected during the final days of the school year, ensuring completion of the grade-level curriculum. Grades 5 and 6 were selected because they represent consecutive stages in upper primary education at which students have accumulated substantial experience with early algebra concepts. According to the Cypriot mathematics curriculum, students at these grade levels engage with algebraic ideas, such as pattern recognition and generalization, symbolic representation of quantitative relationships, functional thinking, and equation solving. Function graphs are introduced sporadically within relevant units.

Instrument Design and Content

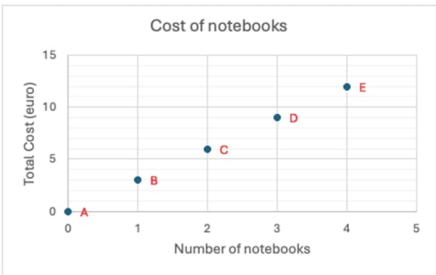
To assess students' performance on function-graph tasks, we developed a 12-item test. Most tasks included subquestions, resulting in a total of 29 scored items. The items were designed to reflect the five conceptual categories summarized in Table 1. The test included both multiple-choice and open-ended items. The categories in Table 1 are not intended to represent a developmental hierarchy but rather to characterize the conceptual demands of function-graph tasks.

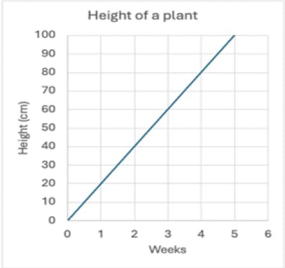
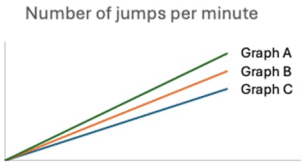
Scoring Procedures - Analysis

Student responses were evaluated for correctness and strategy. Correct responses were awarded one point, partially correct responses were awarded 0.5 points, and incorrect or no response received zero points. For items requiring an explanation, responses were analyzed via deductive qualitative content analysis. Explanations were coded according to theoretically grounded categories rather than derived inductively from the

data (e.g., Chimoni et al., 2018). That is, each explanation was assigned to the category that best represented the student's strategy. For example, in Task 2A (i.e., toy train task), responses were coded as (1) Recursive Inferences when students used step-by-step addition (e.g., "He adds 2 blocks each time"); (2) Condensed Coordination of Quantities when students coordinated values (e.g., "Point C (2, 5)"); (3) Functional Inferences when students identified a correspondence rule (e.g., "Multiply the number of extensions by 2 and add 1"); or **(**4) No obvious strategy when an answer lacked a clear or valid explanation.

Table 1. Example tasks for each conceptual category

Conceptual Category / Task Characteristics	Example Tasks
Coordinate Quantities – quantitative Identifying, plotting, or coordinating specific discrete data points on a graph.	Task A1a: The graph represents the cost of buying notebooks from a bookshop.  **What does point C show?** (a) The cost of buying 10 notebooks is 2 euro; (b) The cost of buying 2 books is 6 euro; (c) The cost of buying 1 book is 10 euro; (d) Someone can buy 2 books; (e) Someone paid 6 euro.
Coordinate Quantities - qualitative Interpreting the meaning of a specific data point on a quantitative graph.	Liam is building a toy train using wooden blocks. He starts with a locomotive and adds a number of blocks to the train each time he extends it. The graph shows the total number of blocks based on the number of extensions. Task A2b: **What does Point A represent?** (coordinate quantities-qualitative)
Functional Relationships Generalizing correspondence relationships, applying rules, or making predictions for large values.	Task A2c: **How many wooden blocks are needed for building a train with 71 extensions?** Explain your thinking. (functional relationships)

Covariational relationships Identification of unit rates or changes in one variable to explain changes in another.	Task A4c: The graph presents the growth in the height of a plant for 5 weeks. To calculate the height of the plant in Week 10, Ann wrote down the following: $80 + (6 \times 20)$. **Explain Ann's thinking.**	
Graph as Object Utilizing qualitative or quantitative graphs to reason about the graph as an abstract mathematical object.	Task A4a: Three friends, Emma, Liam, and Sophia, decide to have a jump rope competition: Emma can jump rope 25 times per minute; Liam can jump rope more times than Emma per minute; Sophia can jump rope less times than Emma per minute. The graph presents the number of jumps per minute for the three friends. **Which graph corresponds to each friend?** Explain your thinking.	

To examine performance, tasks were grouped by the conceptual category they addressed. Responses were scored with a maximum of one per task group. Mean scores were calculated, and performance was classified into three bands based on the proportion of correct responses: low (below 50%), moderate (50–74%), and high (75%+). These bands were used to compare performance patterns across task groups rather than to represent developmental levels of understanding.

RESULTS

Students' performance varied across task groups (see Table 2). Mean scores indicated clear differences in how students engaged with these task types. Performance was highest on condensed coordination of quantities–quantitative tasks (M = .70, SD = .24), followed by covariational tasks (M = .55, SD = .33), graph as object tasks (M = .54, SD = .28), and condensed coordination of quantities–qualitative tasks (M = .52, SD = .25). In contrast, students' performance on functional inference tasks was substantially lower (M = .28, SD = .24), indicating particular difficulty with tasks involving generalizing functional relationships.

Examining performance distributions across bands further illustrates differences. On qualitative condensed coordination tasks, nearly half the students (46.5%) were in the low-performance band, while approximately one quarter (24.1%) were in the high-performance band. In contrast, performance on quantitative condensed coordination

tasks was stronger, with only 17.2% of students in the low band and nearly half (47.8%) in the high-performance band.

Table 2. Performance distribution across low, moderate, and high bands

Performance bands	Cond.-qualit.	Cond.-quant.	Functional relations	Covariation relations	Object
Less than 50%	46.5%	17.2%	75.9%	33.2%	39.6%
50%- 74%	29.4%	35%	18.8%	34.7%	35.3%
75% and above	24.1%	47.8%	5.3%	32.1%	25.1%

Functional inference tasks posed most challenging for students. More than three quarters (75.9%) of students fell in the low band, and only 5.3% reached the high band. Performance on covariational reasoning tasks was more evenly distributed, with roughly one third of students in each band: 33.2% in the low band, 34.7% in the moderate band, and 32.1% in the high band. A similar pattern was observed for tasks addressing the graph as a mathematical object, where 39.6% of students were in the low band, 35.3% in the moderate band, and 25.1% in the high band. Overall, these results indicate that students were generally more successful on tasks requiring coordination of quantities, particularly when quantitative values were foregrounded, than on tasks requiring generalizing a functional relationship. Covariational reasoning and reasoning about the graph as a mathematical object showed intermediate performance, with substantial variability across students.

Table 3 summarizes students' strategies and correctness across two parts of the toy train task. In the condensed coordination–quantitative part (Task A2a), 159 of 237 responses were correct. Among correct responses, functional inference was the most frequent strategy (81; 51%), followed by recursive (18; 11%) and condensed coordination strategies (14; 9%). Forty-six correct responses (29%) were provided without explanation. Incorrect responses (78) were primarily associated with no explanation (47; 60%) and the multiply-only-by-2 strategy (18; 23%). This strategy indicates recognition of a recursive rule (each extension adds two blocks) but failure to coordinate this rule with the initial value, resulting in an incorrect answer. No incorrect responses involved functional inferences. Although the answer could be directly read from the graph, many students relied on functional inferences, suggesting that they attended to the underlying quantitative relationship rather than relying solely on perceptual graph reading.

In the far extension task (Task A2c), performance declined (98 correct; 139 incorrect). Correct responses were predominantly supported by functional inference (81; 83%). Few correct responses involved other strategies (5; 5%) or no explanation (12; 12%).

Incorrect responses were mainly characterized by no explanation (79; 57%) and multiply-only-by-2 strategies (38; 27%). As in the previous task, these responses reflect recognition of a recursive rule without accounting for intercept, insufficient to determine the far extension. While recognition of a recursive rule supported partial reasoning in both tasks, only functional inferences supported successfully generalizing. Tasks requiring extrapolation thus more clearly distinguished students who reasoned covariationally from those who reasoned recursively.

Table 3. Students' strategies in Task A2a & A2c

	Strategy					
Correctness	Recurs.	Cond. Coord.	Funct. Infer.	Multiply by 2	Other Strat.	No Explan.
Task A2a						
Wrong	-	-	-	18(23%)	13(17%)	47(60%)
Correct	18(11%)	14(9%)	81(51%)	0	0	46(29%)
Task A2c						
Wrong	-	-	8(6%)	38(27%)	14(10%)	79(57%)
Correct			81(83%)	0	5(5%)	12(12%)

In the covariation task involving plant growth, which required identifying the rate of change from a linear graph showing the height of a plant over five weeks (see Table 1), 176 of 237 were correct and 61 were incorrect. Most responses, including the majority of correct ones (133; 76%), were provided without explanation. Among responses that included an explanation, correct responses most commonly reflected recursive reasoning (27; 15%), indicating recognition of step-by-step additive change across equal intervals (e.g., "the plant grows 20 cm each week from one week to the next"). Fewer responses referenced a specific point on the graph (11; 6%). Incorrect responses were predominantly associated with no explanation (52; 85%). The high proportion of correct responses without explanation suggests that many students inferred the rate of change from the graph's structure without explicitly articulating a recursive or covariational strategy. This distinction matters because, although both attend to change, recursive reasoning focuses on successive values, whereas covariational reasoning reflects a generalized coordination of change in variables.

In the graph-as-object task, 162 of 237 students (68%) correctly matched graphs to verbal descriptions. Among correct responses, most students provided no explanation (93; 57%) or relied on informal comparisons (e.g., identifying the graph with more or fewer jumps per minute) without explicit reference to slope (46; 28%) or contextual quantities (23; 14%). Notably, no students referred to slope or rate of change. These findings suggest that students relied primarily on global visual comparison, with visual

features supporting correct matching while masking a limited conceptualization of slope as a quantitative measure.

DISCUSSION

Students were generally more successful on tasks requiring coordination of quantities, particularly with quantitative values, consistent with research showing productive engagement with graphs through local, point-based, and correspondence-oriented reasoning (Duijzer et al., 2019). In contrast, tasks requiring generalizing functional inferences posed challenges. Although many students recognized recursive patterns of change, such reasoning was often insufficient for extrapolation or for coordinating rate and initial value, aligning with prior work distinguishing recursive reasoning from more advanced functional and covariational approaches (Carlson et al., 2002; Duijzer et al., 2019; Pittalis et al., 2020). Performance on covariational and graph-as-object tasks was more variable, with students frequently relying on global features of graphs without explicitly articulating rate-based or covariational relationships. This pattern reflects findings that learners may attend to overall trends before conceptualizing slope as a quantitative measure (Leinhardt et al., 1990) and supports views of functional thinking as dynamic and context-dependent, with multiple forms of reasoning coexisting rather than developing linearly.

Beyond documenting student performance patterns, a key contribution of this study lies in synthesizing complementary theoretical frameworks into a coherent set of task categories that capture distinct ways upper elementary students reason with function graphs. The results indicate that function graphs are a powerful tool for investigating a wide range of students' conceptions of functions and their understanding of functional relationships. At the same time, the findings reveal that certain features of function graphs—particularly their visual nature—can sometimes cue correct responses that are not grounded in conceptual understanding. By combining quantitative performance data with qualitative analyses of students' explanations and strategies, the study makes visible this distinction between surface-level success and deeper functional reasoning.

Taken together, these findings show how different task categories foreground distinct reasoning demands and illuminate students' functional thinking. By operationalizing theoretical perspectives through a structured set of task categories appropriate for Grades 5 and 6, the study offers a practical analytic framework for research and instruction that supports nuanced interpretation of students' engagement with function graphs (Kieran, 2022), across varied classroom contexts.

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