EFFECTIVENESS OF METACOGNITIVE INSTRUCTION ON CHILDREN'S DECLARATIVE KNOWLEDGE DURING MATHEMATICS PROBLEM SOLVING

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The purpose of the present study was to examine the changes in children's declarative knowledge related to mathematical problem solving following metacognitive instruction. Participants were 38 Greek Year 3 children, half of which received metacognitive instruction (experimental group), while the others did not (control group). Results indicated that students who were assigned to metacognitive instruction outperformed their counterparts in declarative knowledge, whereas no significant differences between the two groups were observed prior to the intervention. The educational implications of these findings are discussed.

INTRODUCTION

In recent decades, metacognition has become a central educational goal in mathematics education worldwide, given its strong connection to problem solving (Lee, Yeo, & Hong, 2014) as well as its crucial role in later success in mathematics (Desoete, & De Craene, 2019). Metacognition, considered as "thinking about thinking" (Anderson, 2002), enables learners to monitor and regulate their performance (Flavell, 1979). It comprises three main facets: metacognitive knowledge, metacognitive experiences and metacognitive skills (Kim, Park, Moore, & Varma, 2013). Metacognitive knowledge, also referred to as knowledge of cognition, is defined as the knowledge, beliefs or perceptions individuals hold about their own cognitive functioning. It involves three types of knowledge: declarative, procedural and conditional (or contextual) knowledge. Declarative knowledge refers to "knowing what", including awareness of what one knows and does not know, what one needs to know, and one's strengths and weaknesses as a learner. Thus, declarative knowledge relies on reflection and, therefore, can be changeable and fallible. Cornoldi (1998) claimed that declarative knowledge underrelies a further categorization of knowledge, which he termed "connotative knowledge", as it involves making meaningful connections between pieces of information.

Declarative knowledge, like all kinds of metacognitive knowledge, is influenced by person, task and strategy variables; in other words, it is related to components that are essential for the learning process (Ellis, Denton, & Bond, 2014). The person variable includes both intra-individual and inter-individual differences, that is, beliefs about oneself as a learner and beliefs about others in comparison to oneself, respectively. The task variable refers to characteristics of a task, including its complexity, context and

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demands. Awareness of these characteristics facilitates task completion and success. The strategy variable involves knowledge about the selection, application, modification, revision, and transferability of various strategies across different problem-solving situations (Tarricone, 2011).

In view of the long-standing recognition of problem-solving competencies in mathematics education research, a variety of instructional models have been developed to foster students' metacognition during problem solving. For example, Mevarech and Kramarski (1997) proposed the IMPROVE method to support students in activating and developing metacognitive processes throughout four types of self-addressed metacognitive questions: comprehension, connection, strategic and reflection questions. Hohn and Frey (2002) developed the SOLVED training program to help elementary students effectively engage in metacognitive activities during problem solving. De Corte, Verschaffel and Masui (2004) introduced the instructional method CLIA as a framework for a learning environment that promotes metacognitive skills. Yimer and Ellerton (2006) emphasised the shift from procedural doing to metacognitive thinking and identified a five-phase model of problem solving. Similarly, Lee, Yeo and Hong (2014) developed a metacognitive scheme focusing on the understanding and planning stages of Polya's four-stage problem-solving process. Empirical implementations of metacognitive instructional programs revealed positive outcomes for students. Students have been found to enhance their metacognitive knowledge and skills, achieve higher levels of problem-solving success, improve the ways in which they initiate the problem-solving process, and make more effective decisions regarding strategies selection and use.

Researchers have emphasized the importance of metacognitive models, highlighting that without metacognitive skills students cannot effectively use their knowledge, as problem solving involves a purposeful interplay between cognition and metacognition (Wilson, & Clarke, 2004; Vissariou, & Desli, 2019). When assessing metacognition, "off-line" measures are often used, with self-report questionnaires being the most common ones. These measures, which include statements assessing metacognitive knowledge and metacognitive skills, are considered easy to administer, time-efficient, and valid and reliable tools (Gascoine, Higgins, & Wall, 2017). However, research on metacognitive knowledge has predominantly focused on secondary school students.

Given that metacognitive training in mathematical problem solving improves students' cognitive and metacognitive performance and recognising that metacognitive knowledge is equally important as the other main facets of metacognition, the present study aimed to examine the impact of metacognitive instruction on third graders' declarative metacognitive knowledge in non-routine problem solving. The research questions were as follows: a) to what extend does metacognitive instruction improve third graders' declarative metacognitive knowledge? b) how is declarative metacognitive knowledge associated with school performance?

METHOD

Participants

Thirty-eight Year 3 children (20 male) from two state primary schools in Greece participated in the study. The children, whose mean age was 8 years and 3 months at the start of the study, varied in both socioeconomic status and school performance. Based on their mathematics performance, as assessed by their teachers, the students were classified into three sub-groups: low, average, and high achievers. Participants from one school received metacognitive instruction for four months (experimental group), whereas participants from the other school did not (control group).

Research design and data collection instrument

For the purposes of this study, a questionnaire-based quasi-experimental design was employed. All participants completed a self-report questionnaire adapted from the instruments developed by Sperling, Howard, Miller and Murphy (2002) and Yildiz, Akpinar, Tatar and Ergin (2009), which aimed to assess children's declarative metacognitive knowledge. The questionnaire consisted of 16 statements addressing a range of potential metacognitive actions related to declarative knowledge, both before and after the problem-solving process (Figure 1).

Figure1: Declarative knowledge statements

1. I know when I understand a problem.
2. I know what the teacher expects me to do when I solve a problem.
3. In a problem, I know which methods I will use to solve it.
4. When I read a problem, I know if it's easy or difficult for me.
5. I know what I need to do before I start solving a problem.
6. I know the factors that influence my success or failure in solving a problem.
7. I know what information I need to solve a problem.
8. I think, 'Do I understand what the problem is asking me?'
9. I read the problem as many times as necessary until I fully understand it.
10. I know what information I don't need in a problem.
11. I get confused and cannot decide what to do to solve a problem.
12. I solve a problem correctly when I solve it with someone else.
13. I know what is difficult for me when I do not solve a problem correctly.
14. To solve a problem correctly, it is important to know how to perform operations.
15. I solve a problem correctly when I know the solution methods.
16. When I read a problem, I think, 'I can't solve it.'

Children were asked to respond to the statements using a five-point Likert scale, ranging from 1 ("never") to 5 ("always") based on their approach to solving mathematics problems. The content validity and reliability of the questionnaire were

evaluated in a pilot study, after which adjustments were made to ensure the items were appropriate for the age and cognitive level of the participants.

Procedure

At the beginning of the school year, all participants were asked to complete the questionnaire individually in their classroom (pretest) during a single teaching session (45 minutes). Subsequently, the experimental group participated in a four-month intervention based on a new "multifunctional" model of metacognitive instruction, which extends the STARtUP metacognitive teaching scheme proposed by Lee, Yeo and Hong (2014). This model focuses on the initial stages of the problem-solving process (Understanding and Planning) and is particularly suitable for rich mathematical tasks, such as non-routine problems. Each week, students in the experimental group engaged in solving two mathematical tasks accompanied by written responses to metacognitive questions (e.g., "What information is given?", "Which strategy might be needed to solve the problem?", "What am I supposed to find?", etc.). Worksheets were collected after each task, and students received feedback on their solutions. Over the course of the intervention, students were presented with a total of 32 non-routine problems. The control group received traditional instruction, following the mathematics textbooks prescribed by the Greek Ministry of Education. After the intervention, both groups completed a post-test identical to the pretest.

RESULTS

Declarative knowledge

To examine whether the experimental group differed from the control group in their responses to statements of declarative knowledge prior to instruction, an independent samples t-test was conducted. The analysis revealed no statistically significant differences, $t(36)=1{,}83$, $p=.075$; students in both groups responded similarly (experimental group: $M=2{,}99$, $SD=,59$, control group: $M=2{,}69$, $SD=,36$), with most of responses falling between "rarely" and "sometimes". In contrast, the post-instruction measurement revealed statistically significant differences, $t(36)=5{,}96$, $p<.001$, with experimental group students responding more positively ($M=3{,}51$, $SD=,58$) compared to control group students ($M=2{,}55$, $SD=,39$). Their responses were predominantly between "sometimes" and "often". Notably, the highest mean scores were observed for statements 9 and 12: most experimental group students stated that they often or always *read a problem as many times as necessary until fully understood* and *solve problems correctly when collaborating with someone else* ($M=4{,}47$ and $M=4{,}32$, respectively). These findings indicate that understanding a problem thoroughly and engaging in collaborative problem solving were particularly important for participants in the experimental group. For the control group, the lowest mean scores were observed for statements 3, 5, 6 and 8. Most students stated that they rarely *knew which method to use for solving a problem* or *how to initiate the problem-solving* process ($M=1{,}79$).

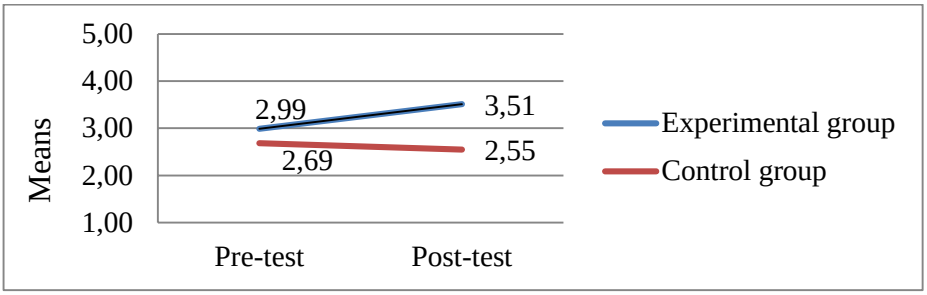
Furthermore, they almost never *understood the goal of a problem* (*M*=1,79) and rarely *recognized the factors that influence their success or failure in problem solving* (*M*=1,37). Table 1 shows these findings.

Table 1: Means (and SDs) of declarative knowledge items before and after instruction

	Experimental group		Control group	
Statements	Mean (Before)	Mean (after)	Mean (before)	Mean (after)
1	3.11 (.94)	3.42 (1.35)	3.16 (1.01)	2.79 (.79)
2	2.74 (1.24)	3.47** (1.17)	2.53 (.61)	2.53 (.61)
3	2.16 (.9)	3.37*** (1.16)	1.89 (.57)	1.79 (.63)
4	3.58 (1.22)	4.00 (1.05)	4.74 (.45)	4.26 (.65)
5	3.32 (1.1)	3.21*** (1.03)	2.00 (.74)	1.79 (.63)
6	2.47 (.96)	3.05*** (.85)	1.84 (.83)	1.79 (.79)
7	3.16 (1.07)	3.95*** (.85)	2.16 (.69)	2.16 (.60)
8	1.84 (1.01)	3.37*** (.89)	2.21 (.71)	1.37 (.49)
9	3.74 (1.1)	4.47*** (.84)	2.53 (.96)	2.16 (.89)
10	3.05 (1.08)	3.16*** (.89)	2.05 (.91)	2.05 (.78)
11	3.00 (1.1)	3.21* (.98)	3.95 (.97)	3.89 (.74)
12	3.68 (1.53)	4.32 (.82)	4.47 (.61)	4.58 (.61)
13	2.95 (1.13)	3.16** (1.17)	2.00 (.82)	2.05 (.78)
14	3.68 (1.34)	3.95 (.97)	4.37 (.77)	3.84 (.89)
15	2.74 (1.37)	3.89 (1.1)	3.79 (.76)	3.79 (.79)
16	3.37 (1.26)	3.37* (1.46)	4.42 (.69)	4.21 (.71)

*p<.05, **p<.01, ***p<.001

Figure 2: Children's declarative knowledge before and after the instruction



To examine the effectiveness of the teaching method, children's responses were subjected to a mixed-model ANOVA, with instruction (2: experimental and control group) as the between-subjects factor and time of testing (2: before and after the instruction) as the within-subjects factor. The interaction instruction by time of testing was significant, $F_{(1,\ 36)}$=18.533, p<.001, η_p^2=.340, indicating that the change in children's responses overtime differed between the experimental and control groups. More specifically, metacognitive instruction led to an improvement in the experimental group's declarative knowledge, whereas traditional instruction did not produce any improvement in the control group. This interaction is illustrated in Fig. 2.

Declarative knowledge and school performance

To examine whether children's responses varied according to their school performance, an Anova was conducted. Statistically significant differences were observed for the total sample, $F_{(2,35)}$=11.089, p<.001. Tukey post-hoc tests showed that these differences were due to students with high performance, who showed significantly higher mean scores than those with average (p<.01) and low (p<.001) performance. When the analysis was conducted separately for each group, the original results were confirmed. In the experimental group, statistically significant differences were observed, $F_{(2,16)}$=9.358, p<.01, with high-performing students scoring significantly higher means than those with average (p<.05) and low (p<.01) performance. Similarly, in the control group, statistically significant differences were found, $F_{(2,16)}$=13.036, p<.001. Tukey post-hoc tests showed that high-performing students again scored significantly higher than their average- (p<.01) and low-performing peers (p<.001). These findings are presented in Table 2.

Table 2: Comparisons of declarative knowledge by school performance

	School performance		Total Sample	Experimental Group	Control Group
Tukey HSD	Low	Average	.308	.352	.328
		High	.000	.002	.000
	Average	Low	.308	.352	.328
		High	.005	.023	.004
	High	Low	.000	.002	.000
		Average	.005	.023	.004

DISCUSSION

The present study aimed to examine the effectiveness of metacognitive instruction was on third graders' declarative knowledge during mathematical problem solving. The intervention appeared promising, as the results revealed two main findings. First, although children's declarative knowledge in both groups was approximately equal at pretest, children in the experimental group showed significant improvement following

the metacognitive instruction, whereas traditional instruction in the control group did not produce any notable change. This finding aligns with previous research suggesting that metacognitive skills must be explicitly taught to develop (Desoete, & De Craene, 2019; Schunk, 2008). In addition, children in the experimental group not only demonstrated higher levels of declarative knowledge after the intervention but were also able to initiate the problem-solving process more effectively. For instance, they reported being able to understand the problem and its goal, select appropriate strategies, comprehend the problem context, etc. These findings support earlier studies indicating that elementary students often struggle with initiating the problem-solving process (e.g., Lee, Yeo, & Hong, 2014). Second, students' level of declarative knowledge was found to be associated with their school mathematics performance, with high-performing students benefitting more from the instruction than their average- or low-performing peers. This effect may be related to the use of non-routine problems, which favour divergent thinking and may have allowed high achievers to profit more from the intervention. Nevertheless, the fact that all students, regardless of performance level, demonstrated substantial improvements in declarative knowledge and notable increases in their problem-solving statements suggests that metacognitive instruction can also be effective for children with lower mathematics performance, consistent with previous research (e.g., Cornoldi, Carretti, Drusi, & Tencati, 2015).

These results should be interpreted with caution, as a key limitation of the present study is the small sample size. Future research with a larger sample could provide more conclusive evidence. Additionally, from a methodological perspective, it is important to note that a self-report questionnaire as the one used in the present study may provide only limited insight into children's declarative metacognitive knowledge. As Desoete (2008) notes, "how you test is what you get" (p. 204), suggesting that multi-method approaches, such as on-line procedures, could offer additional valuable information regarding third graders' declarative knowledge. Last, it is likely that children in the control group were not exposed to non-routine problems and were therefore not expected to show improvements in declarative knowledge. Consequently, examining changes in children's declarative knowledge during routine problem-solving tasks could provide a more comprehensive understanding of how such knowledge develops.

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