FROM MANIPULATION TO CONSTRUCTION: THE ROLE OF SEMIOTIC CONVERSIONS IN BRIDGING THE GAP BETWEEN THE DRAWING AND THE FIGURE

Eleonora Faggiano and Roberto Capone

University of Bari Aldo Moro, Italy

To address the persistent gap between the drawing (perceptual) and the figure (theoretical) in geometry learning, we examine how a four-phase sequence, implemented with 20 undergraduates, based on the synergy between manipulative and digital artefacts, supports robust construction in Dynamic Geometry Environments. We integrate Duval's semiotic registers and the Instrumental Approach to investigate how an intermediate writing instructions task triggers semiotic conversions from perceptual to verbal register and impacted on instrumental genesis: students who identified functional dependencies developed robust schemes of use that withstand the drag-test, while perception-based approaches led to "soft" constructions, confirming the connection and underscoring the teacher's role.

INTRODUCTION

Nowadays, there is a consensus that reasoning and proof should be a natural ongoing part of classroom discussions (Mariotti, 2006). Despite the central importance of geometric reasoning in the mathematics curriculum, students frequently face difficulties when moving from visuo-spatial perception to the formal application of geometric properties. Alongside the introduction of powerful exploration tools (Laborde, 2000; Santos-Trigo & Espinosa-Perez, 2002) that opened new frontiers, linking informal argumentation with formal proof, Dynamic Geometry Environments (DGEs) have brought complex new cognitive demands to the forefront (Healy & Hoyles, 2002; Strässer, 2002). Although dynamic geometry has the potential to encourage both exploration and proof, as underlined also by Mariotti (2006) using Laborde's words, a DGE alone, "without an adequately organized milieu would not prompt the need of proof" (p. 194). In a DGE, a construction is only valid if it is robust, meaning it must withstand the drag-test by maintaining its structural properties under manipulation (Laborde, 2005; Lopez-Real & Leung, 2006). The drag-mode has been seen as a mediator between the concepts of "drawing", perceived as a purely visual entity, and "figure", defined by theoretical properties and relations of dependency (Hölzl, 1996). However, research shows that students often produce "soft" constructions that rely on visual coincidence rather than functional dependency, highlighting a gap where students remain "stuck" between the drawing and the figure, (Jones, 2000).

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M.S. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: Research Reports*, pp. 419-426. PME.

To bridge this gap, the present study investigates the instrumental orchestration (Trouche, 2004) of a teaching activity based on the synergy between artifacts (Faggiano, Montone & Mariotti, 2016) of different natures: tangible (a physical tangram), static (static drawings and written instructions), and dynamic (GeoGebra). Our didactic hypothesis is that the transition between these artifacts acts as a "cognitive engine". Specifically, the intermediate task of writing construction instructions compels a semiotic conversion (Duval, 2006) from perceptual to discursive register, forcing students to shift from "looking" the drawing to "identifying" geometric properties of the figure. This explicit articulation provides the necessary foundation for instrumental genesis in the DGE, ensuring that the development of schemes of use is grounded in theoretical reasoning (Guin & Trouche, 1998; Vérrillon & Rabardel, 1995). Consequently, this research seeks to address the following questions:

RQ1: How does an intermediate task, requiring conversions between semiotic registers (from manipulative-perceptual to verbal), support students in making geometric properties explicit, thus facilitating the transition from the drawing to the figure?

RQ2: How does the process of making these properties explicit influence the development of the schemes of use necessary for creating robust constructions in a Dynamic Geometry Environment?

To answer these questions, we adopted a qualitative approach analysing data coming from an undergraduate mathematics course. The methodology relies on the triangulation of multimodal data, including audio-video recordings of interactions, screen captures of GeoGebra work, and students' written productions (drawings and instructions). This allows us to reconstruct the students' cognitive journey and figure out how the identified properties correlate with their construction strategies in the dynamic environment.

THEORETICAL FRAMEWORK

To analyse the cognitive transition from empirical perception to theoretical reasoning in geometry, this study integrates Raymond Duval's theory of semiotic registers with the Instrumental Approach.

According to Duval (2006), mathematical objects are accessible only through semiotic representations, such as natural language or geometric figures. He distinguishes between two types of transformations: treatments, which occur within a single register (e.g. a logical deduction within a discursive register), and conversions, which involve changing the register while maintaining the same denoted object (e.g. translating a physical configuration into a written set of instructions). Duval identifies conversion as the deciding factor for learning. In geometry, a significant cognitive gap arises when students fail to coordinate the visual register (the drawing, perceived as a physical entity) with the discursive register (the figure, defined by theoretical properties). Without this coordination, students remain at a perceptual level, confusing the representation with the object itself.

To understand how students interact with Dynamic Geometry Environments (DGEs) like GeoGebra, we refer to the Instrumental Approach (Vérrillon & Rabardel, 1995; Guin & Trouche, 1998). This theory differentiates between the "artefact" (the software tool in itself) and the "instrument" (a psychological construct developed by the user). This evolution, or "instrumental genesis", occurs as learners develop "schemes of use", organized patterns of thought and action, to solve specific geometric tasks. In a DGE, instrumental genesis is evidenced by the production of robust constructions that withstand the drag-test (Lopez-Real & Leung, 2006). Dragging serves as a tool for semiotic mediation (Mariotti, 2002): motion dependency on the screen must reflect the underlying logical dependency of the geometric properties. If students lack a theoretical grasp of these properties, they produce "soft" constructions based only on visual appearance, which fail to preserve their structure under manipulation.

Finally, a central pillar of this research is the synergy between artefacts (Faggiano, Montone & Mariotti, 2018). We hypothesize that the semiotic potential of a tangible artefact (e.g. a physical tangram) can be enhanced through an instrumental orchestration that includes a digital artefact (GeoGebra) but also an intermediate verbalization task. Writing construction instructions acts as a "cognitive engine", forcing students to make geometric properties explicit. This transition ensures that the subsequent instrumental genesis in GeoGebra is grounded in theoretical definitions rather than trial-and-error, transforming the digital environment from an opaque artefact into a transparent instrument for reasoning.

METHODS

This study employs a qualitative research design to explore the cognitive and instrumental processes of students as they navigate a multi-layered geometric task. The methodology is designed to capture the triangulation of multimodal data, allowing for a reconstruction of the students' "cognitive journey" from physical manipulation to robust digital construction, to formal proof. This paper mainly focuses on the role of verbal representation in the transition from the physical manipulation to the robust construction, bridging the gap between the drawing and the figure.

The research was conducted within an introductory mathematics education course for 20 undergraduate mathematics students. While all participants held the foundational Euclidean knowledge expected at the university level, their proficiency with DGEs was intentionally heterogeneous: some were familiar with GeoGebra from high school, while others were novices. This diversity provided a rich field for observing the instrumental genesis process across varying levels of technical familiarity.

The four-phase sequence

The study's core is an instrumental orchestration designed to exploit the potential of a synergy between artifacts of different natures in highlighting the difference between a drawing and a figure and bridging the cognitive gap between them. The sequence

focuses on a proof without word of the Pythagorean Theorem and is structured into four distinct cognitive phases.

In the first phase students manipulated a tangible artifact, a physical wooden "Pythagorean tangram". The goal was to foster sensory-motor exploration, requiring them to arrange a set of puzzle pieces to cover squares built on the hypotenuse and the catheti of a right triangle. In the second phase, students were required to reproduce a static drawing of the tangram and, crucially, to write verbal construction instructions for it. This critical intermediate task was designed to trigger a semiotic conversion from the perceptual to the discursive register, forcing students to shift from visual recognition to the explicit definition of geometric properties. In the third phase, students transitioned to GeoGebra to create a robust construction of the tangram proof. The requirement was that the construction must withstand the drag-test, meaning it must be governed by the theoretical properties previously identified rather than mere visual alignment. A final discussion, orchestrated by the teacher in the last phase addressed the obstacles encountered during the instrumental genesis and finally promoted a reflection on the differences between the "drawing" and the "figure".

Data collection and data analysis

To respond to the research questions, we collected multimodal data, including audio-video recordings of interactions, screen captures of students' GeoGebra workflows, written productions (drawings and instructions), and researcher field notes.

To answer to RQ1, we analysed the evolution of students' productions, to pinpoint the key moments of semiotic conversions. Specifically, we looked for the transition where informal descriptions evolved into formal definitions. To address RQ2, we correlated the geometric properties articulated in the second phase with the schemes of use developed in GeoGebra. By analysing screen recordings, we identified whether students used "soft" strategies based on visual appearance or robust strategies that reflect an advanced state of instrumentation. The drag-test served as the ultimate indicator of whether the student had successfully transformed the software into a transparent instrument for reasoning.

FINDINGS

The analysis of multimodal data suggests a clear cognitive trajectory scaffolded by the synergy between artefacts. The results are organized according to the two research questions, tracing the students' evolution from perceptual manipulation to robust digital construction.

From perceptual recognition to verbal description (RQ1)

The initial interaction with the tangible tangram was dominated by the manipulative-perceptual register. Students' verbalizations were primarily indexical and informal, focusing on physical actions such as "this piece fits here" or "turn it around". At this

stage, geometric relations remained implicit, embedded in the sensory-motor experience of the tangram.

In the second phase, the requirement to produce written instructions to replicate the tangram's dissections of the three squares, fostered a semiotic conversion from the manipulative-perceptual register to the verbal register. To make their construction replicable, students were forced to transition from looking at the tangram to defining the necessary geometrical relationship to reproduce such a tangram. Data shows that this conversion compelled students to identify and name geometric properties that were previously only perceived. For instance, an angle that was visually identified as a "right angle" was re-articulated as the result of the construction of a "perpendicular line", and the coincidence of overlapping edges of the puzzle pieces was re-defined through the construction of congruent segments. This explicit articulation resulted by the conversion from the manipulative-perceptual register to the verbal register, provided the necessary theoretical foundation, effectively bridging the gap between the drawing and the figure before entering the digital environment.

From explicit properties to robust schemes of use (RQ2)

In the third phase with GeoGebra, data shows that the quality of the semiotic conversion performed in the second phase significantly affected the students' instrumental genesis.

Groups of students that successfully articulated geometric properties in the writing of the instructions approached the construction in GeoGebra with a deliberate plan. These students developed robust schemes of use by consciously selecting tools that matched their instructions. For example, as the following excerpt shows, to report a specific length, they used the "compass" tool rather than visual placement and identified vertices through the "intersection" of objects.

Student 1: We must find the point. That is, draw the compass from here to here. And the circumference intersects the square at the point we need.

Student 2: That's right, now hide the circumference... and we must make the segment. From here to here.

This kind of translations of properties into instrumented actions, schemes of use, allowed them to produce robust constructions that maintained their structural integrity under the drag-test. For these students, the DGE was transformed from an opaque artefact into a transparent instrument for reasoning. It is worth of note that in most of the cases successful groups of students were not familiar with DGEs and particularly with the notion of robust constructions.

Conversely, students who remained at a perceptual level in the second phase struggled to use the artefact and develop the necessary schemes of use. Lacking explicit properties to guide them, they treated the DGE as a malleable "sketchpad". As in the case of the following excerpt, their constructions were based on visual coincidence (e.g., placing points or segments by eye so they "looked right"). Consequently, their

constructions were “soft” and invariably failed the drag-test, leading to frustration and demonstrating an incomplete instrumental genesis.

Student 3: The vertex must coincide with that of the other segment... but you must draw first one of the two...

Student 4: Yes, but then I get closer. Isn’t there a tool that says, “it has to match”?

These results illustrate that the synergy between artefacts can support students in bridging the gap between the drawing and the figure, however the instrumental orchestration of the activity remains crucial. In the last phase, indeed, the collective discussion proved to be essential for students with stalled instrumentation. By making the schemes of use an object of shared reflection, the teacher scaffolded the completion of the genesis, allowing struggling students to re-interpret their “soft” constructions through the lens of formal geometric functional dependencies.

To summarise, data confirm that semiotic conversion (fostered by the writing of the instructions) is not just a procedural task, but the moment when geometry becomes “theoretical” for the student. This cognitive “bridge” is what allows students to move from using the mouse as a brush (soft constructions) to using the DGE as a logical system (robust constructions), from drawings to figures.

CONCLUSIONS AND FUTURE WORKS

This study investigated how an instrumental orchestration, based on the synergy between artifacts, supports the transition from visuo-spatial perception to formal geometric reasoning. Our findings offer an early nuanced understanding of the cognitive and instrumental processes involved in bridging the well-documented gap between the drawing (as a perceptual entity) and the figure (as a theoretical object).

The analysis confirms that the intermediate task between the physical manipulation and the work within the Dynamic Geometry Environment, requiring the production of written instructions, acted as a “cognitive engine” for learning. According to Duval, conversion between registers is the crucial factor for conceptual understanding. In our sequence, the transition from the tangible/perceptual register (the tangible tangram) to the verbal/discursive register (written instructions) forced students to overcome the “visuo-spatial field” and engage with the “theoretical field”. By requiring students to define construction rules, the task compelled them to identify and name geometric properties (e.g., perpendicularity, congruence) and functional dependencies that had remained implicit during physical manipulation.

When successful, the process of making properties explicit allowed the students to dissociate the mathematical object from its specific physical representation and directly determined the success of the instrumental genesis in the DGE. Conversely, for students whose conversion was incomplete, the instrumentation process was “stalled”. Their reliance on visual coincidence led to constructions that failed the drag-test. This failure seems to confirm the close connection between the ability to perform a semiotic conversion and the development of robust schemes of use. Moreover, it highlights that

a DGE alone, without an organized milieu that scaffolds the conversion of registers, may remain an opaque artefact that fails to bridge the gap between the drawing and the figure and to prompt the need for proof. This consideration also brings to the fore the teacher's role. In the final collective discussion, the teacher acted as a cultural mediator attempting to guide students towards the theoretical perspective necessary for a systematic view of geometry. In particular, the teacher's actions served as a critical scaffold for students with stalled instrumentation. By transforming individual semiotic conversions from the perceptual register to the verbal register into an object of shared reflection, the teacher's orchestration allowed students to reinterpret their digital failures through the lens of mathematical dependencies (Faggiano, 2022).

Our study contributes to the literature by providing a concrete model of synergy between artifacts that addresses a cognitive gap in geometry education. While many studies diagnose the difficulty of moving from drawing to figure, we demonstrate that non-digital, intermediate tasks involving semiotic conversion are crucial for grounded instrumental genesis. In line with the notion of Didactic Cycles of the Theory of Semiotic Mediation (Bartolini Bussi & Mariotti, 2008) which provides the theoretical foundation for the synergy between artefacts (Faggiano, Montone & Mariotti, 2018), future research could investigate if a shared reflection developed before students engage with the DGE, can more effectively promote the semiotic conversions from the perceptual register to the verbal register, ensuring that the DGE would finally become for all a transparent and powerful instruments for reasoning.

Acknowledgments

Authors were partially supported by the Italian Research Group GNSAGA-INDAM.

References

Bartolini Bussi M. G. & Mariotti M. A. (2008). Semiotic Mediation in the Mathematics Classroom: Artefacts and Signs after a Vygotskian Perspective, In L. English et al. (eds.), *Handbook of International Research in Mathematics Education* (pp. 746–783). LEA.

Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics*, *61*(2), 103–131. https://doi.org/10.1007/s10649-006-0400-z

Faggiano, E. (2022). The Semiotic Bundle as a reflective tool in pre-service mathematics teachers' education. In *Frontiers in Education*, 7, (p. 1043710), 1–12. https://doi.org/10.3389/feduc.2022.1043710

Faggiano, E., Montone, A. and Mariotti, M. A., (2016). Creating a synergy between manipulatives and virtual artefacts to conceptualize axial symmetry at primary school, In Csíkos, C., Rausch, A., & Szitányi, J. (Eds.). *Proceedings of the 40th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 235–242). Szeged, Hungary: PME

Faggiano, E., Montone, A., & Mariotti, M. A. (2018). Synergy between manipulative and digital artefacts: a teaching experiment on axial symmetry at primary school. International *Journal of Mathematical Education in Science and Technology, 49*(8), 1165–1180.

Guin, D., & Trouche, L. (1998). The complex process of converting tools into mathematical instruments: The case of calculators. *International Journal of Computers for Mathematical Learning, 3*(3), 195–227. https://doi.org/10.1023/A:1009892720043

Healy, L., & Hoyles, C. (2002). Software tools for geometrical problem solving: Potentials and pitfalls. *International Journal of Computers for Mathematical Learning, 6*(3), 235–256. https://doi.org/10.1023/A:1013305627916

Hölzl, R. (1996). How does 'dragging' affect the learning of geometry. International Journal of *Computers for Mathematical Learning 1*, 169–187 https://doi.org/10.1007/BF00571077

Jones, K. (2000). Providing a foundation for deductive reasoning: Students' interpretations when using dynamic geometry software and their evolving mathematical explanations. *Educational Studies in Mathematics, 44*(1-2), 55–85.

Laborde, C. (2000). Dynamic geometry environments as a source of rich learning contexts for the complex activity of proving. *Educational studies in Mathematics, 44*(1), 151–161.

Laborde, C. (2005). Robust and soft constructions: Two sides of the use of dynamic geometry environments. In Chu et al. (Eds) *Proceedings of the 10th Asian technology conference in mathematics* (pp. 22–35). Cheong-Ju: Korea National University of Education.

Lopez-Real, F., & Leung, A. (2006). Dragging as a conceptual tool in dynamic geometry environments. *International Journal of Mathematical Education in Science and Technology, 37*(6), 665–679. https://doi.org/10.1080/00207390600712539

Mariotti, M. A. (2002). Justifying and proving in the Cabri environment. *International Journal of Computers for Mathematical Learning, 6*(3), 257–281.

Mariotti, M. A. (2006). Proof and proving in mathematics education. In A. Gutiérrez & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education: Past, present and future* (pp. 173–204). Sense Publishers.

Santos-Trigo, M., & Espinosa-Perez, H. (2002). Searching and exploring properties of geometric configurations using dynamic software. *International Journal of Mathematical Education in Science and Technology, 33*(1), 37–50.

Strässer, R. (2002). Cabri-Geometre: Does dynamic geometry software (DGS) change geometry and its teaching and learning? *International Journal of Computers for Mathematical Learning, 6*(3), 319–333. https://doi.org/10.1023/A:1013361712895

Trouche, L. (2004). Managing the complexity of human-machine interactions in computerized learning environments: Guiding students' command process through instrumental orchestrations. *International Journal of Computers for Mathematical Learning, 9*(3), 281–307. https://doi.org/10.1007/s10758-004-3468-5

Vérrillon, P., & Rabardel, P. (1995). Cognition and artefacts: a contribution to the study of thought in relation to instrument activity. *European Journal of Psychology in Education, 10*(1), 77–101. https://doi.org/10.1007/BF03172796