ARTIFICIAL INTELLIGENCE-SUPPORTED MATHEMATICAL TASK DESIGNS BASED ON PRE-SERVICE MATHEMATICS TEACHERS' BELIEFS ABOUT THE NATURE OF MATHEMATICS

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This study examines how pre-service mathematics teachers' (PMTs') beliefs about the nature of mathematics impact their interactions with Artificial Intelligence (AI) during task design. Using a multiple case study approach, 16 PMTs were initially divided into three belief profiles (Integrated Connectionists, Balanced Relationalists, and Procedural-Hybridists). Qualitative analysis of orientation strategies and task outcomes from nine focus cases revealed that PMTs with high relational beliefs utilized AI as a critical co-designer to enhance cognitive demand; conversely, those with stronger transactional beliefs employed AI primarily as a tool for task automation and shaping. The results suggest that mathematical beliefs act as a significant filter for pedagogical action in AI-integrated lesson planning.

INTRODUCTION

Beliefs about the nature of mathematics—concerning its purpose and qualities (Ernest, 1989)- also form the basis of beliefs about teaching and learning mathematics (Dede & Karakuş, 2014). Thus, they are considered a central factor in shaping teachers' practices. Research indicates that teachers' perspectives on mathematics significantly influence how they structure learning environments and select tasks (Thompson, 1992; Ernest, 1989). These beliefs operate largely implicitly, but become visible through tangible outcomes, such as when designing tasks or explaining mathematical concepts. In this study, beliefs about the nature of mathematics, classified in various ways, are examined through four perspectives (tool, aim, progress, and function) developed in collaboration with Turkish pre-service mathematics teachers. The questionnaire used was developed based on these categories and examined the pre-service teachers' beliefs under segregated and interrelated dimensions. The segregated view emphasizes rule-following, procedural correctness, and the separation of mathematical topics, i.e., the static aspect of belief (purpose and means views). In contrast, the interrelated view emphasizes conceptual connections, meaning making, and the coherence of mathematical ideas, i.e., the dynamic aspect of belief (progress and function views) (Ernest, 1989). Importantly, these views are not mutually exclusive; teachers may hold hybrid or even internally conflicting belief systems. Recent research reveals that digital technologies, particularly AI-based tools, make these belief structures more visible in new ways (Lin et al., 2025; Sui & Chang, 2026). When teachers interact with AI

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systems to design instructional activities, they must clearly state both what mathematical content will be taught and how students are expected to engage with that content (Ruthven, 2009). This process reveals pedagogical and epistemological assumptions that are often implicit in traditional lesson planning (Trouche, 2004). Therefore, AI guidance functions not merely as a technical interaction, but as a reflective design space where teachers' beliefs about mathematics and learning are put into practice.

Previous research has highlighted that digital tools shape not only what is taught but also how mathematics is conceptualized and experienced by students (Trouche et al., 2012). In this sense, AI-assisted task generation provides a particularly revealing context in which teachers' epistemological beliefs toward mathematics become visible through the guidance they provide and the activities they design (Bernardi et al., 2025; Li et al., 2025).

From a design perspective, AI-mediated task creation is a tool that enables teachers to structure tools, tasks, and student interactions to achieve didactic goals (Silvola et al., 2025). However, unlike traditional digital tools, generative AI requires teachers to state their epistemic intentions explicitly (Cheung et al., 2025). Consequently, tensions between procedural control and conceptual clarity can manifest directly in the language of commands and the structure of AI-generated activities.

In this context, this study aims to examine how PMTs' beliefs about the nature of mathematics are reflected in the teaching activities they create using AI. It examines how different belief categories regarding the nature of mathematics are reflected in the structure of instruction, pedagogical intentions, and the nature of proposed classroom activities. In doing so, the study aims to illuminate the role of AI-mediated task design as both a mirror and a mediator of teachers' belief systems. Therefore, the following research questions were formulated:

- 1What are the belief profiles of PMTs regarding the nature of mathematics?
- How are the belief profiles of PMTs reflected in the prompt strategies they exhibit when interacting with AI for activity design?
- What kinds of differences do the mathematical tasks created by PMTs show among these belief profiles?

Based on this perspective, the present study conceptualizes AI prompts and develops mathematical activities that mediate between PMTs' beliefs about mathematics and their envisioned pedagogical practices. By analysing these products, it becomes possible to examine how different belief profiles position future mathematics teachers in relation to AI, students, and mathematical knowledge within their instructional designs.

METHOD

Research Design

This research adopts a multiple case study design (Yin, 2003), aiming to examine how PMTs' beliefs about the nature of mathematics are reflected in their interactions with AI when designing mathematics learning activities. The research focuses on pre-service teachers with diverse mathematical beliefs, and the analysis unit comprises activities developed by these teachers using AI. The study focuses on how AI-generated materials are expressed through AI prompts and the resulting activity designs, within the process of their creation and output.

Participants

The participants were 16 pre-service mathematics teachers (PMTs), all of whom were in their senior year and enrolled in a specific course that included AI tools, as part of a mathematics teacher education program at a public university. All participants had completed core coursework in mathematics education, including teaching methods and curriculum design, and were familiar with basic instructional planning.

The study was conducted within a course context in which AI tools (specifically a large language model) were introduced as a supportive resource for lesson and activity design, rather than as an authoritative source. Participants were asked to interact with the AI system to design mathematics classroom activities aligned with specific curriculum outcomes.

Data Sources

The Belief Questionnaire about the Nature of Mathematics (BQNM)

This study used a questionnaire developed by Akyıldız and Dede (2019) in collaboration with pre-service mathematics teachers to determine participants' beliefs about the nature of mathematics. The 41-item, five-point Likert-type questionnaire consists of segregated (13 items) and interrelated (28 items) belief dimensions. The internal consistency (Cronbach's alpha) for the questionnaire, segregated dimension, and interrelated dimension are 0.87, 0.74, and 0.90, respectively. These values indicate that the scale maintains internal consistency and can be used in the current study.

AI Prompts

Each pre-service teacher created prompts for the AI tool specifying the type of math task to be designed, instructional constraints, student roles, and expected learning outcomes. These prompts served as explicitly stated representations of pedagogical intent.

Designed Classroom Tasks

Tasks generated through AI interaction, edited, and finalized by the participants, formed the third data source. These works provided insights into how beliefs and guidance were implemented in concrete instructional designs.

Data Analysis

Data analysis proceeded in three stages. Firstly, participants were grouped into three belief categories based on cluster analysis (Hair et al., 2006) of the segregated and interrelated belief dimensions on the questionnaire. First, the number of clusters was determined using Ward's technique, a hierarchical clustering analysis method. Then, the K-means technique, a non-hierarchical clustering analysis method, was used to determine that the participants belonged to three different belief clusters. Participants were then asked to develop AI-supported tasks afterward.

In the second stage, a qualitative content analysis (Mayring, 2014) was applied to the interaction logs and generated instructional tasks of nine focus participants (3 per cluster). The analysis followed a three-stage coding process: Structural coding, identifying the steps of the statistical investigation process; Process coding, Categorizing the nature of the PMTs' interaction with AI. Codes included “Critical Negotiation,” “Directive Instruction,” and “Clerical Automation; Cross-Case Synthesis was conducted to compare how each cluster’s belief profile manifested in their "AI Agency," defined as the degree of pedagogical control and critical inquiry exerted over the AI output.

Data triangulation (scale scores, prompt logs, and task outcomes) and detailed descriptions of focus cases were used to ensure the reliability of the study. Independent coding was performed to reach consensus on pedagogical impact themes, thus increasing the reliability and accuracy of the qualitative findings.

FINDINGS

The findings are presented under three profiles.

Integrated Connectionists: AI as a Critical Partner

Participants in this belief profile (PMT1, PMT3, PMT8) viewed AI as a pedagogical partner that requires questioning. For example, PMT8 rejected the AI's initial ranking, stating: "*The exploration phase seems too complex for students to formulate questions. Wouldn't it be more meaningful in the deepening phase? Do you have an alternative*? ” With this prompt, PMT filtered the AI's suggestions through their own belief systems. Similarly, PMT1 emphasized student autonomy, saying, “*I do not want to limit students by giving them a list... I want to see a data set emerge naturally*.” PMT3 encouraged AI to create “imaginative and engaging scenarios” leading to a “Technology Usage Habits” task that emphasized decision-making and outlier analysis.

Interestingly, even PMTs with strong interrelated beliefs occasionally reverted to procedural patterns when confronted with “class reality.” While PMT1 advocated open-ended data collection, a suddenly restrictive instruction appeared in the given directive: *“I don't want them to get into too many different responses... I want a specific [data] set to form.”*

Balanced Relationalists: AI as a Structured Scaffold

PMTs with this belief profile (PMT2, PMT6, PMT12) used AI to provide procedural clarity while maintaining relational goals. PMT6 focused on instruction timing by intervening in the AI's flow: "*I want to remind my students how to calculate the mean, median, and mode. Could you add this reminder before the analysis phase?*" PMT12 demonstrated a similar pattern by forcing the AI to integrate multiple representations: "*Ask students to express probability using percentages, decimals, or fractions... so they always see that it is between 0 and 1.*" Their approach treats AI as a content organizer providing the link between "how" (calculation) and "what" (representation). PMT2 established a clear teacher identity (e.g., "*I am a math teacher... students should be active*") by ensuring a balanced and step-by-step implementation of the statistical process.

Procedural-Hybridists: AI as a Tool for Practicality

For PMTs with this belief, AI was a means to an end, focusing on compliance and speed. PMT16 aimed to reduce teacher effort by delegating all data analysis and graphing to AI. PMT14's interaction was even more minimalist; it simply provided the curriculum code and asked, "Can you design an assignment for this and convert it into a Word document?" The resulting output was a static, 10-item questionnaire lacking narrative context or inquiry-based structure. PMT10 provided a "checklist-style" instruction, resulting in a linear activity focused on the mechanical completion of 10 steps.

Qualitative evidence shows that as relational beliefs increase, the impact of PMTs on AI shifts from passive consumption (Procedural-Hybridists) to functional adjustment (Balanced Relationalists) and ultimately to critical co-creation (Integrated Connectionists). PMTs with higher procedural beliefs (Procedural-Hybridists) tend to use AI to overcome the cognitive challenge of task design. In comparison, PMTs with higher relational beliefs (Integrated Connectionists) use AI to enrich the cognitive demand of the learning environment.

DISCUSSION

The findings of this study demonstrate that PMTs' interaction with AI is not merely a technical process but is profoundly influenced by their pre-existing mathematical beliefs. This relationship can be discussed through three primary dimensions: Pedagogical Agency, Cognitive Demand, and the Role of AI in Mathematical Inquiry.

Pedagogical Agency and Belief-Driven Filtering

The contrast between Integrated Connectionists and Procedural-Hybridists demonstrates that mathematical beliefs act as a "cognitive filter" (Pajares, 1992) in the guidance process. Procedural-Hybridist teacher candidates, such as PMT8, exhibited high pedagogical effectiveness by questioning the AI's instructional sequence. Their relational beliefs led them to treat AI as a "Pedagogical Partner"" or "sounding board"

for complex ideas (Molenaar, 2022). In contrast, the "Instruction"-oriented guidance style of Procedural-Hybridists, such as PMT16 and PMT14, reflects viewing AI as performing operational tasks. This suggests that when PMTs perceive mathematics as a piecemeal set of procedures, they use AI to automate these pieces rather than integrate them.

Outsourcing and Augmenting Cognitive Demand

A key finding is how PMTs use AI to meet cognitive demands. We observed a trend toward “Pedagogical Outsourcing” in Procedural-Hybridists. By delegating analytical and representational stages (e.g., plotting graphs and finding central trends) to AI to “save time”, PMTs may inadvertently eliminate the inquiry-based essence of the mathematical task. Conversely, Integrated Connectionists used AI for “Cognitive Enhancement.” Instead of delegating thinking, they used AI to create rich, socio-mathematical contexts (e.g., the Canteen Committee scenario), which enhanced the cognitive demands of their future students (Warshauer, 2015).

Defining the Relationship Between AI and Belief

Traditionally, education has focused on integrating technology, pedagogy, and content. However, our results demonstrate that a new dimension—Belief-Mediated Guidance—is important in the age of AI. The ability to write effective prompts is not merely a technical skill; it is a manifestation of the teacher's belief-based mathematical vision. As seen in PMT1 and PMT12, the most effective “AI literate” teachers were those who could translate their relational beliefs into specific, critical constraints for AI.

CONCLUSION AND IMPLICATIONS

This study concludes that AI has the potential to either solidify procedural instruction or catalyze relational inquiry, depending on the teacher's belief system. In teacher training programs, teaching PMTs only how to use AI may be technical competence. Professional development should also focus on questioning and developing relational mathematical beliefs. These beliefs ultimately determine whether AI will be used to replace or expand upon mathematical thinking.

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