TENSIONS AND TASK DESIGN: A CASE IN MATHEMATICS TEACHER EDUCATION

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We present a case of an early-career mathematics teacher educator, Nadia, who brings real world contexts to her work with prospective teachers, focusing on issues of societal importance. Building on research concerning teachers' tensions, we describe tensions that the teacher educator experienced, navigating between her intention to support teachers' mathematical knowledge and students' attention being pulled away from mathematics. We illustrate Nadia's attempts to navigate tensions by amending her task design in order to support her mathematic-related learning goals.

INTRODUCTION

As part of multiple case-studies project, this report focuses on a mathematics teacher educator (MTE), Nadia (pseudonym), and her experiences of tension when introducing prospective teachers to issues of societal importance revolving around mathematics. The experiences and expertise of MTEs have been of interest to the the International Group for the Psychology of Mathematics Education Conference, as evident, for example, in the working session at on "Mathematics teacher educators' knowledge" (Beswick et al., 2014), as well as in research reporting (Barbosa, 2025; Zazkis & Mamolo, 2016). Despite this focus, we found limited research attending to MTEs experience of tension. Tension related to teaching has been described as "a challenging situation with no clear-cut solution, a situation in which competing influences or forces suggest different, often opposing courses of action" (Barwell, et al., 2016, p. 175). Tensions emerge in situations where an unanticipated event leads to uncertainty over what course of action to take next, and provide important context for understanding teachers' actions and decisions (Liljedal et al., 2015). Liljedahl et al. (2023) described cases of teachers living with tensions, trying to resolve tensions, or avoiding tensions.

While most of the research on tension focuses on the teacher, we extend the conversation by focusing on a MTE, Nadia. In particular, we interpret Nadia's attempts to avoid, resolve, and then eventually live-with tensions that emerged with the realization that her learners' attention had shifted away from the intended mathematical ideas of the task. Our research is motivated by studies that have highlighted the relevance of mathematical knowledge for influencing pedagogical actions and decisions (e.g., Zazkis & Marmur, 2018), as well as by research that has highlighted the complexities of keeping prospective teachers' attention focused on mathematics when addressing interdisciplinary contexts related to issues of societal importance (e.g., Mamolo, 2018; Mamolo, et al., 2022). Specifically, this study addresses the

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M.S. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: Research Reports*, pp. 971-978. PME.

following research questions: *What tensions are experienced by a MTE while trying to enhance the mathematical knowledge of teachers in relation to issues of societal relevance? How does the MTE navigate these tensions?*

CONTEXT AND BACKGROUND

Liljedahl et al. (2023) suggest tensions related to teaching can be internal (related to personal experiences, beliefs) or external (related to imposed curriculum, assessment norms). They can ebb and flow over time (Berry, 2007), are complex (Sparrow & Frid, 2001), unavoidable (Mason, 1988), interconnected and recurring (Cuban, 1992). Research has studied tensions experienced by teachers during mathematics-focused professional development (Widder et al., 2025), and during the implementation of a novel task in their lessons (Herbst, 2003). Tensions related to teachers' mathematical knowledge have emerged in situations where the mathematical content was found challenging (Nipper et al., 2011) or where the societal context of the task was deemed inappropriate for mathematics class (Mamolo & Pinto, 2015). Research with teachers who incorporate societal issues into mathematics has highlighted tensions related to balancing attention paid toward the mathematical ideas to be learnt and the societal context to be explored (Bartell, 2013). Research has also shown that prospective teachers can struggle to identify relevant mathematical ideas when contextualized in societal issues (Mamolo, 2018) and that attention can be pulled away from mathematics when trying to navigate interdisciplinary learning spaces (Mamolo et al., 2022).

In the context of teacher education, researchers have examined experiences of tension from the lenses of preparing prospective teachers to work with diverse student populations (Ben-Peretz & Flores, 2017), in current contexts and the unknowable future (Goodwin et al., 2023), and with respect to balancing authority and credibility with building relationships and care (Phillips & Park Rogers, 2020). Within mathematics teacher education specifically, Berry (2007) identified six binary pairs of tensions pulling teachers in opposite directions. The pair of "safety and challenge", for example, is described as a tension between a supportive learning experience and a potentially uncomfortable learning experience. Another pair, "confidence and uncertainty", describes the tension between the desires of MTEs to build teachers' confidence while also preparing them for the contingencies of teaching. While the tensions identified here related to experiences that might face teacher educators in general, we are particularly interested in tensions related to fostering mathematical knowledge in teacher education. According to Watson and Ohtani (2015), task design is at the heart of effective mathematics teaching and learning. We suggest that task design is also at the heart of effective mathematics teacher preparation, and as such, we examine tensions related to task design in mathematics teacher preparation.

THEORETICAL FRAMEWORK

Liljedahl, et al. (2007) introduced a *usage-goal framework* to help MTEs "think more explicitly about how and why we are using mathematical tasks with our students"

(p.248). The framework disaggregates mathematics from pedagogy so as to allow the examination of tasks from different perspectives; presented as a 2x2 array where the content of each cell can be read as "*The use of* ***x*** *to promote understanding of* ***Y***" (p.240), where ***x***, ***Y*** ∈ {mathematics, pedagogy}. Mamolo (2018) extended this framework to include *context*; that is, the use of ***x*** to promote understanding of ***Y***, where ***x***, ***Y*** ∈ {mathematics, pedagogy, context}, as depicted in Figure 1.

Figure 1: Usage-goal framework extended

		GOALS		
		Mathematics (M)	Pedagogy (P)	Context (C)
USAGE	mathematics (m)	**mM**	**mP**	**mC**
	pedagogy (p)	**pM**	**pP**	**pC**
	context (c)	**cM**	**cP**	**cC**

While the framework was introduced in the context of task design, we extend its applicability by using it to analyse the tensions that can emerge between the design intentions and implementation of a task. That is, we apply the framework as a lens through which to interpret what teachers are attending to while exploring a task that was intended to foster mathematical understanding. According to Mason (2003), "learning consists of shifts in the structure of attention: in what is attended to and how" (p. 9), and "attention is crucial to what can be noticed, and what can be learned" (p. 14). Thus, by applying the framework to teachers' focus of attention, we frame the tensions experienced by Nadia during her implementation, reflection, and modification of a task that intended to use a societal context to foster understanding of mathematics.

METHODOLOGY

In this section, we first introduce Nadia and her orientation toward mathematics teacher education. We then describe data sources and data analysis.

Nadia holds a PhD in Mathematics Education with several years' experience teaching middle and secondary school. Nadia described herself as a social constructivist who believes that learners feel the most engaged when they can see a concrete purpose for their learning. She wants the prospective teachers in her classes to gain experiences that are project-based, collaborative, and reflective as she believes that teacher educators need to model the practices for which they advocate. Nadia described herself as a strong believer in bringing real world problems into the mathematics classroom. She believes in exposing prospective teachers to a variety of real world contexts, for which mathematics is relevant, and emphasized the importance of helping teachers see that mathematics is relevant to more than the fields of engineering, technology, science, and finance. She described an important goal of hers in teacher education as helping teachers connect mathematics to issues of fairness and justice.

Our data included Nadia's collected written reflective accounts, teaching-artefacts she developed and shared with us, and series of semi-structured interviews. We focus our findings and analysis on one example of Nadia teaching: a semester-long project called *Math for Society*. Prospective teachers in Nadia's class were required to work in small groups to explore a societal issue of their choice for which incomplete or inaccurate mathematical knowledge could lead to inappropriate and socially detrimental conclusions. The objective of the project was for teachers to create a digital learning tool that uses mathematics to educate the public about their chosen societal issue. Data related to this project were analysed according to the extended usage-goal framework.

FINDINGS & ANALYSIS

With respect to the extended usage-goal framework, the findings reported here focus on Nadia's usage of mathematics, pedagogy, or context for the goal of enhancing prospective teachers' knowledge of mathematics (Figure 1, left column).

The use of pedagogy to support understanding of Mathematics (pM)

During Nadia's interview, she reflected on her experiences introducing societal issues to her mathematics classes.

> Nadia: Sometimes, when I start this topic in class, I can see some teachers. Their shoulders drop, they look away, I think they stop listening. It feels like, sometimes, the air is let out of the room. But that is only some teachers. Other teachers, they sit up straighter, it's like they wake up finally.

Part of Nadia's strategy for (re)directing attention toward mathematics was through her deliberate usage of pedagogical approaches to counter the resistance she described experiencing in class. One such strategy was her use of "touchpoints", where teachers shared their progress and ideas for feedback and discussion.

> Interviewer: Can you tell us more about the *Math in Society* project and why you felt the need to include what you called "touchpoints"?
>
> Nadia: Oh, yes, sure. Touchpoints are like, for me, points in the semester where we touch on the mathematics. I would meet with each group, and they would tell me about their work so far. The first touchpoint, they talked about what was the issue they chose and what math is connected -- usually, percentages, ratios, maybe some statistics. Then the second touchpoint, I ask them about examples of "mathematical misunderstandings". Like, what is the math idea that can get misunderstood by people, and what are the implications to getting the math wrong. The third touchpoint, is to talk about who is the audience, and how will they understand the mathematics.

Nadia described feeling unsatisfied with the touchpoint meetings. She found students more concerned with posing questions about project requirements and deadlines, and she struggled to maintain attention on the intended mathematics. Nadia felt it would be unfair to ignore students' questions, so the touchpoint meetings changed into Q&A sessions. In response to this, Nadia decided to use the projects as springboards for

further classroom learning, and began incorporating aspects of the projects into her other class activities, often pulling directly from the societal contexts introduced by students. We interpret this approach as yet another pedagogical strategy that Nadia employed to redirect her students' attention toward the intended mathematics.

The use of context to promote understanding of Mathematics (cM)

As mentioned, Nadia tried to leverage the project topics selected by her students as contexts for mathematical discussion in class. Nadia described the projects as "very interesting… because, you know, I didn't expect these connections".

> Nadia: For instance, the cancer project was very interesting. They focused on percentages. Like, they said the risk of colorectal cancer increases by 18% for each 50-gram portion of processed meat.... The teachers thought "18% increase" could be misleading for students – maybe they see it as 18% risk instead. But then, the example they used to show this is not correct... They never addressed the idea of 18% risk, not even to do a calculation.

The topic of percentage increase came up as a source of challenge in multiple projects. Nadia lamented that "most groups didn't do any calculations, even when showing this would explain what is going on - which was the point of the project!" In response, Nadia decided to address the topic in class discussion, which she began by sharing the information on colorectal cancer and asking the question "What does an 18% increase mean in this context?" We asked Nadia to reflect on the class discussion that ensued.

> Nadia: Well, [laughs] first thing they said is "oh, I had this-and-this for lunch… oh no". And then, if you can believe, they start talking about how much meat is in their sandwich – maybe is less than 50g and one person says then the statistic is not relevant because no one puts that much in their sandwich. And then, but what about two sandwiches? And is turkey sandwich better than ham sandwich? And what about pork chop? Is it red meat or not red meat? Like, all these things they are wondering about the food, but they are not wondering what it means to have 18% increase.

The use of mathematics to promote understanding of Mathematics (mM)

The digression of the discussion away from mathematics prompted Nadia to again adapt her strategy, this time by using her knowledge of mathematics to help redirect attention toward the mathematical content.

> Nadia: I told them, forget about meat. I asked them, first, what do we need to know to understand what it means to have 18% increase? I thought the answer would be obvious, but it took a bit of time you know. Then someone says "you need a starting number". OK, so I say, "let's say we start with 10%". Easy number, easy to see what will happen.

Nadia's described her attempt to anchor the mathematical discussion with an "easy number" as receiving a mixed reception. While some students began explore the

figures and calculations, other students began a debate on whether or not 10% "was really a number", with students arguing for and against.

> Nadia: They [the students] decided that it was not appropriate to call 10% a "starting number", although they acknowledged that 10% is the same as 0.1 and that 0.1 is a number, but in this context, 10% was a rate and should be called as such or it will confuse students.

For other students, the idea of a 10% cancer rate was so alarming that it derailed the conversation completely. These students spent class time researching cancer data to find accurate rates to share with the class. Nadia used the opportunity to steer the discussion toward making appropriate choices when picking numerical examples, where considerations should include how manageable the calculations are as well as how realistic the figure is for the context ("so as not to spread misinformation").

> Interviewer: Did you ever get back to the topic of percentage increase?
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> Nadia: No. I wanted to though. I tried to come up with a different example – like, salad dressing with low fat, 50% less fat than regular. So, I went to supermarket and took pictures of nutrition labels... Regular dressing was one with 23% fat and 8% trans-fat, and label for 50% less fat dressing says 13% fat and 5% trans-fat. So how is it 50% less? It is not social justice, but it is interesting mathematically. But, I don't know. I didn't like it as much… Anyway, what if they just start talking about ranch vs vinaigrette [laughs]? There are so many other topics to discuss…

DISCUSSION

Our case study explored two related questions: *What tensions are experienced by a MTE while trying to enhance the mathematical knowledge of teachers in relation to issues of societal relevance? How does the MTE navigate these tensions?* We applied the goal-usage framework in a novel way to analyse tensions that surfaced as a MTE navigated competing influences and then responded to the implications of her choices.

In response to the first question, we note that tensions emerged for Nadia as she made choices related to the use of pedagogy, context, and mathematics to promote understanding of mathematics. In response to our second question, we note a partial answer in task design. For instance, tension emerged when the deliberately designed touchpoint meetings turned into general Q&A sessions and Nadia navigated this tension by setting aside her intended learning goals for the touchpoint sessions and designing a follow up task for class discussion. When the class discussion still resulted in minimal mathematical engagement, Nadia attempted to prompt a computational exploration by providing an "easy number" to start with. This tweak led to further tensions, as the connection between 0.1 and 10% was not obvious to some students. The tension between choosing an "easy" number and an accurate number was also felt by Nadia, who navigated it by again tweaking the task, this time through a change in context. This change surfaced a further tension for Nadia, who felt that while the new context of nutrition was important, it was not as impactful as the original discussion.

The experience of tensions begetting further tensions resonates with research on tensions incorporating societal contexts into K-12 mathematics teaching (LeSage & Mamolo, 2023). Nadia's experiences are also in line with research by Liljedahl et al. (2023) who describe an interplay between external and internal tensions. In Nadia's situation, external tensions related to student expectations led to internal tensions regarding Nadia's intended mathematics learning goals and her personal beliefs about the inclusion of meaningful societal contexts.

CONCLUDING REMARKS

In her efforts to navigate tensions, Nadia drew respectively on the uses of pedagogy, context, and mathematics. Her story highlights the complexities and interconnection of tensions that are elicited and experienced in mathematics teacher education as MTEs feel themselves torn between advancing their mathematics-related learning goals and adapting to their students' divergent attention. This case study adds new understanding about the interplay of tensions impacting MTEs and the use of task design in anticipating, mitigating, and navigating tensions.

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