CALCULATING THE IMPOSSIBLE: STUDENT SUSPENSION OF SENSE-MAKING IN AN OVERSPECIFIED AND CONTRADICTORY GEOMETRY TASK

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Grounded in Brousseau's theory of didactical situations, this study highlights epistemic validation, often neglected in geometry compared to arithmetic. Addressing the mechanism of "suspension of sense-making," we investigate how valid calculations can mask impossible objects. We analyzed 109 Indonesian students' reasoning regarding an overspecified task containing mutually exclusive side-angle properties. Results classify participants into "Pythagorean Pragmatists" (~80%) exhibiting metacognitive blindness to fulfill the contract, "Procedural Formalists" (~15%) experiencing dissonance, and rare "Critical Whistleblowers" (<5%) capable of validation. These findings frame errors as rational adaptations to school norms, advocating for diagnostic teaching to foster epistemic agency.

INTRODUCTION

Fostering students' ability to treat mathematics as a consistent logical system remains an enduring challenge. Extensive research documents the "suspension of sense-making" (de Corte et al., 2000; Schoenfeld, 2012), where students prioritize algorithmic execution over logical consistency. We operationally define this phenomenon not merely as a deficit, but as a rational adaptation to the 'didactical contract' (Brousseau, 2002), where students filter out structural constraints to satisfy the perceived demand for a numerical answer (Carotenuto et al., 2021). Cognitively, this functions as "metacognitive blindness" (Alifiani et al., 2023), separating procedural fluency from conceptual meaning.

In the context of high school geometry, this study investigates students' tendency to reduce complex figures into mere calculation triggers. We utilize tasks characterized as 'overspecified' (providing redundant information) and 'contradictory' (containing conflicting premises, such as side lengths incompatible with angles). These tasks are designed to disrupt the standard expectation that all school problems are well-posed. By confronting students with such anomalies, we assess their "epistemic control" (Schoenfeld, 2014), the capacity to validate premises rather than execute procedures blindly.

The complexity of geometry, particularly within Fischbein's (1993) "figural concepts," means that visual validity can often mask conceptual contradictions (Potvin, 2023). Instead of resolving conflicts, students often exhibit metacognitive blindness, reflecting deep-seated beliefs that mathematics is a disconnected set of computations (Schoenfeld, 2012). This study analyzes Indonesian high school students' reasoning on

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an overspecified and contradictory geometry task. Using the didactical contract (Brousseau, 2002) as a primary analytical lens, we aim to demonstrate how the pressure to fulfill the contract actively induces metacognitive blindness, thereby suppressing the validation of geometric premises. Theoretically, this work reframes student errors as rational adaptations to school norms. Practically, the findings underscore the necessity of diagnostic teaching, advocating for the use of 'impossible' tasks not as pitfalls, but as diagnostic instruments to cultivate students' epistemic agency.

Research Questions

Guided by the theoretical framework of the didactical contract and recent cognitive perspectives on suspension of sense-making, this study addresses the following research questions:

RQ1: What response typologies do students exhibit when engaging with an overspecified geometry task with contradictory visual and numerical information, and how prevalent are they?

RQ2: How do the justifications provided by the dominant typologies reflect a rational adaptation to the didactical contract, specifically the activation of alternative sense-making over structural consistency?

RQ3: How does epistemic control manifest in students' reflective responses, distinguishing those who detect the contradiction from those who exhibit metacognitive blindness?

THEORETICAL FRAMEWORK

Brousseau's (2002) theory of didactical situations centers on the didactical contract, implicit rules governing classroom interactions. A pervasive clause dictates that teacher-assigned problems must have a unique, calculable solution using all provided data. This shapes students' epistemological beliefs, often prioritizing algorithmic execution over logical consistency (Schoenfeld, 2012). When faced with contradictory tasks like Baruk's (2016) "Captain's Age," students' illogical responses are not merely errors but a "rational adaptation" to fulfill this contract (Carotenuto et al., 2021). Cognitively, this manifests as "metacognitive blindness" (Alifiani et al., 2023), where students become inattentionally blind to anomalous data to produce a contract-compliant solution.

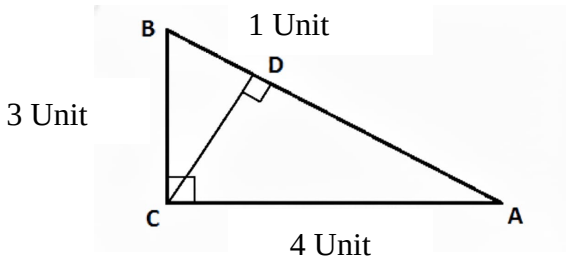
In geometry, Fischbein's (1993) theory of "figural concepts" highlights the symbiosis between figural properties (shape) and conceptual constraints (axioms). Ideally, concepts control figures. However, "pseudo-objects" (Baccaglini-frank et al., 2013), visually valid but conceptually incoherent figures, test this relationship. Under the pressure of the didactical contract, students with fragile figural concepts decouple calculation from deduction (Hadas et al., 2000), performing valid operations on impossible objects. Thus, the contract acts as a catalyst for this cognitive fracture, reinforcing a focus on answer-getting over epistemic validation.

METHODOLOGY

Task Design

The instrument was a geometry task designed to be overspecified and contradictory, adopted from As'ari et al. (2019). The problem presented a right-angled triangle ABC (implied by the Pythagorean triple 3-4-5 sides) with an internal segment $BD = 1$, creating a sub-triangle ADC, as shown in Figure 1.

Figure 1: An overspecified and contradictory geometry task



Given Data Set A (Sides): $AC = 4$, $BC = 3$. Given Data Set B (Angle): angle $BAC = 30°$. If we accept the sides ($AC = 4, BC = 3$), the angle A is fixed: $\tan A = \frac{3}{4} = 0.75$, so $A \approx 36.87°$. If we accept the angle ($A = 30°$), the side BC is fixed: $BC = AC \times \tan 30° = 4 \times \left(\frac{1}{\sqrt{3}}\right) \approx 2.31$. Thus, the problem is mathematically impossible in Euclidean geometry; the sides and angles cannot coexist as stated.

Participants and Research Context

The study involved 109 Grade 11-12 students drawn from five distinct institutions across three cities in West Java, representing a diverse mix of Senior High Schools, Vocational Schools, and Religion-affiliated Schools. All participants had prior trigonometry instruction, and informed consent was obtained. The instrument simulated a formal exam environment to activate the didactical contract. To ensure internal validity, a mandatory 1-minute observation period (image only) preceded the 20-minute execution phase, serving as a control to distinguish inattentional blindness from perceptual oversight. Construct validity was established through multi-method triangulation (Meijer et al., 2002): Planning (Q1) captured initial mental models, Execution (Q2) provided behavioral evidence of metacognitive blindness (Faradiba & Alifiani, 2020), and Reflection (Q3) assessed epistemic control through the presence of epistemic emotions (Muis et al., 2015).

Data Analysis

Employing a deductive approach grounded in the didactical contract and epistemic control, the data analysis progressed through classifying student response typologies (RQ1), examining the rationality of justifications regarding the contract (RQ2), and contrasting reflective statements to assess epistemic monitoring (RQ3). To ensure the

robustness of these categorizations, inter-rater reliability was established using Cohen's kappa (κ). This statistic was selected over simple percent agreement to rigorously account for the possibility of chance agreement among raters (McHugh, 2012). A subset of 35 randomly selected responses was independently double-coded by the first author and an external experienced mathematics teacher from West Java. The initial independent coding yielded a Cohen's κ of 0.91, which is interpreted as "almost perfect" agreement. The few remaining inconsistencies were resolved through a negotiated agreement approach (Campbell et al., 2013; O'Connor & Joffe, 2020), where disparate items were discussed to refine category definitions until 100% consensus was reached. The high degree of consistency observed suggests that the typology variables were sharply differentiated and the coding scheme was rigorously defined, thereby indicating that the collected data are accurate representations of the variables measured.

RESULTS

The analysis of student responses ($N = 109$) indicates a pervasive tendency toward algorithmic execution over logical validation. Despite the explicit visual and numerical contradictions embedded within the task, where the geometric properties of the angle conflicted with the side lengths, the analysis revealed that the vast majority of students prioritized the production of a calculable answer. Based on the coding scheme described in the methodology, the student responses were categorized into three distinct typologies: Pythagorean Pragmatists, Procedural Formalists, and Critical Whistleblowers.

The first and most prevalent category, identified as Typology 1: The Pythagorean Pragmatists (Prevalence: ~80%), is characterized by a specific form of metacognitive blindness where students systematically filtered out the contradictory angle ($30°$) to maintain procedural coherence. Upon recognizing the side lengths ($AC = 4, BC = 3$) as a standard "Pythagorean triple", these students activated a heuristic script that rendered the angular data irrelevant. In the Planning phase (Q1), responses reflected a strong "rational adaptation" to the didactical contract, operating on the assumption that integer values constitute the primary data for the solution. Student S-B5 (Vocational School, Bekasi) explicitly articulated this exclusion strategy, stating: "Information regarding the angle $BAC = 30°$ does not seem necessary... A triangle with sides 3, 4, and 5 is a right-angled triangle (Pythagorean Triple)". Similarly, Student A-S5 (Senior High School, Sukabumi) framed the task solely as an algorithmic execution: "The problem is clear, and the data is complete... Just need to be careful not to mix up the numbers". Crucially, reflection data (Q3) reveals that this exclusion was not accompanied by doubt; rather, students expressed high epistemic confidence. As asserted by Student A-S5, "I am sure," indicating that their sense-making was entirely internally consistent with school mathematics norms: identifying the schema,

executing the algorithm, and obtaining a "clean" integer result, while remaining completely blind to the geometric impossibility created by the $30°$ angle.

Unlike the Pragmatists who successfully filtered out the conflict, the second group, labeled as Typology 2: Procedural Formalists (Prevalence: ~15%), acknowledged the presence of the anomalous information ($30°$ angle) but failed to navigate the resulting structural contradiction logically. Their reasoning exhibited "inattentional blindness" to the logical implications of their calculations, often forcing incompatible methods to coexist in a syncretic manner. Strategies typically involved attempting to synthesize trigonometric rules with Pythagorean properties, regardless of the mathematical incoherence. A representative example is Student J-S4 (Senior High School, Sukabumi), who calculated the side DC using trigonometry ("Determine length of DC using sin $30°$. Result is 2") but simultaneously determined the hypotenuse AB using the Pythagorean theorem ("Determine length of AB... is 5"). Mathematically, these two operations derive from contradictory premises within the triangle's structure, yet the student proceeded to calculate the final area as 4 without detecting the conflict. Consequently, reflective responses in this group were marked by epistemic dissonance rather than confidence. As noted by Student D-S5 (Senior High School, Sukabumi), "Always unsure, tried to prove in several ways but the results remain wrong," a behavior that aligns with a "reinterpreting response" (Potvin, 2023), where students distort data or tolerate incoherence to fulfill the assessment obligation.

Finally, Typology 3: The Critical Whistleblowers (Prevalence: < 5%) comprised a small minority of participants ($n = 3$) who demonstrated the requisite epistemic control to suspend algorithmic processing and validate the task's premises. These students successfully detected that the provided numerical data (sides) and visual data (angle) were mutually exclusive. Student F-MA (Islamic Senior High School, Bandung) provided a rigorous proof by contradiction in his response: "The information seems incorrect... if we use trigonometry $\frac{BC}{AB} = \sin 30°$, the hypotenuse is 6, which contradicts the answer 5 when using Pythagoras". By identifying that the hypotenuse cannot simultaneously possess two distinct values, Student F-MA actively rejected the didactical contract's imperative to "find the answer". Similarly, Student W-B5 (Vocational School, Bekasi) displayed sophisticated reasoning by classifying the contradictory information as a distractor. He wrote: "We didn't calculate the $30°$ angle because we thought it was just a distractor (dummy)... If we calculated with the angle, the values wouldn't meet clearly". Unlike the Pragmatists who ignored the angle unconsciously, Student W-B5 consciously decided to discard it after detecting the incoherence. This explicit validation process distinguishes the Whistleblowers as the only group capable of overcoming the metacognitive blindness induced by the overspecified and contradictory task.

DISCUSSION

The overwhelming prevalence of "Pythagorean Pragmatists" (~80%) and "Procedural Formalists" (~15%) provides compelling evidence of the didactical contract's hegemony. Consistent with Carotenuto et al. (2021), this behavior represents a "rational adaptation" rather than a cognitive deficit; students prioritized the contract's implicit rule that "problems must be solvable" over structural consistency. Consequently, they filtered out the contradictory angle not due to incapacity, but to satisfy the imperative for a "clean" answer, effectively divorcing formal logic from problem-solving (Schoenfeld, 2012). This suggests that the pressure to produce a solution creates a filter where students selectively attend to data that fits the algorithmic script while discarding anomalous information that threatens contract fulfillment.

Specifically regarding the Pythagorean Pragmatists, their behavior exemplifies complete "metacognitive blindness" (Alifiani et al., 2023). Captivated by the "comfort" of integer side lengths (the familiar 3-4-5 triple) they unconsciously filtered out the contradictory $30°$ angle to maintain procedural coherence. For these students, the figure served merely as a prompt for calculation rather than a spatial object constrained by Euclidean axioms. The "comfort" of the integer calculation overpowered the conceptual constraints of the pseudo-object (Baccaglini-frank et al., 2013), allowing procedural knowledge to become completely disconnected from conceptual understanding.

The Procedural Formalists occupied a more complex, transitional cognitive state characterized by "inattentional blindness" to structural logic rather than data presence. Unlike the Pragmatists who ignored the angle entirely, these students acknowledged the anomalous ($30°$), often attempting to use trigonometry in their calculations. However, they failed to grasp the structural implications of this choice, resulting in a syncretic attempt to force incompatible methods (Pythagoras and Trigonometry) to coexist. This led to epistemic dissonance rather than resolution, where students sensed confusion ("Always unsure") but lacked the epistemic control to resolve the contradiction logically.

In contrast, the rare Critical Whistleblowers demonstrated high "epistemic control" (Schoenfeld, 2014), acting as an immune system against this blindness. Unlike their peers, these students suspended algorithmic processing to validate the premises first, successfully identifying that the visual and numerical data were mutually exclusive. Student F-MA’s proof by contradiction and W-B5’s identification of the "distractor" illustrate a sophisticated capacity for "theory change" (Potvin, 2023). They successfully breached the didactical contract, prioritizing mathematical truth over the classroom norm of finding a numerical answer.

CONCLUSION AND IMPLICATIONS

While the phenomenon of "suspension of sense-making" has been extensively documented in arithmetic word problems (de Corte et al., 2000; Schoenfeld, 2012), this study extends the theoretical landscape by uncovering its distinct mechanism within high school geometry. Our findings reveal that in the face of overspecified and contradictory geometric tasks, the "suspension" is not merely a cognitive deficit but a rational adaptation to the hegemony of the didactical contract (Brousseau, 2002; Carotenuto et al., 2021). By integrating the socio-didactical lens with Fischbein's (1993) theory of figural concepts, we demonstrate that the pressure to produce a "clean" answer induces metacognitive blindness, causing students to decouple procedural calculation from visual-conceptual constraints. Consequently, the "Impossible Triangle" task serves a dual purpose: first, as a potent diagnostic instrument to distinguish "Pythagorean Pragmatists" from the rare "Critical Whistleblowers" possessing epistemic control (Schoenfeld, 2014); and second, as a pedagogical catalyst for renegotiating the didactical contract, shifting classroom norms from blind algorithmic execution to epistemic validation (Komatsu et al., 2014).

To address this, we propose Diagnostic Teaching as a strategic means of redesigning the didactical situation (Brousseau, 2002), where the intentional use of contradictory tasks creates productive cognitive conflict that invalidates traditional contract assumptions (Potvin, 2023). This conflict initiates a shift from a 'situation of action' to a 'situation of validation,' effectively transferring the responsibility for mathematical truth to the student. Therefore, teachers must be trained to utilize these tasks not as cognitive pitfalls (Kahneman, 2011) or mere procedural hurdles (Skemp, 1976), but as instruments employing metacognitive intervention strategies to foster epistemic accountability (Alifiani et al., 2023; Bonotto, 2002). Future research could further substantiate this framework by investigating how factors such as gender, academic achievement, and epistemological beliefs influence students' tendency toward blind algorithmic execution.

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