STUDENTS' BELIEFS ABOUT OPEN MODELLING PROBLEMS

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Solving open mathematical modelling problems is challenging for students. Students' beliefs may be a reason for these difficulties. In this study, we analyzed (1) students' beliefs about open mathematical modelling problems and (2) how their beliefs are related to their solution processes. Ten ninth graders solved open mathematical modelling problems and verbalized their solution processes. Inductive qualitative content analysis revealed that students held multiple inappropriate beliefs, for example, that applying a mathematical procedure is more important than critically engaging with the real-world situation. This belief was negatively associated with the correctness of students' solutions to modelling problems. This study contributes to the theory of beliefs by revealing a belief system with respect to open modelling problems.

INTRODUCTION

The ability to solve modelling problems is an important goal of mathematics education in many countries around the world. One important characteristic of modelling problems with high relevance for students' current and future lives is the openness of the problems. To emphasize the importance of openness in modelling, we use the term "open modelling problems" in this study. Many studies have demonstrated that students encounter considerable difficulties when solving modelling problems (Niss & Blum, 2020). An important affective construct that can influence students' difficulties with modelling problems is their beliefs. Students' beliefs about problems (e.g., "every problem has exactly one correct solution") can lead to inappropriate problem-solving behavior. However, little is known about students' beliefs while they are working on open modelling problems. In this study, we investigated students' beliefs while they worked on modelling problems with open goal states. Furthermore, we examined the relationship between students' beliefs and their problem-solving behavior.

THEORY

Students' beliefs

Beliefs are "psychologically held understandings, premises, or propositions about the world that are thought to be true" (Philipp, 2007, p. 259). Students' beliefs are related to other affective constructs (e.g., motivation). For example, if students believe that the ability to solve modelling problems is not important in school mathematics, they might be less motivated when working on modelling problems in class. According to Skott (2015), the concept of beliefs is characterized by four key aspects: conviction, commitment, stability, and impact. Conviction means that a certain belief is

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M.S. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: Research Reports*, pp. 1555-1562. PME.

subjectively true for the person who holds it. Commitment implies that beliefs are value-laden, making them personally meaningful. Stability refers to temporal stability, an aspect that is also applicable to other affective constructs (e.g., interest). Beliefs are considered to be relatively stable over time and hard to change, as they result from prior experiences and can only be changed by new, meaningful experiences (Schukajlow et al., 2025; Skott, 2015). Additionally, beliefs are expected to impact individuals' behavior. For instance, if students hold the belief that memorizing all the mathematical formulas for a specific topic is sufficient preparation for a mathematics exam, they will likely do just that. Important characteristics of emotions (e.g., valence, situational specificity, and the object) are also applicable to beliefs. For example, the belief "I am good at mathematics" has a positive valence, whereas the valences of other beliefs may be negative or neutral. Beliefs are embedded in specific social-cultural situations (e.g., beliefs that are activated in mathematics classrooms) and refer to specific objects (e.g., mathematical modelling problems with open goal states).

Open modelling problems

Open problems are central to mathematics and mathematical learning. The notion of openness can refer to a variety of problem characteristics: problems that have not been solved yet, problems with multiple solution methods, or problems that allow for different interpretations (Silver, 1995). In mathematics education, an important type of open problem is the mathematical modelling problem. Mathematical modelling involves the process of solving real-world problems using mathematical means, requiring demanding transfer processes between reality and mathematics in both directions (Niss & Blum, 2020). Building on approaches from general problem-solving research (Simon, 1973), the openness of modelling problems can refer to the initial state, intermediate state, or goal state of the problem (Schukajlow et al., 2023). The initial state is open if numerical information needed to solve the problem is missing, necessitating assumptions about unknown quantities. An open intermediate state occurs when multiple mathematical models and procedures can be applied to solve the problem. This quality is a genuine attribute of modelling problems, as their intermediate state is always open. A modelling problem with an open goal state involves a vague question that must be clarified during the problem-solving process, or it might not contain a question at all. In this study, we used modelling problems with open goal states. These problems are especially important because real-life situations often involve vague questions. A typical example of a modelling problem with an open goal state is the Filling Up problem (see Figure 1). To solve a problem with an open goal state, different factors that determine the problem's goal state must be identified. In the Filling Up problem, the factors are the differences in price and time between filling up in Trier or Luxembourg. Additionally, Ms. Stein's hourly wage can be included to relate the two factors. One possible answer is: "Filling up in Luxembourg saves about €12, but it also takes an extra 40 minutes. If Ms. Stein worked during that time, she would save more money. It is not worthwhile." During the solution process, additional factors (e.g., environmental aspects) might be considered.

Figure 1: Modelling problem "Filling Up" (adapted from Niss & Blum, 2020, p. 37)

Filling Up Ms. Stein is a car mechanic who earns €30 per hour. She lives in Trier, around 20 km from the Luxembourg border. Ms. Stein often drives her VW Golf to Luxembourg to fill up with gasoline because there is a gas station just across the border. She usually drives at an average speed of 60 km per hour and uses 6 liters of fuel per 100 km. In Luxembourg, a liter of gasoline costs only €1.40, compared with €1.68 per liter at a gas station in Trier across from her garage. Ms. Stein usually needs 45 liters of gasoline to fill her tank.

Is it worthwhile for Ms. Stein to drive to Luxembourg to fill up?

Students' beliefs about open modelling problems

Many studies have discussed how students' and teachers' inappropriate beliefs can lead to unrealistic solutions when solving mathematical word problems (e.g., Reusser & Stebler, 1997). In these studies, students showed a strong tendency to neglect real-world knowledge when solving word problems in school. As students are often exposed to closed word problems in mathematics classrooms (Van Dooren et al., 2019), they may hold inappropriate beliefs about the openness of mathematical modelling problems. Previous studies have outlined the importance of students' beliefs when working on mathematical modelling problems (Kaiser & Maaß, 2007). For example, in a study involving modelling problems with an open initial state but a closed goal state, almost all of the students (N = 293) reported the belief that word problems provide all the information needed to find a solution. This problematic belief was negatively related to their performance in solving the modelling problems (Schukajlow et al., 2025). Conversely, having appropriate beliefs about modelling problems might help students when they are solving such problems. For instance, if students are convinced that it is important to consider the real-world context of modelling problems when solving them, students might think more carefully about whether their solutions are realistic. However, little is known about students' beliefs when solving modelling problems with open goal states that require them to identify the factors that should be considered during the solution process.

RESEARCH QUESTIONS

This study was conducted within the framework of the project known as Open Modelling Problems in Self-Regulated Teaching – Open Goal State (in German: Offene Modellierungsaufgaben in einem selbständigkeitsorientierten Unterricht – Offener Endzustand). This project is aimed at investigating cognitive and affective conditions for solving modelling problems with open goal states. The research questions in this study were:

RQ 1: What beliefs about mathematical modelling problems do students hold when solving modelling problems with open goal states?

RQ 2: How are students' beliefs related to their solution processes for modelling problems with open goal states?

METHOD

Participants and data collection

Ten ninth graders (four female, all aged 14 or 15) from two German middle-track secondary schools (German Realschule/Gesamtschule) voluntarily participated in this study. All participants had good or excellent grades in mathematics. To gain insights into the students' beliefs and problem-solving behavior, we conducted individual sessions with each participant involving three stages: (1) problem-solving, (2) a stimulated recall interview, and (3) a semi-structured interview. In Stage 1, each participant solved four modelling problems with open goal states. The sessions were videotaped, and participants were asked to think aloud while working on the problems.

Data analysis

We transcribed and sequenced the video material. We assigned sequences of the stimulated recall interview to the corresponding sequences from the problem-solving stage. Using qualitative content analysis (Mayring, 2014), we analyzed the transcripts for all ten participants for the modelling problems. We identified and paraphrased the sequences in the transcripts in which beliefs about open modelling problems showed up. Then, we used the paraphrases to inductively build categories of beliefs about open modelling problems. For example, we paraphrased the sequence "Because we mostly have it like this, when there are any numbers or important pieces of information that have to do with numbers, they are usually important" as "All numerical information given in the problem statement is important." This statement was coded as the belief "To solve modelling problems, all numerical information in the problem statement must be used." After analyzing students' beliefs about modelling problems, we examined the extent to which the identified beliefs were related to students' problem-solving processes. For example, regarding the aforementioned belief, we investigated whether students who expressed this belief used all numerical information when solving the modelling problems.

RESULTS

We present the results of the inductive coding for the first two modelling problems (Filling Up and Milk) (see Table 1). For research question 1, we found both appropriate and inappropriate beliefs in the problem-solving processes of six different students. We identified eight distinct beliefs and categorized them into three types of beliefs. The first category comprises beliefs about the information given in modelling problems. Two participants (Marc and Patrick) expressed the inappropriate belief that, to solve modelling problems, all the numerical information in the problem statement must be used. For example, Patrick stated, "Well, basically, I had already solved the problem in my head, but I saw that I still had one number left over." Interestingly, Marc showed the appropriate belief that modelling problems may contain irrelevant

information, but he also showed the inappropriate belief that it is intended to distract attention. Anne demonstrated the problematic belief that whether information in the problem statement is relevant to solving a modelling problem depends on how extensively the information is represented. While solving the Milk problem, she argued that she considered some of the given information because "there is a big text about CO_2 at the top, so I would have thought that this is something important."

Table 1: Results of the inductive coding of students' beliefs

Category of beliefs	Subcategories
Beliefs about the information that is given in modelling problems	To solve modelling problems, all the numerical information in the problem statement must be used.
	Modelling problems contain irrelevant information.
	Irrelevant information in modelling problems is intended to distract attention.
	Whether information in the problem statement is relevant to solving a modelling problem can be determined by how extensively the information is represented.
Beliefs about sense-making in modelling problems	Solving modelling problems requires applying a mathematical procedure to the numerical information given in the problem statement.
	In modelling problems, mathematical processing is more important than making sense of the real-world situation.
Beliefs about clarifying the goal state in modelling problems	In modelling problems, you are not allowed to make your own decisions.
	When solving modelling problems, multiple factors must be considered.

The second category includes beliefs about sense-making in modelling problems. For example, Patrick expressed the problematic belief that solving mathematical modelling problems is not about making sense of the real-world situation. When answering the question posed by the Milk problem, he mentioned: "Probably not, in terms of money, since it is probably just about the math and not the deeper meaning behind it." A similar phenomenon was observed in Jonas' stimulated recall. He said: "So I tried several things. […] I do not know exactly why. I thought maybe it would come up with a number that would somehow fit, but it did not." Consequently, Jonas believed that he needed to apply a mathematical procedure to the given numerical information and did not need to develop an in-depth understanding of the problem. The third category

comprises beliefs about clarifying the goal state of a modelling problem. Mathilda showed the inappropriate belief that making one's own decisions about the importance of factors that determine the goal state is not permitted in the solution process. While working on the Filling Up problem, she commented: "It does not say exactly whether she does this in her free time or if she could be working during that time. If it comes out of her wages, then I would have to factor that in as well. […] But how would I know that?" By contrast, Lukas showed the appropriate belief that multiple factors must be considered when solving a modelling problem.

To answer the second research question, we examined the relationship between beliefs and problem-solving behavior in the six students for whom beliefs were identified. Regarding the first category of beliefs ("beliefs about the information that is given"), we found several associations between beliefs and problem-solving processes. For example, Patrick only identified the time factor as relevant for solving the Filling Up problem because he had numerical information left over: "But the question is, what does that have to do with the hourly wage? […] Is it somehow because she needs a certain amount of time to get there […]?" In Patrick's case, his belief that all numerical information must be used led him to identify the time factor. Marc demonstrated two contrary beliefs, which were also reflected in his solution processes. At the beginning of his solution process for the Filling Up problem, Marc mentioned that the numerical information given in the problem is usually important for finding the solution. Later, however, he deemed Ms. Stein's hourly wage irrelevant to the problem, stating in the stimulated recall that problems can contain irrelevant information to distract attention. Consequently, Marc did not identify the time factor in his solution to the Filling Up problem. For the second category ("beliefs about sense-making"), Jonas tried to calculate the fuel consumption for the trip to Luxembourg to solve the Filling Up problem. Instead of determining what to calculate, he incorrectly combined different numerical values, which confirmed his previously identified belief and led to an incorrect solution. Patrick mentioned that solving modelling problems is more about mathematics rather than making sense of the situation. This belief was also reflected in his solution process. He correctly identified all relevant factors in his solution process to the Milk problem but did not interpret the result of his calculations realistically. For the third category ("beliefs about clarifying the goal state in modelling problems"), Mathilda was unsure about whether Ms. Stein wanted to fill up during or after working hours. Her belief that making one's own decisions is not allowed was reflected in her solution process, as she did not make a decision. Instead, Mathilda added to her answer that it would only be worth it to drive to Luxembourg to fill up if it was outside of her working hours. Lukas' belief that multiple factors must be considered showed up in his solution processes for both modelling problems. He considered multiple factors and the real-world situation of the problems, offering different correct answers depending on individual perspectives. Like Mathilda, Lukas did not provide a clear answer to the questions posed in the modelling problems.

DISCUSSION

The goal of this study was to identify the beliefs that school students hold when working on modelling problems with open goal states. One important finding of this study is that students may hold beliefs that seem to contradict each other (Marc), which may hinder or support their modelling abilities. Hence, future studies should explore the relationship between students' belief systems and their behavior when solving open modelling problems. The results demonstrate that inappropriate beliefs identified in previous studies on word problems (e.g., "Use all numbers that are part of the problem in order to calculate the solution") (Reusser & Stebler, 1997, p. 325) also exist when students try to solve modelling problems with open goal states. One reason for this finding may be the prevalent use of standard word problems in mathematics classrooms. Since beliefs are quite stable and result from previous experiences, the problem-solving culture in mathematics classrooms may prevent students from developing appropriate beliefs about open modelling problems. Another reason these similar beliefs are shown may be the trouble students have in distinguishing between different types of problems. Previous studies demonstrated that noticing the openness of modelling problems is challenging for students (Schukajlow et al., 2023). This difficulty indicates that students treat open modelling problems like standard word problems. Even when working on modelling problems with open goal states as in this study, one student (Patrick) mentioned that answering the vague real-world questions is about mathematics rather than about dealing critically with the real-world situation. Therefore, an important implication of this contribution is that beliefs about modelling problems are persistent and do not change simply by solving problems with vaguely formulated questions. In addition, we identified new beliefs about modelling problems with open goal states. For example, Lukas expressed the appropriate belief that multiple factors must be considered to solve modelling problems.

As observed in previous studies with other types of problems (e.g., Kaiser & Maaß, 2007; Schukajlow et al., 2025), we found several associations between the beliefs we identified and students' problem-solving behavior. One novel contribution of this study is that, for beliefs about the information that is given, though they may have seemed inappropriate at first (e.g., the belief that all numerical information given in the problem must be used), such beliefs may be appropriate for modelling problems that include all numerical information and no information that is superfluous. Patrick might not have identified the time factor as relevant for the Filling Up problem if the hourly wage had been excluded from the problem statement. At the same time, when working on the Milk problem, Patrick demonstrated the inappropriate belief that making sense of real-world situations in modelling problems is not necessary. This belief may have prevented him from engaging critically with the real-world situation. Even though causal inferences cannot be made, these two examples illustrate the crucial role that beliefs play when students solve mathematical modelling problems. Consequently, an implication of this study is that school mathematics should address the importance of realistic considerations in mathematical problem-solving. In terms of clarifying the

goal states of the problems, we observed beliefs that may have prevented students from deciding on the importance of the factors that determine the goal state, resulting in answers that included a decision-making scenario. Therefore, discussing the characteristics of modelling problems (e.g., their openness) in class could help students recognize the different demands of these problems and overcome barriers such as making their own decisions when solving such problems.

Acknowledgments: This study was financially supported by the German Research Foundation (GZs: SCHU 2629/6-1 and KR 5721/3-1).

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