WHY SOME STUDENTS STRUGGLE TO BRIDGE THROUGH 10

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Three Grade 4 students (aged 9-10) were selected for an intervention because of their problematic reliance on counting-based strategies for single-digit addition. Using a multiple baseline design, we examined their daily use of retrieval and derived-fact strategies before and after an intervention over a period of 20 days. Although one participant benefited greatly from the intervention, the other two showed only limited improvement. Using microgenetic methods to analyse data, we examined features of participants' prior performances that appeared to enable or hamper the development of derived-fact strategies. Findings provide evidence of the critical knowledge students need to be able to bridge through 10.

INTRODUCTION

In this paper, we focus on the essential knowledge students need to effectively use decomposition strategies to solve addition problems involving bridging through 10. Decomposition strategies involve decomposing (partitioning) numbers into parts to simplify computations and include the bridging-10 strategy (e.g., 7 + 5 = 7 + 3 + 2 = 10 + 2 = 12). When applying decomposition strategies, students make use of known or retrieved facts (e.g., 7 + 3 = 10) from those stored in their long-term memory to assist deriving answers to sums not yet known. Decomposition strategies form part of a class of strategies commonly referred to as Derived-fact Strategies, where answers are derived from known facts.

The term retrieval-based strategies has been used in the literature to encompass both direct retrieval and derived-fact strategies (Canobi, 2009). We know that the frequent and accurate use of retrieval-based strategies for addition in Grade 1 is one of the strongest predictors of growth in mathematics achievement through to Grade 5 (Geary, 2011). Consequently, it is critical that students acquire proficiency with retrieval-based strategies to solve addition problems early in primary school.

Theoretical perspectives (Shrager & Siegler, 1988) posit that retrieval-based strategies are strengthened via practice solving problems using strategies of choice (including counting-based strategies). Drawing on findings from an exploratory instructional intervention intended to accelerate students' adoption of decomposition strategies for addition problems, we identify and discuss critical aspects of students' associated knowledge, which will facilitate their employment of the bridge-10 strategy

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(sometimes called ‘making 10’). Further, we offer a theoretical account of why certain students fail to benefit from practice in ways that support their use of derived-fact strategies. The investigation was guided by the research question: What foundational knowledge is needed for students to effectively bridge through 10?

BACKGROUND LITERATURE AND THEORETICAL PERSPECTIVE

Young children learn foundational knowledge of numbers through counting and usually progress from simple counting-based strategies to increasingly more advanced counting-based strategies (see Table 1). For single-digit addition, initial strategies generally include Count-all Strategies (e.g., the longsum- or sum-counting strategy) and progress to Count-on Strategies, where children count on from the first operand (count-from-first strategy) or make use of the more efficient min-count strategy by counting on from the larger operand (Hopkins & Lawson, 2002). These early-learned counting-based strategies become ineffective as numbers get larger and there is a need for students to develop increased proficiency using retrieval-based strategies.

Educational standards in many countries identify a stage in early-primary school where students are expected to use retrieval-based strategies to solve single-digit addition problems—usually by around the second grade. Despite these expectations, many students from mainstream classrooms worldwide persist using counting-based strategies for solving addition problems (e.g., Graven & Venkat, 2021; Hopkins et al., 2022) even into the seventh grade (e.g., Hopkins & Bayliss, 2017).

For students to effectively transition from counting-based strategies to derived-fact strategies they must have a deep and systematic understanding of part-part-whole relations of numbers 1-10 and not simply know disconnected facts (Cheng, 2012). An understanding of part-part-whole relations involves, for example, understanding that 6 as the whole can be partitioned into parts representing 4 and 2, 3 and 3 or 1 and 5 (Kullberg & Nord, 2025). Such understanding means that students know multiple combinations of numbers to make other numbers, allowing them the flexibility to select the most efficient strategy to solve addition problems.

Recognition of the importance of students developing a deep understanding of part-part-whole relations and its role in supporting decomposition strategies has motivated numerous instructional intervention studies aimed at strengthening these foundational concepts. Collectively, findings from this body of research reveal several important insights for the current investigation. First, they demonstrate that, with appropriate instruction, young children can learn to partition numbers (Kullberg & Nord, 2025; Cheng, 2012). Second, that learning to bridge through 10 is still quite challenging for many children (Cheng, 2012; Murata, 2004).

Taking these findings into account, we also considered a second important instructional component for facilitating the effective transition to retrieval-based strategies: the provision of accurate and targeted practice (Shrager & Siegler, 1998). According to Siegler’s overlapping waves theory of strategy development (Siegler, 2005),

individuals typically have multiple strategies for solving number problems at any given time. With continued correct practice, students generally shift their strategy use toward more advanced ones, with this shift initially characterised by marked intra-variability in strategy use, until retrieval-based strategies come to dominate performance. The theory, however, offers limited explanation for the persistence of counting-based strategies and for students' difficulty in adopting more advanced strategies, such as bridging through 10, if they are mostly accurate with the strategies they use. We were interested in understanding why many students struggle to bridge through 10 even when interventions provide opportunities for correct practice.

METHODOLOGY

Participants

Participants were three students in Grade 4 (9–10 years old), selected from one metropolitan primary school in Australia. In this context, curriculum documents indicate that by Grade 2, students should be able to recall and demonstrate proficiency with addition and subtraction facts within 20. The three participants were among a group of 81 students from the same school who completed an individual interview to determine the strategies they used for solving single-digit addition. They were selected because they used counting-based strategies more frequently than their peers.

The Intervention

To support students learning to bridge through 10, an exploratory instructional intervention was designed with the intention of accelerating students' adoption of derived-fact strategies for addition. The intervention provided targeted practice that facilitated students' use of retrieval and derived-fact strategies for addition problems. To help strengthen problem-answer associations via this practice we employed a novel tool called the Keyboard. The Keyboard encourages students to visualise strategic jumps when learning to solve arithmetic problems with sums of 1-10, and to utilise the bridging-10 strategy for solving problems with sums over 10. For further details of the Keyboard's design, theoretical underpinnings and application see Hopkins et al. (2023) and Russo et al. (2025).

We employed a multiple baseline design involving four phases over 20 school days: a pre-intervention phase (4 - 6 days), an intervention phase (7 days), a post-intervention phase (5 days), and a maintenance phase (2 - 4 days). Each phase spanned consecutive school days with one exception: a week separated the last two days in the maintenance phase. All sessions were video recorded.

Participants were individually withdrawn from class to work with a research assistant (RA) for 20 minutes each day. During the pre-intervention, post-intervention and maintenance phases, participants solved the same set of 49 single-digit addition problems ($a + b$ where $2 \leq a$ & $b \leq 9$), which were randomly displayed on a computer screen. Students were instructed to use their own preferred strategies and had access to a Keyboard in the post-intervention phase but not the maintenance phase. During the

intervention, participants were introduced to the Keyboard using games and activities, with minimal explicit instruction on how it might be used to solve addition problems.

Analysis

Microgentic methods were chosen for analysis due to their capacity to capture individual student strategies in depth and to identify systematic patterns in responses, and detect shifts in strategy use of data collected on multiple occasions (Siegler & Cowley, 1991). In line with a microgenetic approach, a combined method of observation (by the RA) and participants' self-reports was used to identify strategies used on a problem-by-problem basis, in the pre- and post-intervention phases. A second researcher separately coded strategy use from the video data. Inter-rater agreement was high (94.2%) and all differences were later resolved with discussion. The scheme used to categorise strategies is outlined in Table 1.

Table 1: Scheme used to record strategies

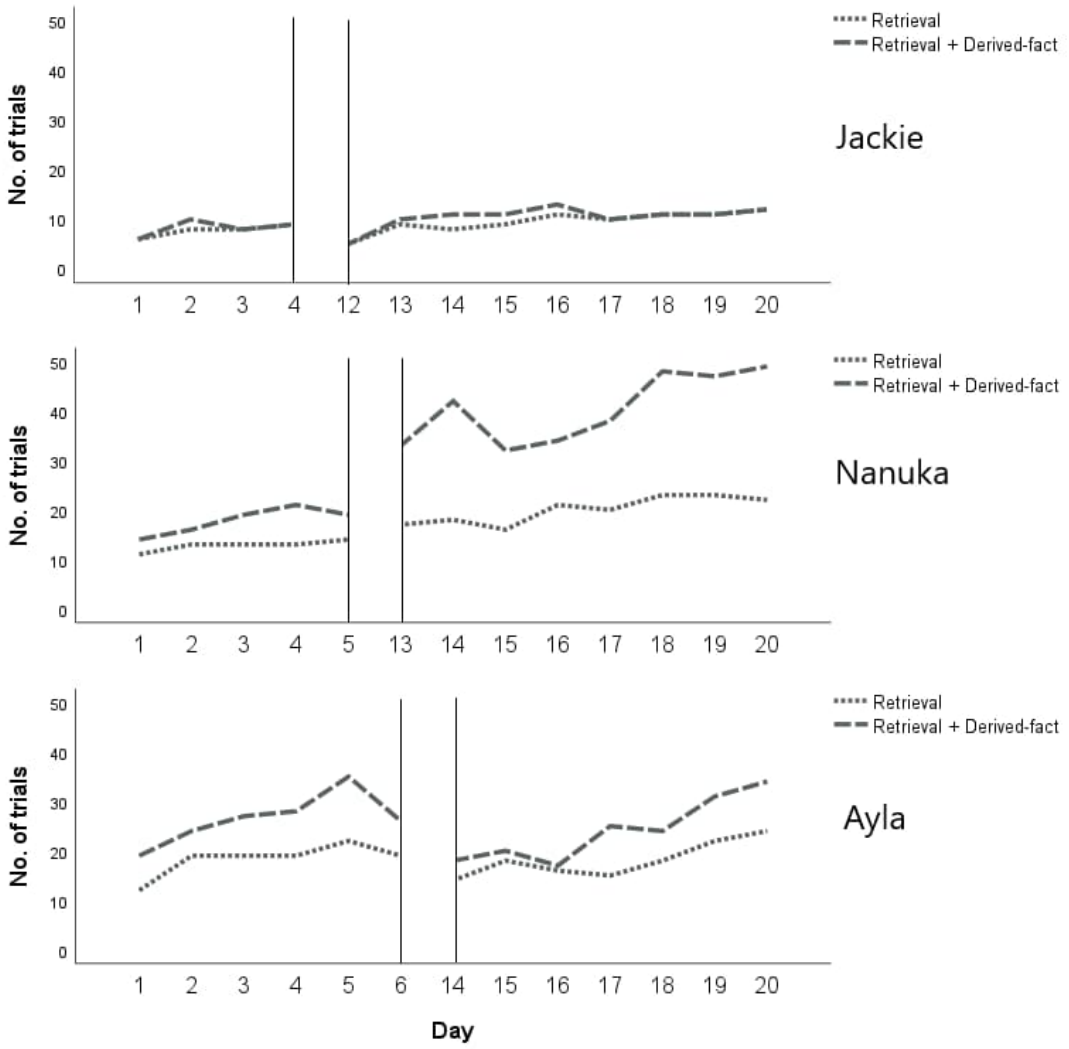
Strategy Type	Strategy Name	Example (observation + self-report)
Count-All	longsum-counting	3 + 4: counts *1, 2, 3* and *1, 2, 3, 4* then *1, 2, 3, 4, 5, 6, 7.*
	sum-counting	3 + 4: counts *1, 2, 3* and *1, 2, 3, 4* then recognises 7 (e.g., finger pattern)
Count-On	count-from-first	3 + 4: starts at 3 and counts *4, 5, 6, 7.*
	min-count	3 + 4: starts at 4 and counts *5, 6, 7.*
Derived-Fact	bridge-10	6 + 7= 6 + 4 (known) + 3
	near-doubles	6 + 7 = 6 + 6 (known) + 1
	overshoot	8 + 9 = 9 + 9 (known) - 1
	balance	6 + 9 = 5 + 10 (known)
Retrieval	-	Just knew it
Other		Not elsewhere recorded

RESULTS

The number of trials solved using retrieval and derived-fact strategies for each participant are shown in Figure 1. (A trial refers to an occasion where a problem was solved by the participant.) Participants made few errors, with no student recording errors on more than 1.3% of trials.

As shown in Figure 1, Nanuka benefitted greatly from the intervention. In the pre-intervention phase, she predominantly used a min-counting strategy to solve the problem set (54% of trials), followed by retrieval (26.1%), and derived-fact strategies (10.2%). After the intervention, during the maintenance phase, her use of retrieval increased to 46.3% and her use of a derived-fact strategy increased to 51.7%.

Figure 1: Strategies used by three participants during the pre-and post-intervention phases (with correct and incorrect trials combined). Phases are staggered due to the multiple baseline design. Two vertical lines represent the 7-day intervention phase.



Unlike Nanuka, Jackie and Ayla benefitted little from the intervention. In the pre-intervention phase, Jackie predominantly used a min-counting strategy to solve the problem set (78.1% of trials): otherwise, she used retrieval (15.8%). Rarely, did she use a derived-fact strategy (1.0%). She chose not to use the Keyboard in the post-intervention phase. Her pattern of strategy use in the post-intervention and maintenance phases was similar to that demonstrated in the pre-intervention phase.

Ayla's performance was different to the other participants in that she predominantly used a sum-counting strategy in the pre-intervention phase, a particularly inefficient counting strategy. She had somewhat ingeniously adapted this strategy for solving problems with sums greater than 10. The derived-fact strategies she used at the start of the study involved knowing double-facts (e.g., 6 + 7 = 6 + 6 + 1). She used the

Keyboard during the post-intervention phase, but when it was no longer available in the maintenance phase, she resorted back to using a sum-counting strategy and near-doubles strategy.

Using microgenetic methods we examined individual student data from the pre-intervention and intervention phases to investigate why the intervention was effective for one participant and not the other two. From this analysis, we hoped to identify factors that seemed critical for learning decomposition strategies more generally and thus move towards developing a more inclusive explanation of what hinders some students from effectively employing them, with a particular focus on the bridging-10 strategy. We identified that Jackie and Ayla were not often retrieving answers to problems with sums less than 10 (e.g., 3 + 5 = 8): retrieval was used for 23% and 48% of these trials respectively. This made it difficult for them to partition addends efficiently when using a bridging-10 strategy [e.g., 7 + 8 = (7 + 3) + 5 requires that students can partition 8 into 3 and 5]. In comparison, Nanuka solved 73% of these trials using retrieval. It is interesting to note that low retrieval of problems with sums equalling 10 (e.g., 4 + 6 and 7 + 3) appeared to be less inhibitive than low retrieval of problems with sums less than 10. In the pre-intervention phase, Jackie, Nanuka, and Ayla respectively retrieved 0%, 15%, and 46% of trials to problems with sums equalling 10. Thus, although Nanuka rarely retrieved add-to-10 facts in the pre-intervention phase, she still benefitted from the intervention and practice, such that by the end of the study, she was using retrieval-based strategies for all the single-digit problems. We concluded from this analysis that even if students understand part-part-whole knowledge (demonstrated by the use of a near doubles strategy), their progress in learning to bridge through 10 may still be impeded if they do not know (i.e., retrieve) facts associated with sums *less than* 10 – more so than if they do not know facts associated with sums *equalling* 10. For this reason, we suggest that a bridging-10 strategy is more difficult than a near-doubles strategy: bridging through 10 requires knowing sums to 10 and sums up to 10, whereas near doubles require knowing doubles and only sums up to 10 that have an operand of one.

DISCUSSION AND CONCLUSION

Studies show that young children can learn part-part-whole relations (Cheng, 2012; Kullberg & Nord, 2025; Murata, 2004) but that learning decomposition strategies is not straightforward and can be quite difficult for many students (Cheng, 2012; Murata, 2004). Adding to previous research about students' learning of decomposition strategies, findings of the current study highlight how important it is for students to know (retrieve) number facts to problems with sums that are less than 10 (e.g., 4 + 3 = 7, 3 + 5 = 8 etc) for bridging through 10, if they are to benefit from practice. For example, for calculating 8 + 7, not only is it helpful for students to know that 8 + 2 = 10, it seems even more important that they know that 2 + 5 = 7 (or 2 + ? = 7). While the importance of knowing facts up to 10 is well recognised (Geary, 2011), these

findings are novel in as much as they evidence how practice-based interventions for learning decomposition strategies may be impeded if these facts are not known.

Findings for Nanuka fits with Siegler's (2005) theory of strategy development. As a result of practice, Nanuka successfully transitioned from using counting-based and retrieval-based strategies for single-digit addition, to retrieval-based strategies exclusively. Furthermore, for this student, the intervention to strengthen problem-answer associations in memory appeared to accelerate the uptake of retrieval-based strategies because of practice. We draw attention to how this case exemplifies the advantages of using a microgenetic approach with a multiple baseline design, to illustrate rates of change in strategy uptake as a result of practice, before and after an invention.

Our findings have practical implications. We advise teachers to not only focus on strengthening students' part-part-whole understanding of number but also their skills with retrieving facts for sums up to 10 (understanding 8 can be composed of 3 and 5 may not necessarily transfer to solving 3 + 5 using direct retrieval). Otherwise, they might not benefit from practice to increase the efficacy of adopting decomposition strategies, specifically for bridging through 10.

In terms of theoretical implications, Siegler's (2005) theory of strategy development does not account for why students who accurately apply counting-based strategies, do not develop retrieval-based strategies for problems with correct practice. We posit that while practice is important to ensure increasing proficiency for basic addition, it is more likely to be effective for problems with sums between 10 and 20, if students are already well on the way to retrieving answers to problems with sums up to 10.

Although the small sample size constrains the generalisability of findings, the microgenetic approach yielded fine-grained data of students' shifts in strategy use. This method revealed individual differences about the elements that supported or impeded students' strategy development—differences that would likely remain obscured in studies involving much larger cohorts. To extend the benefits of practice and support greater numbers of students transitioning from counting-based strategies to retrieval-based strategies, future research (like Kullberg & Bjorklund, 2020) should investigate instructional interventions that allow young children to apply part-part-whole number knowledge to retrieve answers to questions like 2 + ? = 7.

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