DISCERNING MATHEMATICAL IDEAS IN A CHOREOGRAPHY: PRE-SERVICE TEACHERS' INTERPRETATIONS OF BECKETT'S *QUAD*

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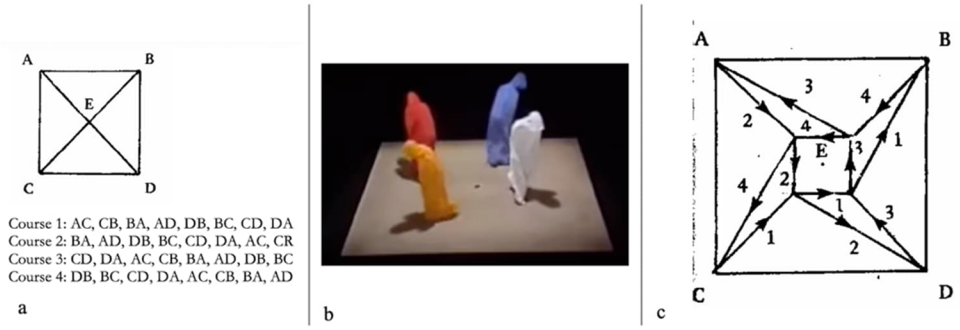
This multiple-case exploratory study examines what mathematical ideas pre-service mathematics teachers (PSTs) discern and represent when interpreting a non-verbal, rule-governed movement system. Twenty-nine PSTs viewed Samuel Beckett's Quad and produced textual and diagrammatic accounts of the choreography. Analysed through the lens of professional vision, the accounts reveal a variety of mathematical ideas alongside systematic gaps and representational tensions, and in some cases show PSTs extending beyond Beckett's movement score by drawing on prior mathematical knowledge. Taken together, the findings suggest that observing embodied movement can serve as a diagnostic context for examining emerging professional vision and for informing pedagogical support in embodied mathematics teacher education.

Movement-based approaches in mathematics education have gained attention for their potential to reveal spatial, relational, and temporal structures through embodied interaction (Abrahamson & Lindgren, 2014; Radford, 2014). However, empirical research on how such embodied resources function in teacher education, particularly in recognising mathematical ideas, is just beginning to accumulate (e.g., Boonstra et al., 2025; Palatnik et al., 2026). As movement-based tasks enter mathematics classrooms, teachers are required to develop attentiveness to the embodied dimensions of mathematical thinking, such as movement patterns, timing, and spatial organisation, as these emerge in students' activities (Flood et al., 2022).

Figure 1. Beckett's *Quad* Score (1984): (a) directional paths, (b) video screen capture of four dancers and the central forbidden zone, and (c) deviations from the centre.



The present study takes a preliminary step toward addressing teacher attentiveness to mathematical thinking expressed through movement by examining how pre-service

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mathematics teachers observe and represent mathematical ideas embedded in a choreography. Rather than positioning the task as instructional content, we use it as an observational and interpretive situation designed to make embodied mathematical ideas available without relying on symbolic representations, with the intention of informing the design of future instructional interventions. To this end, participants watched *Quad* (Beckett, 1984, Figure 1), a rule-governed choreography in which mathematical ideas are expressed through movement, timing, and spatial constraint. While alternative tasks could have been selected, *Quad* provides a site for investigating what mathematical ideas PSTs observe, articulate, and represent.

THEORETICAL BACKGROUND

Drawing on Goodwin's (1994) theory of professional vision, we conceptualise PSTs' accounts of choreography as interpretive acts involving selective highlighting, coding, and representational transformation. Extending this framework to mathematics teacher education, Palatnik et al. (2026) characterise professional vision as an emergent capacity to attend to, interpret, represent, and act on mathematically relevant features of activity. In this study, PSTs' accounts are treated as traces of the mathematical ideas that entered participants' awareness in the embodied interpretive situation and of how these ideas were expressed across representational registers.

Following Thompson (2008), *mathematical ideas* are understood as ways of construing relationships and organisation, rather than as indicators of formal conceptual mastery. This perspective is particularly relevant to the present study, as participants' responses often involved partial and heterogeneous articulations of mathematical meaning, rather than conventional formal statements. In this study, the term mathematical idea is used to denote features of mathematical structure that become available through observing and coding the choreography and are expressed in textual or diagrammatic form.

A systematic review of embodied teaching and learning in higher education suggests that existing research often foregrounds participation and reflective articulation, focusing on cognitive and discursive aspects of embodiment, while the sensory and bodily dimensions through which disciplinary ideas may become available remain comparatively underexplored (Hegna & Ørbæk, 2021). At the same time, work on responsive teaching for embodied learning shows that attentiveness to movement-based dimensions of mathematical thinking, such as spatial relations, coordination, and temporal structure, does not necessarily emerge spontaneously from participation alone but can be explicitly learned and supported (Flood et al., 2022).

A growing body of research has explored the pedagogical potential of movement and embodied activity in mathematics education, particularly in relation to students' learning. Prior work suggests that embodied engagement can function as a generative context for mathematical meaning-making in classroom settings (e.g., Apsari & Abrahamson, 2024; Gerofsky, 2025; Efron & Palatnik, 2025). However, this work has focused primarily on students' experiences. The present study extends this line of

inquiry into mathematics teacher education by examining how PSTs highlight, code, and represent mathematical ideas emerging from embodied activity, and how such observation can be understood in terms of emerging professional vision. Accordingly, our research question is:

What mathematical ideas do PSTs observe and represent when describing a choreography through textual and diagrammatic representation?

METHODOLOGY

This exploratory study adopts a qualitative multiple-case design (Stake, 2013). Drawing on professional vision as an analytical lens (Goodwin, 1994; Palatnik et al., 2026), we examine how PSTs engaged in acts of interpretation and representation when observing *Quad* (Video excerpt analysed in this study: https://youtu.be/LPJBIvv13Bc). Beckett's movement score and written instructions (Figure 1 a, c), to which they were not exposed, thus functioned as an ideational horizon, indicating which aspects of the choreography were potentially available for interpretation. Participants' textual and diagrammatic responses were analysed not as measures of accuracy, but as traces of how mathematical ideas were taken up and externalised across representational forms. Each participant was treated as an individual case of interpretive engagement. Cross-case comparisons were then conducted to identify recurrent patterns across responses.

Focal Task and Procedure

Twenty-nine PSTs participated individually in an asynchronous session centred on viewing a 5-minute version of Samuel Beckett's *Quad*. Participants completed two sequential tasks: a written textual description of the choreography and a wordless diagram intended to further explicate its organization. Task prompts invited participants to describe spatial organisation, temporal order, coordination, and regularities in the movement, supporting their attempts to articulate and represent mathematical ideas expressed in the choreography.

Data Analysis

Analysis proceeded through iterative cycles of close reading, analytic categorisation, and cross-case comparison (Stake, 2013). Textual descriptions and diagrams were treated as complementary representational sites through which mathematical ideas could be expressed. Guided by the framework of professional vision (Goodwin, 1994), we examined which mathematical ideas participants highlighted and elaborated in their accounts. For each mathematical idea, presence was recorded binarily (present = 1, absent = 0), independent of representational register. An idea was thus considered present if it appeared in either textual or diagrammatic form, allowing us to identify which mathematical ideas entered participants' awareness.

FINDINGS

The findings address the research question in two steps. First, we examine the mathematical ideas observed and represented across participants, regardless of the

representational mode. Second, we analyse how these ideas were distributed and expressed across textual and diagrammatic registers.

Table 1 summarises the mathematical ideas expressed by PSTs, indicating whether they are in Beckett's movement score, textually or diagrammatically represented

Table 1: Top 11 mathematical ideas expressed by the 29 PSTs.

#	Mathematical Idea	In Beckett's Score?	Expressed by PSTs	In Text	In Diagram
1	square	Yes	28	26	23
2	side	Yes	28	21	27
3	series	Yes	28	26	11
4	centre deviation	Yes	26	19	22
5	diagonal	Yes	25	20	23
6	four figures/colours	Yes	24	21	22
7	(counter) clockwise motion	Yes	21	11	18
8	straight lines	No	16	10	13
9	angles	No	13	11	7
10	vertices	No	13	12	11
11	length of side	Yes	2	2	1

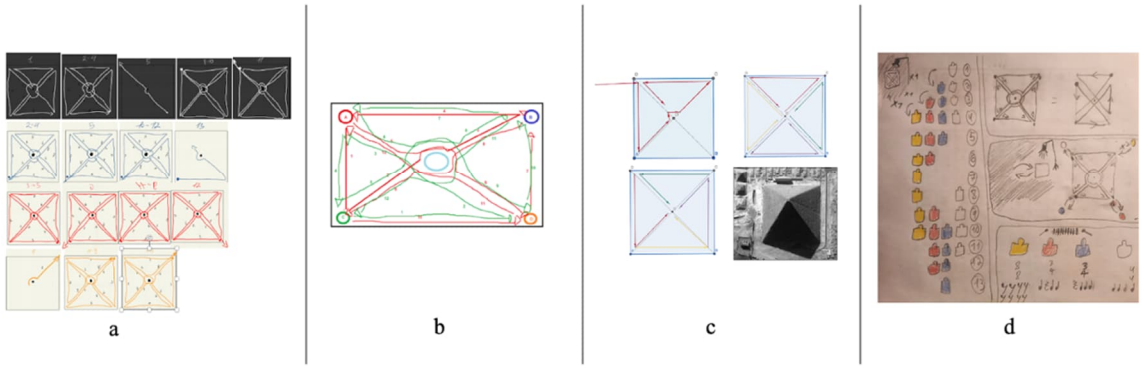
Distribution of mathematical ideas across participants

The analysis revealed a systematic distribution in the mathematical ideas expressed by participants. Several ideas explicitly specified in Beckett's score were widely represented across participants, including the square (#1 row in Table 1) configuration, boundary-constrained movement (#2), serial or rule-based organisation (#3), systematic deviation from the centre (#4), diagonals (#5), and presence of four figures (#6) (Figure 2a), appearing textually and diagrammatically.

Other ideas were considerably less frequently represented. Notably, although boundaries were often described, only two participants explicitly referred to the length of the sides (#11), despite this feature being specified in Beckett's score.

At the same time, several mathematical ideas not explicitly specified in Beckett's score appeared frequently in participants' responses. These included directional orientation (#7), straight-line trajectories (#8), angles (#9), and vertices (#10). These ideas were articulated across both textual and diagrammatic registers, suggesting that participants recruited familiar geometric ideas to organise and interpret the observed movement.

Figure 2. Diagrammatic examples of representational variation.



Uneven distribution across participants

The range of representations also varied markedly across participants. Out of a total of 21 mathematical ideas identified across the dataset (13 specified in Beckett's score and 8 generated by participants), seven students (~24%) expressed 20–25 ideas across textual and diagrammatic registers, whereas five (~17%) participants expressed only 8–12 ideas. The remaining participants occupied intermediate positions within this range. These differences are not treated here as indicators of individual competence, but rather as illustrating the breadth of professional vision expressed within the cohort under identical observational conditions.

Representational gaps and misalignments

Beyond overall frequencies, the findings reveal systematic gaps and misalignments in how mathematical ideas were represented. Some ideas were expressed in ways that departed from geometric precision, including the depiction of rectangles in place of squares by 6 students (~ 20%), the addition of circular forms around the centre, and the use of angles that do not correspond to the choreography's prescribed paths (Figure 2b). Rather than being treated as errors, these accounts indicate selective emphasis on aspects of the movement, such as trajectory, orientation, or flow, over strict geometric form. Other ideas were notable by their absence. As mentioned above, side length, although specified through the number of steps in Beckett's score, was represented by two PSTs (~ 6%), suggesting that some quantitative aspects of the movement were less readily expressed through observation alone.

In addition, a small number of participants produced representations that extended beyond the two-dimensional organisation of the choreography. These included three-dimensional interpretations (Figure 2c), descriptions involving right-angled or equilateral triangles, a single PST who measured durations, and, in one case, the use of musical notation to organise and measure the movement temporally (Figure 2d). Although these responses were too infrequent to form stable categories, they illustrate

moments of extensions in which participants introduced additional mathematical dimensions not specified in the choreography.

Register tension: textual and diagrammatic representations

Several mathematical ideas proved unevenly expressible across representational registers. Series, order, and cyclicity were articulated textually by 26 participants (~90%), yet appeared diagrammatically in only 11 cases (~38%). Conversely, some ideas were represented diagrammatically without being explicitly mentioned in the text. For instance, seven (~24%) participants diagrammed systematic deviation from the centre without textual description, while four (~14%) described this feature textually without diagrammatic representation. These patterns suggest that textual and diagrammatic accounts supported different representational choices rather than redundant or equivalent ones. Even when the same idea appeared in both registers, the forms did not always align (e.g., square configurations were described textually while diagrammed as rectangles, Figure 2b).

DISCUSSION

Taken together, the findings suggest that observing a rule-governed choreography can function as a diagnostic window into PSTs' emerging professional vision for embodied mathematical ideas. When watching Beckett's *Quad*, participants' professional vision was manifested in the ways they highlighted features of the movement and coded them using familiar mathematical ideas (e.g., square, diagonal, order). These ideas were then expressed across textual and diagrammatic representations. Participants were able to identify and represent a wide range of mathematical ideas without textual explanation or symbolic notation, drawing on perceptual and spatial aspects of the movement to support interpretation. In this sense, participants' accounts can be understood as traces of how mathematical ideas became perceptually salient and how they were transformed through acts of interpretation and representation. The findings align with embodied perspectives on mathematical cognition, which argue that mathematical ideas can emerge through perceptual and spatial engagement with movement (Abrahamson & Lindgren, 2014; Radford, 2014). Notably, most participants identified nearly all mathematical ideas elicited by the choreography, although not all were externalised in both their textual and diagrammatic representations. The patterned distribution of what was highlighted, unevenly represented, or left implicit points to attentional resources and areas where pedagogical support may be productive (Flood et al., 2022).

From a geometric perspective, the findings also point to a tension between conventional forms of geometric representation and the embodied geometry enacted in *Quad*. Geometry is typically encountered in school mathematics as a two-dimensional abstraction expressed through static diagrams. In contrast, *Quad* presents a rule-governed geometric system that unfolds through movement in space and time, giving rise to a fundamentally spatio-temporal form of geometry. This view resonates with traditions such as *'turtle geometry'*, which conceptualises geometric form as emerging

from sequences of movement rather than from static figures (Papert, 1980). While many PSTs were able to identify familiar geometric ideas, such as squares, diagonals, and vertices, the uneven coordination of spatial and temporal features suggests that perceiving geometry as an embodied, dynamic system poses additional demands. These demands reflect not a lack of geometric knowledge, but the absence of representational tools for attending to geometry as it is enacted in space and time.

Beyond documenting what PSTs identified, the findings invite a reframing of embodied movement as an epistemic site for examining how teachers conceptualise geometry. The distinction between identifying isolated geometric elements and coordinating these into a coherent spatio-temporal organisation suggests that professional vision in embodied contexts involves more than recognition of familiar structures; it involves perceiving relationships as they unfold dynamically across space and time. From a teacher education perspective, this shifts the focus from using movement merely as a pedagogical tool toward cultivating teachers' capacity to see geometry in movement. Developing this form of perception may require deliberate representational and perceptual supports, rather than relying on embodied engagement alone, positioning embodied geometry as both an approach to instruction and a productive context for investigating the development of professional vision.

At the same time, the findings revealed systematic gaps and representational tensions. Certain quantitative ideas, such as side length and explicit temporal measurement, were rarely expressed despite being specified in Beckett's movement score, while others were unevenly expressed across textual and diagrammatic registers. These mismatches suggest selective representational choices rather than failures of perception, with ideas prioritised according to what was more readily expressible within a given register. From the perspective of professional vision (Goodwin, 1994; Palatnik et al., 2026), such partial representations reflect the challenges of engaging with dynamic movement and indicate where further pedagogical support may be needed.

The present analysis focused on participants' final textual and diagrammatic products rather than on the process of their construction. Future research may examine how mathematical ideas emerge during the act of representation and whether additional features come into focus when PSTs are asked to reconstruct or enact the choreography. Task design, including the sequencing of diagrammatic and textual production, may also shape these processes. Together with more formalised systems of movement notation, such as Eshkol–Wachman Movement Notation (1958), these approaches may support the elaboration of spatial and temporal relations and the development of professional vision in embodied mathematics teaching.

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