SENSE-MAKING IN MATHEMATICAL MODELLING: INVESTIGATING COLLEGE STUDENTS' COMPETENCIES IN A FOOD SECURITY MODELLING TASK

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This study investigates how mathematical sense-making manifests during the modelling process among Technical and Vocational Education and Training College mathematics students. Using a qualitative case study design, thirty third-year national certificate vocational students collaborated in groups to solve a food security modelling task. Qualitative data comprised written group solutions, modelling competency rating summaries by mathematical modelling expert raters, and transcripts of audio-recorded discussions of students solving the modelling task. Findings revealed significant disparities in sense-making across modelling competencies: stronger in assumption-making but limited in computing and interpreting, and mathematical reasoning.

INTRODUCTION

This study aimed to investigate how mathematical sense-making manifests within the mathematical modelling process among third-year Technical and Vocational Education and Training (TVET) College mathematics students. Mathematical sense-making in mathematical modelling is defined as the deliberate coordination of assumptions, representations, and contextual constraints to produce, interpret, and justify solutions across the stages of the modelling cycle (Biccard, 2018; Leong & Tan, 2020; Schoenfeld, 2013). Although mathematical modelling situates mathematics in authentic contexts and promotes higher order thinking skills, empirical evidence highlights persistent difficulties during the modelling process, often resulting in models that fail to integrate contextual constraints meaningfully—revealing gaps in sense-making (Cevikbas et al., 2022; Edo et al., 2013). These difficulties are particularly salient in South Africa's TVET Colleges, where modelling tasks intersect with socio-economic contexts such as food security, making the investigation of mathematical sense-making within the modelling process both pedagogically significant and timely (Vimbelo & Bayaga, 2024). This study responds to this gap by investigating mathematical sense-making as it occurs in practice within the modelling process rather than treating it as an additional outcome. Using Leong and Tan's (2020) Analytical Scale of Rubric for Assessing Mathematical Modelling Competencies (ASRAMMC) to analyse students' group-written solutions, the study seeks to investigate how sense-making manifests during the modelling process. Therefore, the guiding research question is: How do TVET College third-year students demonstrate

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sense-making during the mathematical modelling process when solving a modelling task on food security?

THEORETICAL FRAMEWORK

This study adopts the mathematical modelling process as its theoretical framework, conceptualised as an iterative cycle in which real-world situations are analysed, mathematised, computed, interpreted, validated, and revised (Niss et al., 2007). Within this framework, mathematical sense-making is understood as an inherent feature of modelling—emerging as students coordinate assumptions, representations, contextual constraints, and mathematical ideas to construct, interpret, and justify solutions (Biccard, 2018; Schoenfeld, 2013).

To operationalise this framework, the study employs Leong and Tan's (2020) ASRAMMC, which aligns the modelling cycle with three dimensions of sense-making: (1) making assumptions (interpretive sense-making)—identifying salient variables and formulating context-sensitive assumptions; (2) computing and interpreting solutions (analytical sense-making)—developing and executing models and interpreting results relative to contextual feasibility; and (3) mathematical reasoning (justificatory sense-making)—explaining, justifying, and validating modelling choices (Kaiser & Brand, 2015).

The above three modelling competencies delineate how sense-making unfolds across the modelling cycle: interpretive sense-making structures the problem, analytical sense-making mediates mathematisation and interpretation, and justificatory sense-making integrates the modelling stages into a coherent return to context. Leong and Tan's (2020) rubric thus functions as a theoretically grounded analytic tool for investigating how students formulate assumptions, perform and interpret computations, and justify decisions. This framework underpins the study's analysis of students' sense-making as it manifests through the modelling competencies—within a food-security modelling task.

METHODOLODY

This study employed a qualitative case study design to enable an in-depth investigation of students' sense-making practices in a naturalistic classroom setting when solving a modelling task on food security. To implement this design, the following subsections provide details on (1) the context and participants, and (2) the data collection and analysis used in the study.

Context and Participants

The study was conducted at a South African TVET College. Thirty third-year national certificate vocational students solved a modelling task (see Figure 1) situated within a food security context. For in-depth analysis, two groups—Group001 and Group002—were purposefully selected, and their written solutions were evaluated using the ASRAMMC.

Figure 1: Modelling task on Estimating the cost of growing spinach at home

Food insecurity remains a pressing challenge in South Africa, particularly within low-income communities where many households rely on social grants for survival. Rising food prices, limited access to fresh produce, and the cost of transportation to and from town compound the difficulty of maintaining a nutritious diet. Home gardening offers a low-cost, sustainable approach to enhancing household food security and nutrition, particularly when focusing on nutrient-dense crops like spinach, which provides essential vitamins A, C, and K. Research supports home gardening as a viable method for addressing malnutrition and improving resilience in resource-constrained settings. Technical and Vocational Education and Training (TVET) students studying Agriculture, Hospitality, and Community Development are ideally positioned to bring practical solutions to their communities by designing home gardening systems. These systems must incorporate horticultural knowledge, budgeting, spatial planning, and sustainability principles. Thembi's grandmother, who depends solely on a social grant, has been diagnosed with a deficiency in vitamins A, C, and K. Her doctor recommends that she regularly eat spinach, a rich source of these nutrients. However, she finds that the cost of traveling to town and purchasing spinach is unaffordable. Behind her RDP house, she has an unused patch of land that could potentially be used for gardening. Due to financial constraints, she cannot afford to cultivate the entire space and needs help deciding how much she can realistically use. She has asked you, a TVET student, to help her design a cost-effective home garden to grow spinach. **You are tasked with developing a mathematical model that estimates the total cost of growing spinach at home**.

Resource: *https://mdseedlings.co.za/*

The modelling task required students to estimate the cost of growing spinach at home for Thembi's grandmother, dependent on a social grant (state allowance), incorporating variables such as seed price, soil preparation costs, and water usage, among others. This context was chosen for its alignment with the students' curriculum, the appropriateness of its mathematical content, and its potential to foster sense-making through authentic modelling decisions.

Data Collection and Analysis

Qualitative data were drawn from the documentary analyses of the students' written solutions from the modelling task, which provided evidence of group sense-making. In addition to these group-written solutions, students' group discussions during the activity were audio-recorded and transcribed, offering further insight into how they justified and explained their modelling decisions. Both written solutions and transcribed discussions were analysed using Leong and Tan's (2020) ASRAMMC. The rubric assessed three modelling competencies—making assumptions, computing and interpreting solutions, and mathematical reasoning—on a four-point analytical scale (Level 1: Emerging level; Level 2: Basic level; Level 3: Proficient Level; and Level 4: Advanced level). Two mathematical modelling expert raters independently rated students' group work, and inter-rater reliability was established through consensus meetings.

Group-written solutions and transcribed discussions illustrated how sense-making manifested or faltered during the modelling process. The ratings, supported by documentary and verbal evidence, strengthened the reliability of the findings. Building on this methodological foundation, the next section presents and interprets the findings, demonstrating how sense-making unfolded through students' engagement with the modelling task.

RESULTS AND DISCUSSION

This section concurrently presents and discusses the study's results, focusing on the sense-making demonstrated by Group001 and Group002. In Table 1, sense-making, as operationalised through mathematical modelling, is presented in the column labelled competency—consisting of making assumptions, computing and interpreting, and mathematical reasoning—based on Leong and Tan's (2020) analytical scale. Table 1 below indicates a summary of the agreed-upon ratings by two mathematical modelling expert raters.

Table 1: Summary of modelling competency ratings for Group001 and Group002

Competency	Group001	Group002
Making Assumptions	4	3
Computing and Interpreting	3	2
Mathematical Reasoning	2	2

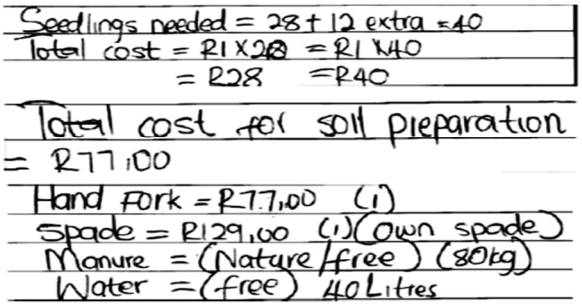
Both groups' mathematical modelling competencies range from level 2 – 4, with Group001 exhibiting higher levels of modelling competencies in making assumptions and computing and interpreting. While both groups showed basic mathematical reasoning competency.

Making assumptions (interpretive sense-making)

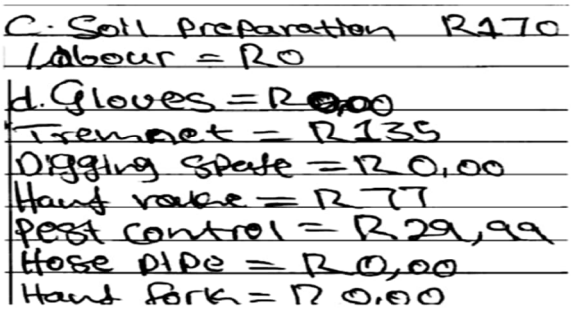
Leong and Tan (2020) assert that making assumptions refers to students' ability to identify relevant variables and formulate assumptions grounded in real-world contexts. Group001 demonstrated an advanced level of modelling competency, while Group002 was considered proficient in making assumptions. Figure 2 illustrates the contrast in assumption quality between the two student groups.

Figure 2: Comparison of assumptions by Group001 and Group002

Group001



Group002



Group001 presented a comprehensive, context-sensitive set of assumptions that preserved salient socio-economic constraints in a social grant-dependent household. For example:

Group001: …Remember this lady is poor… we want to minimise costs because this one is receiving a social grant…

Such reasoning informed their specification of variables (e.g., access to water as non-costed, locally sourced manure) and their decision to anticipate seedling mortality. For example:

Group001: …we know that if we plant 28, the chances are high that some can die, which means we have to replace…we add extras…

These choices produced a robust context-specific model that supported horizontal mathematisation from reality to mathematics (Niss et al., 2007). While Group002 listed multiple variables, key cost decisions were under-justified (e.g., zero-priced items without a rationale), yielding a less robust context-specific model. For example:

Teacher: What about the estimates of preparation costs, including fertiliser or manure—justify your choices and quantities?

Group002: Okay, Okay… maybe two kilograms… we are not sure… because the garden is too small…

This lack of justification illustrates the partial substantiation of assumptions, a challenge well documented in novice modellers where students recognise contextual features but cannot yet anchor them to defensible simplified assumptions (Kaiser & Brand, 2015; Maaß, 2006).

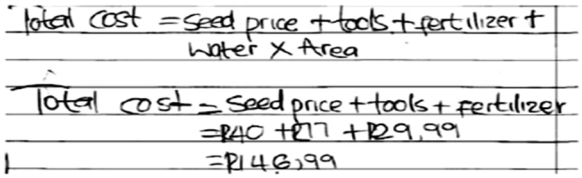
Although Group002 did not fully achieve this modelling competency, sense-making in this phase is still evident, particularly in how both groups negotiated contextual constraints to formulate assumptions that enable the interpretation of the situation. For example, Group001's inclusion of socio-economic factors reflects interpretive reasoning that transforms lived experiences into mathematised variables (Schoenfeld, 2013). By contrast, Group002's partial substantiation of assumptions indicates limited sense-making, as their decisions lacked defensible connections to real-world constraints—a pattern consistent with findings that justification is central to modelling coherence and making sense of the context (Kaiser & Brand, 2015). Thus, interpretive sense-making is revealed through the making assumptions stage of the mathematical modelling process (Schukajlow et al., 2015).

Computing and interpreting solutions (analytical sense-making)

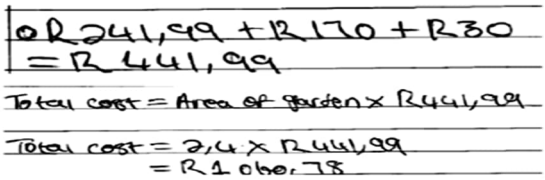
This competency involves developing a mathematical model, performing computations, interpreting results, and considering contextual constraints. Group 001 and Group 002 showed a proficient and basic level of competency, respectively. Figure 3 below presents both student groups' computations and solutions.

Figure 3: Students' cost models and computations for the food security activity

Group001



Group002



Group001 developed a mostly coherent cost model. Although they could differentiate between fixed and variable costs, they exhibited a fragmented understanding of the area concept, particularly when linking water usage to area (e.g., multiplying water by area). When the teacher asked:

Teacher: Should we multiply water by area?

Group001: The length was 2 metres, and the breadth is 1.2 metres… so the area equals 2 × 1.2, which is 2.4… maybe we can take half of that…

This exchange illustrates their struggle with spatial reasoning and proportionality despite efforts to compute cost correctly. Importantly, Group001 correctly executed their computations and assessed the feasibility of their results against contextual constraints, such as social grant affordability (e.g., total cost of R146.99 was affordable) and anticipated yield variability. This aligns with research indicating that while students often demonstrate proficiency in structuring cost models, they frequently struggle with spatial reasoning and proportionality, particularly when transitioning between contextual and mathematical representations (Niss et al., 2007).

In contrast, Group002 demonstrated limited integration of variables into a cost model, exhibited minor computational errors (e.g., multiplying the cost by the area of the garden), and showed inadequate differentiation between fixed and variable costs, resulting in a fragmented cost model. Although they attempted to generalise their approach, for example:

Group002: …We must come up with a formula… say our formula is equal to the metre square multiplied by the total cost… area per metre square… then multiply by cost…

This effort lacked conceptual clarity and did not resolve the underlying misunderstanding between fixed and variable costs. These shortcomings distorted decision logic and reduced cost sensitivity—critical to feasibility in this context (e.g., the total projected cost was approximately R1,060.78, which is almost half of the monthly social grant allowance of R2,315). The observed difficulties are consistent with findings that the interpretation and validation phases are particularly challenging in modelling activities, with students often resorting to procedural fluency rather than criterion-based evaluation (Cevikbas et al., 2022; Edo et al., 2013).

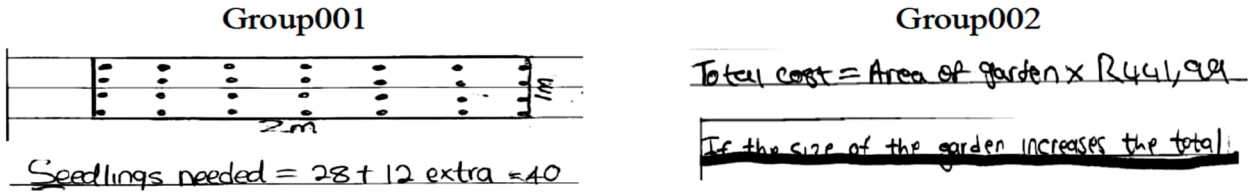
Sense-making in this phase is revealed through students' efforts to connect numerical outputs to contextual viability. Group001's evaluation of costs against socio-economic

constraints demonstrates analytical sense-making, as they moved beyond mere computation to assess feasibility within authentic conditions (Schoenfeld, 2013). Conversely, Group002's reliance on procedural calculations without validating results against real-world situations reflects limited sense-making, a challenge echoed by Cevikbas et al. (2022) and Edo et al. (2013). Thus, analytical sense-making emerges when students interpret and justify computations in relation to contextual constraints, transforming isolated calculations into meaningful decisions (Kaiser & Brand, 2015).

Mathematical reasoning (justificatory sense-making)

Mathematical reasoning refers to explaining reasoning logically, justifying choices, and connecting processes coherently (Leong & Tan, 2020). Group 001 was at the basic level of competency, while Group 002 also remained at the basic level. Figure 4 presents both student groups' reasoning and justification.

Figure 4: tudents' reasoning excerpts illustrating justification and coherence



Both student groups demonstrated limited cyclical reasoning and inappropriate use of mathematics at critical stages (e.g., Group002 conflated fixed and variable costs—"area × R441.99"; Group001 drew 28 seedlings but calculated for 40). Mathematical reasoning was at a basic level and lacked evidential support, as seen in Group002's incomplete statement, "If the garden size increases the total…,". This aligns with Leong and Tan's (2020) view that justification is the critical "why", integrating modelling processes into a unified argument—a process requiring deliberate metacognitive planning (Kaiser & Brand, 2015; Schoenfeld, 2013). Sense-making in this phase emerged only at a basic level when students attempted to justify decisions and connect assumptions, computations, and interpretations. Group001 made attempts to explain cost decisions, indicating partial justificatory sense-making, whereas Group002's fragmented reasoning reflected minimal sense-making, consistent with research on novice modellers' struggles to articulate trade-offs (Biccard, 2018). Thus, justificatory sense-making entails constructing logical explanations that unify modelling processes. The next section presents the study's implications and conclusion, synthesising key insights and outlining recommendations.

IMPLICATION AND CONCLUSION

This study offers a practical example of assessing sense-making through Leong and Tan's (2020) ASRAMMC. Findings reveal that TVET College students' sense-making was disparate—stronger in assumption-making but limited in computing and interpreting, and mathematical reasoning competencies. These gaps highlight the need

for mathematics teachers, curriculum designers, and researchers to: (1) facilitate the process of justifying and validating assumptions; (2) scaffold mathematical reasoning and cost analysis to move beyond procedural fluency; and (3) cultivate argumentation and metacognitive monitoring to integrate modelling processes with coherent reasoning. By exposing these gaps, the study provides a basis for future research to design interventions that strengthen justificatory sense-making and proportional reasoning in mathematical modelling.

References

Biccard, P. (2018). Mathematical sense-making through learner choice. *Pythagoras, 39*(1), 1-9. https://hdl.handle.net/10520/EJC-11fe0312ac

Cevikbas, M., Kaiser, G., & Schukajlow, S. (2022). A systematic literature review of the current discussion on mathematical modelling competencies: State-of-the-art developments in conceptualizing, measuring, and fostering. *Educational Studies in Mathematics*, *109*(2), 205-236. https://doi.org/10.1007/s10649-021-10104-6

Edo, S. I., Hartono, Y., & Putri, R. I. I. (2013). Investigating Secondary School Students' Difficulties in Modeling Problems PISA-Model Level 5 and 6. *Indonesian Mathematical Society Journal on Mathematics Education*, *4*(1), 41-58.

Kaiser, G., & Brand, S. (2015). Modelling competencies: Past development and further perspectives. In Stillman, G. A., Blum, W., & Biembengut, M. S. (Eds.), In *Mathematical Modelling in Education Research and Practice: Cultural, Social and Cognitive Influences*, (pp. 129-149). Cham

Leong, K. E., & Tan, J. Y. (2020). Exploring secondary students' modelling competencies. *The Mathematics Enthusiast, 17*(1), 85-107. https://doi.org/10.54870/1551-3440.1481

Maaß, K. (2006). What are modelling competencies?. *Zentralblatt für Didaktik der Mathematik 38*(1), 113–142. https://doi.org/10.1007/BF02655885

Niss, M., Blum, W., & Galbraith, P. (2007). Introduction to modelling and applications in mathematics education. In *Modelling and applications in mathematics education*. 14th ICMI Study, (pp. 3-32). Cham

Schoenfeld, A. H. (2013). Mathematical modeling, sense making, and the common core state standards. *Journal of Mathematics Education at Teachers College*, *4*(2), 6-17. https://doi.org/10.7916/jmetc.v4i2.620

Schukajlow, S., Kolter, J., & Blum, W. (2015). Scaffolding mathematical modelling with a solution plan. *ZDM Mathematics Education 47*(1), 1241-1254. https://doi.org/10.1007/s11858-015-0707-2

Vimbelo, S., & Bayaga, A. (2024). Transforming mathematics education in TVET Colleges through humanising pedagogy: An exploration of teaching approaches, student engagement, and real-life examples. *IETE Journal of Education*, *65*(2), 139-154. https://doi.org/10.1080/09747338.2024.2324808

Yazan, B. (2015). Three Approaches to Case Study Methods in Education: Yin, Merriam, and Stake. The Qualitative Report, 20(2), 134-152.