TASK DESIGN PROMOTING A SYNERGIC LINK BETWEEN RESEARCH AND PRACTICE IN TEACHER EDUCATION

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This paper investigates how tasks designed by mathematics teacher educators can support prospective mathematics teachers in using research-based theoretical ideas within teaching-related practices. Drawing on the Meta-Didactical Transposition model, seven meta-tasks were identified and characterised in relation to the research and teaching practices they mobilise. An a priori analysis of meta-tasks was developed to highlight the boundary objects involved and the potential actions at syntactic, semantic and pragmatic levels that may foster internalisation processes.

INTRODUCTION

Recent developments in mathematics teacher education aim to develop courses and teacher education activities that are research-informed and practice based (e.g. Jackson et al. 2024). The interplay between research in mathematics education and the practice of teaching has been a main issue of concern in the mathematics education research community. Ruthven and Goodchild (2015) point out the dialogic cycle between scholarly knowledge and craft knowledge as a way of linking academic research with teaching practice. Concerning practicing teachers, there are studies where the research-practice relationship is addressed in collaborative settings where teachers with the support of didacticians inquire into mathematics teaching (Jaworski, 2006). However, currently research shows that research in mathematics education is not used by prospective mathematics teachers (PMTs) as a resource to interpret phenomena of mathematics teaching and learning (Rottem et al., 2024). There are attempts to support PMTs to use research in mathematics teaching but in a rather linear way: research results are presented in the context of a course and PMTs are expected to link research with mathematics teaching practices in the field experiences (Lendínez et al., 2024).

In our study we are interested on how the link between research and practice relationship is promoted by mathematics teacher educators (MTEs) through the design of tasks for PMTs. For this purpose, we introduce the notion of *meta-tasks*, defined as structured activities that are designed to promote a synergic link between research in mathematics education and teaching practice, by orchestrating the mobilisation of research and teaching practices around specific objects. In this paper we address the following research question: *How can the meta-tasks designed by MTEs be characterised and what potential do they have to support PMTs in mobilising research-based theoretical ideas within teaching-related practices?*

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THEORETICAL BACKGROUND

The study draws on the Meta-Didactical Transposition (MDT) model (Arzarello et al., 2014; Cusi et al., 2022), developed to analyse processes emerging in collaborative settings involving communities of researchers and teachers that jointly engage in work around the teaching and learning of mathematics. MDT characterises these interactions through the notion of didactical and meta-didactical *praxeologies*, intended as systems of tasks and techniques to carry out the tasks, and the discourses developed to justify the chosen techniques. These systems support both action in teaching (didactical praxeologies) and reflection on teaching (meta-didactical praxeologies). Praxeological components may appear as external to a community and progressively become internal as they are adopted and re-contextualised to support the community's own activity. The process through which external components become internal components, referred to as *internalisation*, is not conceived as the simple acquisition of external knowledge, but as the transformation of praxeological components through discussion between different communities, appropriation and re-organisation.

Within MDT.2 (Cusi et al., 2022), the notion of *boundary object* (BO) (Star & Griesemer, 1989) has been incorporated to deepen the analysis of how internalisation processes unfold when different communities interact (Robutti et al., 2019). BOs are objects that are sufficiently flexible to be used by different communities while maintaining a recognisable identity. In MDT.2, BOs are interpreted as objects of joint actions carried out by actors belonging to the researchers' community or to the teachers' communities. BO's boundaries are therefore conceived as the collaborative context within which these actors develop shared aims and engage in forms of exchange that make the object relevant for their own practice. Working on a BO can lead actors to reinterpret, adapt or integrate aspects of praxeologies coming from other communities, thereby promoting internalisation.

To investigate how internalisation takes place, MDT.2 incorporates Carlile's (2004) characterisation of three levels of action on BOs, which can be observed in the discourses developed around them: (i) *syntactic level* (transfer), in which actors already share a common vocabulary and exchange concrete and syntactic components of the BO; (ii) *semantic level* (translation), in which actors negotiate meanings related to the BO, refining interpretations and aligning perspectives; (iii) *pragmatic level* (transformation), in which BO-related components become integrated into actors' practices by means of social mediation, generating new shared knowledge.

METHODOLOGY

The study was conducted by two MTEs, the authors of this paper, who collaborated in the co-design of a set of meta-tasks aimed at promoting a synergic link between research in mathematics education and mathematics teaching practice. The meta-tasks were implemented in two university courses for secondary school PMTs, one in Italy and one in Greece, both spanning approximately 13 weeks. Participants were PMTs enrolled in mathematics education courses within master's degree courses in

Mathematics. The PMTs had no prior teaching experience in school, which situated them at an initial stage of entering the professional field of mathematics teaching. The courses aimed to develop inquiry-oriented perspectives on mathematics teaching and included activities that required PMTs to analyse classroom data, read research papers, design tasks or lessons, and discuss their design choices with peers and with the MTE. The full data corpus includes (i) video recordings of the lessons, (ii) PMTs' written productions and reflections, and (iii) semi-structured interviews carried out at the end of the courses.

To address the research question, the present report focuses on the a priori analysis of the meta-tasks, with the aim of characterising their design and to highlight their potential to support PMTs in mobilising research-based theoretical ideas within teaching-related practices. The analysis was characterised by three main phases: (1) identification of the practices that the meta-tasks were intended to mobilise; (2) identification of broader categories of meta-tasks, which we refer to as *core meta-tasks*; (3) analysis of the core meta-tasks, by drawing on the theoretical lenses provided by MDT.2, to highlight how they create opportunities for PMTs to engage in actions on specific BOs (e.g. theoretical constructs, research data, analytical tools, tasks or lesson designs…) at different levels (syntactic, semantic, pragmatic), thus supporting the potential internalisation of praxeological components.

RESULTS

The first phase of the analysis enabled us to characterise the meta-tasks as configurations of teacher education activities that orchestrate the engagement of PMTs in practices that can be associated either with the research community (e.g., reading research literature, using theoretical lenses to analyse data from practice) or with the teaching community (e.g., designing tasks, analysing lessons). We identified 13 practices mobilised by the meta-tasks, distinguishing between 7 research practices (R) and 6 teaching practices (T). An overview of these practices is provided in Table 1.

Table 1: The research practices and the teaching practices mobilised in the meta-tasks

Research practices	Teaching practices
R1: Searching for research papers	T1: Identifying key teaching and learning issues
R2: Reading research papers	T2: Selecting mathematical tasks
R3: Presenting theoretical ideas	T3: Designing lessons
R4: Reviewing	T4: Enacting lessons
R5: Designing research	T5: Assessing students' work
R6: Using theoretical and research ideas for data analysis	T6: Evaluating/Analysing lessons
R7: Justification of research choices	

The second phase of the analysis led to the identification of a set of *core meta-tasks* that characterise how the sequence of tasks was designed, within the two courses, to support PMTs in mobilising research-based theoretical ideas within teaching-related practices. The seven core meta-tasks identified by our analysis are summarised in Table 2, together with the specific R/T practices each of them mobilises. Each core meta-task indicates a specific configuration of PMTs' engagement in R and T practices, and different points of entry into the relationship between research and practice.

Table 2: The core meta-tasks and the R and T practices mobilised by them

Core-Meta-tasks	R and T practices
MT1: From the analysis of classroom phenomena to the emergence of theoretical ideas	PTs analyse classroom data provided by the MTE (T6). MTE presents theoretical ideas (R3) and highlights the links (R7) between T6 and R3 or asks PMTs to make these links explicit (R7)
MT2: From the introduction of theoretical ideas to the analysis of classroom phenomena	MTE presents research findings and theoretical ideas (R3). PMTs analyse classroom data (T6) using theoretical tools (R6). MTE highlights the links (R7) between R3 and T6
MT3: theoretical ideas as means for designing and justifying the teaching design	PMTs use theoretical models (R6) to design tasks or hypothetical classroom dialogues (T3). PMTs justify the links (R7) between R6 and T3
MT4: peer review	PMTs review peers' designs (R4-T6). PMTs reflect on the links (R7) between R4 and T6
MT5: guided readings of research papers	PMTs read and present a research paper (R2-R3) to identify key teaching and learning issues (T1). MTE asks guiding questions promoting PMTs' reflection on R2 and on the links (R7) between R2-R3 and T1
MT6: design/re-design of tasks/activities using theoretical ideas	PMTs select mathematical tasks (T2). Then PMTs design a lesson (T3) using theoretical ideas discussed in the course but selected by their own (R6) and justify their design on theoretical basis (R7). MTE and PMTs provide feedback (T5) about the link (R7) between T3 and R6
MT7: design of a research lesson	PMTs engage in designing a research lesson (R5-T3). PMTs present their research lesson in the course (R3). MTE and PMTs provide feedback (T5) about the links (R7) between R5 and T3

Across the seven core meta-tasks, the BOs for PMTs primarily consist of: (i) research products (data, constructs, analytical tools) originating in the research community, and (ii) teaching artefacts or practices (tasks, lesson plans, student work) originating in the

teaching community. To show the result of the third phase of the analysis, in this paper we focus on three core meta-tasks (MT1, MT3 and MT7), selected for their relevance in illustrating different ways in which the mobilisation of R and T practices can be orchestrated while PMTs work on these BOs, thus supporting the potential internalisation of praxeological components.

Core meta-task MT1

In MT1, PMTs are provided with classroom data (e.g., video excerpts, transcripts of students' dialogues, students' written solutions) related to mathematical tasks enacted in instructional settings. The MTE asks PMTs to analyse these data to identify key issues concerning students' reasoning, teacher-student interactions, or the evolution of mathematical ideas during the lesson. After group work and collective discussion, the MTE introduces specific theoretical constructs developed by mathematics education research to frame the phenomena identified by PMTs and makes explicit the links between the constructs and the classroom data previously analysed.

On the one hand, the work on research data is predominantly carried out by the PMTs, who are asked to analyse and interpret these materials to identify relevant features of the teaching-learning processes. This activity initially takes the form of an exploratory analysis in which PMTs rely on their own categories and vocabulary to describe and interpret what they observe. At this stage, the discourse remains at a *syntactic level*, as it does not yet require a negotiation of meaning nor alignment with theoretical constructs. On the other hand, the work on theoretical constructs as research products is initially developed by the MTE, who introduces and connects them to the phenomena identified by the PMTs. Through this mediation, the MTE relates the PMTs' interpretations to the interpretations produced within the research community, thus activating a *semantic level*, where meanings are negotiated. The MTE's intervention enables the alignment between the ways PMTs interpret the research data and the ways in which such data are interpreted through specific theoretical lenses. This mediation opens the possibility for a *pragmatic level*, in which theoretical constructs begin to be integrated into the PMTs' emerging praxeologies as tools for analysing and justifying interpretations of classroom phenomena. In this sense, MT1 does not merely expose PMTs to theoretical ideas; it uses theory as a means to reorganise their interpretations of classroom data, thereby supporting potential internalisation of theoretical components originally external to the PMTs' praxeologies.

Core meta-task MT3

In MT3, PMTs are asked to design teaching artefacts such as sequences of mathematical tasks, lesson plans, or hypothetical classroom discussions, and to justify their design choices by explicitly referring to theoretical constructs or analytical tools introduced during the course. The design work is usually carried out in small groups and is followed by collective discussion, where PMTs present their designs and receive feedback from peers and the MTE, who focuses on how theoretical ideas were used to guide and justify design decisions.

Unlike MT1, where theoretical ideas are first mediated by the MTE, here PMTs must mobilise theoretical constructs autonomously to justify their design decisions. At the beginning of the activity, PMTs need to make sense of the theoretical constructs and negotiate how these constructs inform design choices. For this reason, the discourse develops at a *semantic level*, as meanings associated with the theoretical constructs are interpreted, refined, and discussed among peers in relation to the artefacts being designed. When the products of the design process are discussed with peers and the MTE, interests may diverge, revealing different priorities attached to the design (e.g., mathematical focus, student activity structure, or didactic goals). Such divergences may lead to a reorganisation of the design and to a re-alignment of theoretical justifications. These moments indicate a shift to a *pragmatic level*, in which theoretical constructs begin to function as tools for guiding design decisions and for justifying them in accountable ways. The requirement to justify design decisions makes visible both (i) the semantic work on meanings and (ii) the emergence of pragmatic concerns linked to practice, thus supporting potential internalisation of theoretical components as elements of the PMTs' emerging praxeologies.

Core meta-task MT7

MT7 represents the least structured core meta-task. Here PMTs are asked to design a research lesson, which requires them to formulate a research question, choose an appropriate methodological approach, select analytical or theoretical tools, and design a lesson that embodies and responds to such choices. PMTs are responsible for each step of the design process, and therefore for the first time they produce their own research product. This product, together with the associated theoretical and methodological decisions, functions as the main BO of the activity.

Because PMTs must autonomously coordinate research practices with teaching practices, the activity operates from the outset at a *pragmatic level*. It is here that internalisation processes can be most clearly displayed, as theoretical and methodological components originally external to the PMTs' praxeologies may become integrated as elements that organise and justify their professional decision making. Moreover, MT7 maximises the degrees of freedom available to PMTs, who are no longer responding to predetermined prompts but engage in an open inquiry process. This shift marks a change in the nature of the boundary: rather than working on artefacts produced by the research community, PMTs now generate artefacts that could, in principle, circulate within it. Therefore, PMTs begin to act as legitimate producers of research-informed teaching designs, rather than as external observers or users of research products.

DISCUSSION

The a priori analysis of the activities designed by the two MTEs showed that meta-tasks can be conceptualised as design units that orchestrate the mobilisation of R and T practices around specific objects. The characterisation of the seven meta-tasks indicates that different entry points into the research-practice relationship can be

promoted in the courses. The three meta-tasks analysed in detail illustrate how such design units may have the potential to support PMTs in mobilising research-based theoretical ideas within teaching-related practices.

This analysis also highlights the role that specific objects, such as classroom data, theoretical constructs, design products and research lesson plans, may play as potential BOs that mediate the relationship between R and T practice. Through the lenses provided by MDT.2, these artefacts can be interpreted as objects of joint action that enable PMTs to operate at different levels of discourse (syntactic, semantic, pragmatic), thus supporting potential internalisation processes through which praxeological components originating in research can become gradually integrated into PMTs' emerging professional systems of practice.

The sequence of meta-tasks designed within the courses can be seen as promoting a form of *boundary navigation*. At the beginning of their programme, PMTs do not fully belong to either the research community or the community of teachers; rather, they occupy a position "on the boundary" between communities. Meta-tasks can be understood as design units that foster micro-movements across this boundary. By engaging with objects that originate in research (e.g., theoretical constructs, methodological tools) and in teaching (e.g., classroom data, lesson plans), PMTs may progressively experience a "zig-zag" trajectory through which they explore, negotiate and use practices, languages and rationales characteristic of both communities. In these terms, the potential role of meta-tasks is not merely to convey theoretical ideas, but to structure opportunities for PMTs to inhabit and navigate the boundary between research and teaching in productive ways.

The present report focused on an a priori analysis of the design of meta-tasks and of their potential to support PMTs in mobilising research-based theoretical ideas within teaching-related practices. Future work will analyse the implementation of these meta-tasks by examining PMTs' written productions, oral presentations and interactions with the MTE and peers during the courses. This subsequent analysis will aim to document how PMTs actually work on the artefacts that may function as BOs, how they move across levels of discourse (syntactic, semantic, pragmatic) and whether evidence of internalisation processes can be observed in their emerging praxeologies. In this sense, the a priori analysis presented here constitutes a basis for a subsequent empirical investigation into the effects of the implementation of meta-tasks in initial teacher education.

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