INDETERMINATE OPERATIONS AS A ROUTE INTO EARLY ALGEBRA: A MULTI-MODAL SEMIOTIC ANALYSIS OF A CHINESE PRIMARY CLASSROOM

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This study examines 9–10-year-old students' early algebraic reasoning in representing and solving single-variable linear equations. Adopting Radford's Theory of Objectification, a teaching–learning activity was co-designed in a Chinese primary school with concrete and iconic pan-balance representations. Video and worksheet data from 30 students revealed diverse representations of equality (numbers, letters, Chinese characters, shapes, and balance diagrams), including the culturally embedded Heavenly Stems (天干), a potentially novel finding in English language research. Gestures and multimodal actions supported an embodied understanding of equivalence via symmetry and compensatory operations. I argue for an extension to Radford's framework for algebraic thinking, to include the indeterminate operations.

INTRODUCTION

Starting from the twenty-first century, many research findings have motivated scholars (e.g., Kieran, 2004; Kaput, 2008; Radford, 2014; Blanton et al., 2019) to investigate mathematical activities that can better prepare primary students for the transition from arithmetic to algebra, moreover, studies have found that students are capable of grasping algebraic concepts even before the completion of arithmetic (e.g., Schmittau, 2004; Mellone et al., 2020). However, much of the existing research has focused primarily on identifying broad design principles for instruction. As Kieran (2022) points out, further research is needed to more precisely define the nature of instructional practices that effectively cultivate distinct aspects of early algebraic thinking in young learners.

In Chinese primary mathematics education, there is still a tradition of "arithmetics first, algebra later" approach. According to my examination of the widely used People's Education Press (PEP) mathematics textbooks (2022), although elements of algebraic thinking are introduced in the upper primary grades (year 5 or 6)—particularly in topics such as "using letters to represent numbers" and "simple equations"—the curriculum in grades 1 to 3 remains dominated by arithmetic-based instruction. The gap between lower-grade arithmetic and upper-grade early algebra persists, and several studies have documented the existence of this disconnection for students. For example, as one of the activities in early algebra, the understanding of equations is emphasized early in the fifth grade in China, while recent studies (e.g., Xie and Cai, 2022) showed that there were significant numbers of fifth graders using procedural strategies in solving equations, those students had lower success rates of solving equations with letter

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symbolic unknowns, even after formal instruction of equation solving, and they still tended to think arithmetically and tended to use inverse strategies when encountering letters. This suggests a deficiency in understanding equal signs, unknowns, and equation solving within the context of Chinese mathematics education. This reseach report aims to investigate such emergence of algebraic understanding in learning linear equations with a single variable in a Chinese classroom.

I have the following research question: What student activities can be identified and characterized as algebraic thinking, particularly in the context of learning equations with a single variable?

THEORETICAL FRAMEWORK

For this study, I adopt the framework of algebraic thinking proposed by Luis Radford (2014), which comprises three key dimensions: indeterminacy, referring to the interpretation of unknown quantities; denotation, referring to the formal or informal symbolization of unknowns; and analyticity, referring to operating on unknowns as if they were known. The aim of this study is to identify and investigate students' progressive development in algebraic thinking, particularly in relation to their evolving understanding of the equal sign within linear equations containing unknown variables. Grounded in dialectical materialism and Luis Radford's Theory of Objectification (Radford, 2021), this process of learning and becoming is explored through multimodal social interactions within a sensuous cultural-historical activity. These interactions may include the co-expressive use of speech and gestures, involving the teacher, peers, signs, or artifacts.

In the Theory of objectification, Radford (2021) conceptualizes knowledge as a pure possibility that becomes accessible and integrated into students' awareness through a process of objectification. This process requires active engagement among teachers, students, and mathematical tasks, emphasizing tangible and experiential involvement alongside abstract reflection. Participants use various types of semiotics to articulate their intentions and ideas, drawing on culturally and historically established patterns of thought and behavior. Through the process of objectification, teachers and students collaboratively construct shared and complex meanings, engaging in multimodal actions that foster intersubjective understanding. To observe the emergence of algebraic thinking, co-expressed through meaningful activities, in this study, several designed teaching-learning activities were implemented, guided by the theory of objectification and developed by me, in close collaboration with a teacher in China. One of the key focuses of the activity was fostering a deeper understanding of the equal sign through both concrete and iconic quantitative representations, thus in this study we devised both concrete and iconic pan balance models for learning equations.

DATA COLLECTION AND ANLYSIS

In this study, data collection was conducted in a Chinese primary mathematics classroom with a size of 30 students. The classroom activities took the form of workshops, with three one-hour classes. Classroom activities, including small group

work and whole-class discussions, were observed using video cameras to capture interactions between the collaborating teacher and the students. There were 6 groups of students with 5 students in each group, based on the room size and number of groups, 7 static tripod mounted cameras were used to document whole class instructions, group discussions and classroom dynamics, ensuring a detailed view of both teacher and student interactions, with one main camera set at the back of the classroom facing at the instructor, and the other six cameras facing the small groups. For the process of data analysis, fully transcribed videos, students' written work, and field notes were collected. Weekly research meetings were held, enabling the collaborating teacher and me to review the data, identify key moments, and select episodes that are most relevant to addressing the research aim.

For analyzing gestures, I adopted the classification of gesture by McNeill (1992) into deictic, iconic and metaphoric. Deictic gestures are pointing gestures used to indicate specific objects, events, or abstract concepts, a metaphorical entity is often required when deictic gesture is referring to an abstract concept. Iconic gestures are used to represent concrete object, events or actions, they are closely connected with the semantic content of the speech, these gestures visually mimic or resemble the features of the object or action being described. Metaphoric gestures are used to represent abstract ideas, events, objects, or actions rather than concrete ones, they are often connected with metaphorical content of speech.

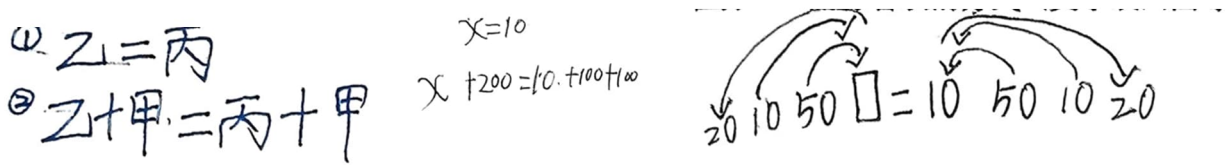
RESULTS AND FINDINGS

In the classroom activity, the teacher introduced a cup filled with sand as the unknown weight, measured as 10 grams, and posed the question, "What operations on the balance will retain its equality?". Episodes are drawn from the session in which students were asked to perform algebraic operations on the equation $x = 10$, first using a real pan balance, then discussing their reasoning in small groups, and finally recording their work on a worksheet. The teacher then selected several student solutions and invited those students to present their approaches at the board. In this activity, the subsequent operations were deliberately structured to preserve equality, which is a foundational algebraic principle, by ensuring that identical transformations are applied to both sides of the equation.

Representing equality on the worksheet

Following the operations on the concrete pan balance, students began to represent and manipulate the iconic pan balance on their worksheets. They first formulated their ideas independently and then collaboratively shared and discussed their proposed methods within their groups. Analysis of the worksheets revealed three primary modes of expression: description in Chinese; alphabetic symbols in English; and graphical diagrams. There was a high degree of variability in how students represented their ideas within each group. The following figures exemplify each type of representations.

Figure 1. Written representations by students.



Notably, the first figure, which exemplifies representation using Chinese characters, employs a traditional method of representing variables known as the Heavenly Stems (天干). The Heavenly stems consist of ten symbols (甲 , 乙, 丙 , 丁 , 戊, 己 , 庚 , 辛 , 壬 , 癸), they have been used in traditional Chinese mathematics and systematic labelling for centuries. While their primary historical role was in calendrics, astrology, and philosophy, in Chinese mathematics textbook the heavenly stems function as placeholders or labels to denote generic entities in mathematical or logical problems. They serve as default labels for unknowns in textbooks, examinations, and puzzles (e.g., mathematics problems). This usage perhaps reflects a cultural preference for systematic, historically rooted symbols that are easily recognizable. I do not believe the use of Heavenly Stems has been reported on before in English language research on learning algebra and I return to their potential significance in the conclusion.

In the third figure, the representation is derived from operations on the concrete pan balance. The student first added a 50-gram weight to both sides of the balance, maintaining equilibrium, and then added 10 grams again while preserving balance. The arrows in the representation signify the operations of adding or subtracting weight. Notably, the student also drew representations of removing 20 grams from both side, although this operation did not occur during the concrete activity, it suggests that the student inferred that removing equal weights from both sides would also preserve balance. This inference appears to stem from conjecturing the underlying quantitative relationship observed during the process of restoring equilibrium, where the restoration occurred as students in the small group took turns operating the pan balance, including removing 10 grams and 50 grams from both sides. The conjecture of subtracting the same weights on boths sides of the pan balance could be an evidence of algebraic thinking (Coles&Ann, 2022). To further examine whether students shift from concrete sensory experience toward grasping underlying quantitative relationships, I will consider multimodal reasoning, encompassing not only written representations but also oral and gestural expressions (Radford, 2021).

To analyze students' multimodal reasoning, three episodes will be presented, each combining gestural figures, verbal utterances, and semiotic alignment. The presentation format is as follows: Gestural figures will be presented and labelled to visually map to its correspondence utterance; utterance timeline is represented in three tiers: Original Chinese texts (CHS) Preserves linguistic authenticity; Interlinear

Morphemic Glossing (IMG) breaks down morphemes to align with gestural timing and semiotic meaning (Lehmann, 2004); English Direct Translation (ENG) provides semantic clarity. Note that the purpose of using IMG is to synchronize gestural and verbal components temporally, and highlight how morphemes map to specific gestures.

Episode 1: Using arrows to represent operations

In the first episode, the student A wrote a representation on the board and began describing the operations.

Figure 1: Student A reprenting the operations using arrows, from A to F

Table 1: Semiotic alignment of student A's motimodal description

CHN	使/沙子里面/加上/20克, 在/ 10克/这里/加上20克
IMG	Let /the sand' inside/add 20 grams, on/ 10 grams/ here /add 20 grams.
FGR	/2A----------/2B(s)-----2B(f)/2C----------/2D---/2E(s)----- 2E(f).
ENG	Add 20 grams on the inside of the sand, add 20 grams on the 10 grams.

I will elaborate on each figure. Figure 2A: Student A performs a deictic gesture, targeting the sand/cup region on the board, however due to the camera angle, the deictic gesture may not exclusively highlight the cup. If the student's finger aligns closer to the numeral 20 (adjacent to the cup), the gesture could instead initiate a procedural chain: pointing to 20 (the addend) and later "arrowing" it toward the cup. This would frame the arrow in 2B as operation to the cup, transferring 20 to the cup's existing $100 + x$ grams—which aligns with the student's verbal emphasis on "addition."

Figure 2B(s)-2B(f): Student A then performs a gesture of arrow: A "gesture of an arrow" typically refers to a bodily movement or hand configuration that mimics the form, direction, or symbolic meaning of an arrow. Arrows are universally recognized as symbols of direction, movement, targeting, or transition, and their gestural representations often carry similar connotations depending on context. In here the gesture of arrow 2B and 2E are metaphorical since they represent a movement of objects, typically meaning by an operation of addition in mathematics.

Figures 2C & 2D: Student A used deictic gestures targeting the numerals 10 and 20 on the right side of the equation, synchronizing with the utterances: "On 10 grams". Figure 2E(s)-2E(f): The student A reproduces the curved arrow gesture from Figure 2B, now directed at the numeral 10 on the right side. This recurrence reinforces the metaphorical mapping of addition as a spatial transfer (e.g., "adding to" as "moving toward").

It is notable that, instead of horizontal movement in bidirectional deictic gesture, a symmetry is observed between the arrow gestures on both sides of the equation. I will define the sequences of arrowing gestures as symmetrical arrowing gestures, they operate as a gestural metaphor of maintaining the equivalence, these symmetrical arrows express operational balance that shifts from embodied understanding from structural to procedural equivalence.

Episode 2: Using metaphors to represent operations

In the second episode, student B demostrated a series of gestures without writing anything on the board.

Figure 2. Student B reprenting the operations using gestures, from A to B

Table 2. Semiotic alignment of student B's motimodal description

CHN	这个是减，然后这个是加.
IMG	This is /minus------/and this is /plus-------/
FGR	--------/4A(s)-4A(f)/----------/4B(s)-4B(f)/
ENG	This is minus, and this is plus.

It is notable that symmetry is observed in both balance metaphor and operational metaphor, Student B creates both structural symmetry and operational symmetry i.e., the position of palms and the hand movement, where symmetry acts as a simultaneous operation on both sides of equations and is reinforced dynamically by bidirectional gestures. I label this type of gesture as a symmetrical metaphoric gesture.

The symmetrical metaphoric gestures of addition and subtraction are also interconnected. The final position of the addition gesture overlaps with the initial position of the subtraction gesture, which is coherent with the procedural logic of preserving equilibrium when operating on the balance: first adding then removing weights in a pan-balance system. The procedural logic of first adding then removing weight was observed in group activities, when students take turns to operate in the small group, one must reset the balance to x=10 grams by removing all added weights, and the action of adding or removing weight on both sides created a need of developing a symmetric gesture of operating on pan balance.

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SYNTHESIS AND CONCLUDING MARKS

In this study, I observed students representing different types of operations on equalities, initiated by the equation x = 10. Although no indeterminate quantities were explicitly involved, the teacher created a space for students to explore indeterminate operations while maintaining the underlying principle that, regardless of what is added or subtracted, balance must be preserved in the end. Based on the findings of this study, I argue activities that involve *operational* indeterminacy, the signification of operations, and embodied actions representing operations also constitute algebraic activity and hence I argue that incorporating operations (as well as quantities) is an extension to Radford's framework of algebraic thinking (Radford, 2014) and offers a route into early algebra that is perhaps under-recognised.

Driven by the small group discussion, students' presentational variations (Figure 1) demonstrate thier capacity to derive and apply the operational logic of equivalence preservation across various notations, including the traditional Chinese characters the Heavenly Stems. As Davydov cautions through Radford (2020, p. 330), "The use of letters should not be seen as something that makes the child enter automatically into the realm of algebra". Thus, frequency of letter-symbolic representation cannot simply reveal algebraic capability, relational understanding emerges through internalizing the transformative logic of equality itself, in this activtiy, it emerges from the concrete-sensual experience of operating pan-balances. I argue that Heavenly Stems are potentially a resource for the Chinese learner, in offering a culturally recognised way of symbolising something indeterminate (like an "x" but a symbol that is familiar to children in a range of contexts). Such culturally constituted symbolization can contribute to forms of algebraic thinking. Further research is needed to explore the extent of the use of these symbols in learning algebra in China and their affordances.

Symmetres in both students written and gestural representations were frequently observed. The equals sign here functioned not merely as a symbolic notation, but as a mediation between concrete sensual experiences and symbolic representation, it seemed positioned as the kinetic midpoint of the pan-balance metaphor, hence the equals sign materialized students' sensual understanding. In other words, the equals sign came to represent equivalence as dynamic equilibrium, maintained through symmetrical action. Students' interactions with pan balances established such experience of symmetry and asymmetry—where any action on one side require compensatory action on the other to maintain equilibrium and imbalance triggered immediate correction. The idea of "restoring the equality" is connected to Davydov's idea, that algebraic thinking does not begin with manipulating symbols, but grasping

universal relationships (Radford, 2020). I note the starting point of instruction, for the Chinese teacher, was an explicit articulation of equivalence ("What operations on the balance will retain its equality?"). The beginning of instruction was an abstract principle, that was made more concrete through the course of activity. Starting with abstract, universal relationships seems a relatively under-used and under-theorised approach to early algebra, barring work inspired by Davydov (e.g., Schmittau & Morris, 2004). Further research is also needed on how algebraic thinking develops from such beginnings.

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