SEMIOTIC NOTICING: A TOOL FOR RESEARCHERS

Miglena Asenova[1], Agnese Del Zozzo[2], and Marzia Garzetti[3]

[1]University of Milan, [2]University of Trento, [3]SUPSI Locarno

This theoretical and methodological paper introduces the construct of researcher's semiotic noticing, defined as the operational use of Semiotic Interpretative Knowledge to analyse students' written productions. Semiotic noticing is grounded in Duval's semio-cognitive theory and focuses on three analytical functions: the identification of distinctive features, treatments, and conversions. Drawing on Rabardel's theory of instrumental genesis, these functions are transformed from a theoretical artefact into an analytical tool through the abstraction of a scheme of use that emerges from the detailed analysis of three students' written solutions.

RATIONALE

Research on teacher noticing has highlighted that professional vision is not limited to "seeing" events or students' work, but involves selecting what is relevant and interpreting it in principled ways (Jacobs et al., 2010; Mason, 2002, 2021). Within this field, aspects related to noticing and interpreting have been further specified through constructs such as Interpretive Knowledge (IK), which captures the knowledge entailed in making sense of students' mathematical productions by inferring meanings, strategies, and underlying understandings (e.g., Ribeiro et al., 2016). In previous work (Asenova et al., 2023, 2025), we addressed a more specific need, linked to the construct of Semiotic Interpretative Knowledge (SIK), that is, the interpretative knowledge related to sign use and transformation. When the object of interpretation is a student's written production, a substantial part of what can be meaningfully noticed and interpreted is semiotic, that is, tied to the forms of representation chosen, the transformations performed within and across representations, and the meanings emerging from such transformations. Although IK and SIK have primarily been used to characterise teachers' in-the-moment interpretations in professional development contexts, they also invite reflection on how researchers themselves come to notice and interpret. As Mason (2021) emphasises, noticing can be understood as a phenomenologically grounded method for researching one's own research practice. Whereas practitioners' noticing is oriented toward acting within and developing one's own teaching practice, researchers' noticing is oriented toward the development of research practice itself, foregrounding reflection on the theoretical constructs and methodological choices that shape how phenomena are noticed, interpreted, and theorised. In this theoretical and methodological paper, we introduce *semiotic noticing* as a construct in continuity with (S)IK, but focused on aspects concerning the researcher's practice. The aim is to provide a systematic analytical lens to account for

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how attention is directed to semiotic features before interpretations are generated and validated, and how these processes can be described through *schemes of use* (Rabardel & Béguin, 2005) that stabilise over time. We are guided by the following research question: *How can semiotic noticing be defined and operationalised through identifiable schemes of use of SIK when analysing students' written productions in the research context?*

First, we provide a theoretical foundation and introduce the method. Then, we present our analysis of three students' written solutions to tasks and define *semiotic noticing* as the enactment of SIK through a stabilised scheme of use. Finally, we discuss the results and draw conclusions.

THEORETICAL FOUNDATION

According to Mason (2021), "The discipline aspect of the Discipline of Noticing is about how to sensitise oneself to situations so as to enable an action that might otherwise be overlooked" (p. 235). As we focus on semiotic aspects of noticing, we need to theoretically anchor what Mason calls "sensitising" in semiotics. Semiotics has a long tradition in mathematics education research (Presmeg et al., 2016). Our contribution aims to show how semiotic theory can be operationalised as an analytic lens for noticing by identifying stable schemes of use. For this reason, we draw on selected elements of Duval's semio-cognitive approach, which offer the "technical tools" to structure these schemes. Duval (1993, 2017) emphasises that mathematical activity crucially depends on coordinating multiple semiotic systems with established sign production and transformation rules—called registers—and on operating transformations within the same register (called *treatments*) and between registers (called *conversions*). Following Duval, D'Amore (2003) highlights that learning is grounded in activating three semiotic functions: (i) selecting and mobilising distinctive features that make a mathematical object recognisable within the representation in a semiotic register; (ii) performing treatments (e.g., symbolic manipulation, reconfiguration of a diagram); and (iii) performing conversions (e.g., from the symbolic to the graphical register). In our approach, these three semiotic functions are treated as an initially theoretical artefact: a set of conceptual resources available in the literature that does not yet determine how it is used in the interpretation of students' work. This stance requires a distinction that is foundational to our framework. On the one hand, we focus on a *semiotic layer*, linked to the forms, structures, and transformations visible in the production and the meanings they afford within particular semiotic registers. On the other hand, we focus on a *conceptual-interpretive layer*, which involves making inferences about how a student may have reasoned. The semiotic layer is not opposed to the interpretative one; rather, it constrains and orients interpretation by making visible what is treated as relevant, what is transformed, and what is coordinated. This distinction allows us to describe interpretive activity without prematurely collapsing semiotic evidence into conceptual conjectures. Following Mason (2002), the semiotic layer can be seen as a particular type of *accounting of*, as

it refers to a description of what is noticed from semiotic point of view using basic semiotic tools (the three semiotic functions), whereas the interpretation layer can be seen as an instantiation of *accounting for*, as it refers to an explanation or justification of (semiotic) noticing through a coherent narrative.

The coordination of these two layers is anchored in the enactment of SIK as "the knowledge needed (...) in order to interpret students' answers (...) when conceptual knowledge is hindered, and thus remains hidden behind difficulties related to patterns of sign use and production" (Asenova et al., 2023, p. 11). SIK is in continuity with the construct of IK (e.g., Ribeiro et al., 2016), but it foregrounds what must be known for interpretations to be grounded in semiotic evidence: knowledge of registers and their affordances, sensitivity to distinctive features as made salient by representations, and competence in tracking and coordinating treatments and conversions.

METHOD

To frame the transition from the semiotic functions as a theoretical artefact to an analytic lens, we draw on Rabardel's theory of instrumental genesis (Rabardel & Béguin, 2005). In this framework, an artefact becomes an instrument through a dual process: *instrumentation*, in which the user's activity is shaped by the artefact, and *instrumentalisation*, in which the user adapts, selects, and organises features of the artefact for their purposes. The outcome of this genesis is the emergence of *schemes of use*: organised and partly stabilised ways of acting that coordinate goals, rules of action, and operational invariants. We apply instrumental genesis to a specific situation: the appropriation of a theoretical artefact (Duval's semiotic functions) for the activity of interpreting students' written productions in research. This move is coherent with previous uses of instrumental approaches in mathematics education that emphasise how material or symbolic tools become functional through the development of schemes (e.g., Rabardel & Béguin, 2005). Here, the "tool" is not a physical artefact but a conceptual one; nonetheless, the same issue arises: a theory can remain a set of notions about representations unless researchers develop workable schemes that make those notions operative in analytic practice. Within our framework, schemes of use are conceived as researchers' schemes: structured ways of directing attention, coordinating semiotic functions, generating hypotheses, and checking coherence between interpretations and semiotic evidence. The focus on schemes serves two purposes. First, it explains why the semiotic functions can become "goggles" for noticing: they become an instrument to see with only through stable schemes that enact them, allowing what Mason (2021) calls *accounting of* and *accounting for*. Second, it provides a way to communicate the construct beyond metaphor, by focusing on the recurrent organisation of analytic actions that constitute semiotic noticing.

We chose three examples of tasks with students' resolution taken from literature or from other scholars' research and teaching practice, and analysed them as we usually do in our research practice on SIK, accounting for instrumentation. Each of us independently produced a text for one of the examples in which she explicitly described

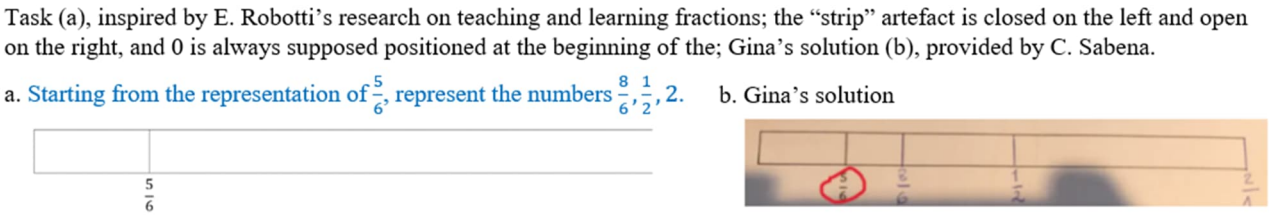
the trajectory of semiotic analysis followed for the individual protocol. We then analysed the texts, searching for invariants indicating schemes of use, accounting for instrumentalisation. The three examples were selected because they involve different school levels and are semiotically rich, as they invite the use of at least two different semiotic systems in addition to natural language; the solutions are unusual, whether correct or incorrect.

ANALYSIS

In the following, we summarise our analysis in the three texts corresponding to examples A, B, and C. An extended version is available at https://tinyurl.com/MAMPME49.

Example A: Fraction with iconic representation and Gina's solution (Figure 1).

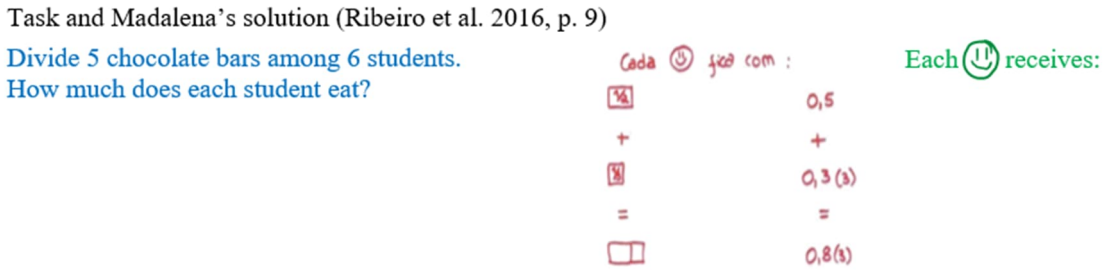
Figure 1: The task is blue-colored in a., Gina's solution in b.



The task involves four numbers expressed in the arithmetic register and an iconic strip representation providing a common reference. It requires identifying the reference unit and interpreting fractions as part–whole relationships. The researcher solved the task, establishing that the correct order of the numbers is $\frac{1}{2}, \frac{5}{6}, \frac{8}{6}, 2$, and that their positions on the strip should be determined proportionally to the unit inferred from the given placement of $\frac{5}{6}$. In analysing Gina's solution, the researcher identified conversions between arithmetic and iconic registers, as she associated each number with a position on the strip, writing it near the corresponding part–whole representation. The researcher also observed a treatment in her use of $\frac{2}{1}$ instead of 2 and in the addition of partition lines. The analysis revealed an instability in the conversion system, as different placements implicitly relied on different reference units. This instability was further supported by the absence of an explicit reference unit and by the positioning of $\frac{2}{1}$ so as to close the strip into a rectangle. Gina also followed the order of the numbers as presented in the task that constituted a relevant semiotic cue.

Example B: Word problem on fractions with Madalena's solution (Figure 2).

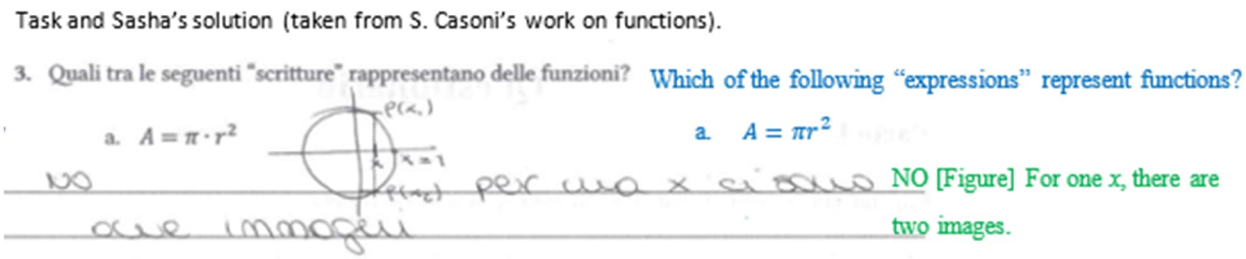
Figure 2: The task is blue-coloured and translation of Madalena's solution in green.



The task presents two integers, 5 and 6, and the mathematical object at stake is fraction as division: a fixed quantity of five chocolate bars is to be equally shared among six students. The researcher solved the task, obtaining $\frac{5}{6}$ in the arithmetic register of fractions. In analysing the student's solution, the researcher identified three registers: the arithmetic register of fractions (e.g., $\frac{1}{2}$ and $\frac{1}{3}$), the arithmetic register of decimals (e.g., 0.5, $0.\overline{3}$, $0.8\overline{3}$), and an iconic register based on rectangular representations of chocolate bars. In the iconic register, the division is represented by partitioning rectangles, while fractions initially appear as part–whole relationships (halves and thirds) and decimals as rational-number representations. The researcher identified treatments through explicit operator-based relations, most notably the equality $0.5 + 0.\overline{3} = 0.8\overline{3}$ in the decimal register, and a corresponding additive structure suggested iconically by adjacent rectangles positioned next to an equality sign. Conversions were inferred from spatial correspondences across registers: the work is organised in adjacent columns, supporting links between $\frac{1}{2}$ and 0.5 and between $\frac{1}{3}$ and $0.\overline{3}$. The correspondence between the iconic and fractional registers is reinforced by the insertion of fraction symbols within rectangles whose relative sizes are consistent with the indicated partitions. The analysis also highlighted a representational absence: the fractional expression of the total result, $\frac{5}{6}$, does not appear explicitly and is present only in the decimal and iconic registers. This suggests a lack of a stable treatment for adding fractions, which appears to trigger a shift to the decimal register, where addition becomes operable and produces the result. In this interpretation, the rectangles support the structure of the whole, whereas decimals provide the computational route when fractional addition is not constructed.

Example C: Task on functions with Sasha's solution (Figure 3).

Figure 3: The task is blue-coloured and the translation of Sasha's solution is in green.



The task provides the expression of the area of a circle as a function of the radius, formulated in the algebraic register, and asks whether it can be considered the expression of a function. The researcher identified the expected solution as affirmative: recognising a functional relationship between the radius r and the area A , according to the definition of a function, allows the situation to be expressed algebraically (e.g., $A(r) = \pi r^2$) and, if needed, graphically as a parabolic branch. In the student's solution, however, the dominant register is the graphical one: the student draws a circumference in the Cartesian plane accompanied by point-like inscriptions and a written explanation. A canonical trajectory would foreground the distinctive feature of a function as a relation between two sets. Identifying the expression as referring to the area of a circle and recognising radius and area as independent and dependent variables would support the conclusion that it defines a function. Instead, the student foregrounds a different distinctive feature, interpreting a function as a curve in the Cartesian plane. The central conversion takes place from the algebraic register to the graphical one, linking the expression $A = \pi r^2$ to a circle. Within the graphical register, the student performs targeted treatments by highlighting two points on the circumference that share the same abscissa, implicitly constructing the configuration required for what is referred to in math classrooms as a "vertical line test" argument. This reasoning is then converted into the verbal register through the explicit statement. The analysis highlights a problematic transformation: the selection of the circumference as the graph preserves a semantic link to the familiar object 'circle' but not the mathematical object defined by the algebraic expression. Moreover, an explicit stabilisation of independent and dependent variables as a functional relation (e.g., $A(r)$) is absent, and the graph is ultimately superfluous with respect to the algebraic meaning of the expression. The researcher interprets the solution as coherent within the student's semiotic framing of function. The student begins with a negative stance and develops an internally consistent argument grounded in a conception of function as a graph that must satisfy the vertical line test. The error thus does not indicate a lack of familiarity with functions per se, but rather a predominant focus on functions as curves in the plane rather than as relationships between variables.

RESULTS AND DISCUSSION

By comparing our texts, we identify a scheme of use that accounts for the instrumentalisation of the semiotic functions into an analytic tool for semiotic noticing. To outline this scheme, we proceeded as follows. For each text produced, the three authors initially selected the relevant actions by identifying linguistic markers related to the semiotic functions, such as representations, registers, distinctive features, conversions, treatments, or verbs linked to the manipulation of the task or the solution. Subsequently, the authors made explicit the sequence of actions carried out. Comparing these sequences led to the identification of a succession of actions that characterise the semiotic analysis process undertaken.

The process was articulated in seven steps (0 to 6):

Step 0: Identifying the semiotic registers and representations in the task and then solving it.
Step 1: Identifying the semiotic registers in the student's solution.
Step 2: Making the distinctive features explicit, with reference to the mathematical object addressed by the task, and identifying which of its aspects and properties are reflected (and reflected back) in the representations proposed in the task and produced by the student.
Step 3: Identifying treatments and conversions.
Step 4: Searching for possible missing representations or for incorrect or superfluous conversions and treatments, in relation to the task goal and the expected result.
Step 5: Interpreting the solution by giving an overall meaning to the set of representations used by the student and constructing one or more narratives that answer the question "How could the student have reasoned?".
Step 6: Reiterating Steps 1–5 to check the coherence of the narrative.

The emergence of the scheme of use shows that semiotic noticing can be defined as the instrumented form of researchers' noticing when SIK is enacted through a stabilised scheme of use. Drawing on Rabardel's instrumental approach, we treat Duval's semiotic functions as an initially theoretical artefact; they become an instrument for researchers' activity of analysing written productions only when coupled with a utilisation scheme. Our analysis suggests that semiotic noticing can be operationalised through the seven-phase scheme of use identified above. Semiotic noticing is therefore not "paying attention to representations" in a generic sense; rather, it is SIK-in-action, that is, a disciplined interpretative activity in which semiotic evidence is systematically produced, connected, and used to constrain and refine interpretations of students' reasoning. The enactment of semiotic noticing can support a fine-grained identification of possible subsequent steps after interpreting a student's solution. For instance, noticing specific semiotic choices prompts questions and allows for the identification of targeted prompts that can be used to design focused interviews aimed at clarifying the underlying reasoning. The results of our research contribute to the literature on noticing in two directions. First, they provide a methodological tool for research on noticing, extending ways of enacting *accounting of* and *accounting for* (Mason, 2002, 2021) by "wearing semiotic goggles," which ensure that actions become available that enable researchers to act differently based on what they notice from a semiotic point of view. Second, they offer a starting point for researchers' training on semiotic noticing as the enactment of SIK, in line with results on SIK enhancement in teacher education (Asenova et al., 2025).

CONCLUSIONS

In this paper, we have provided a definition and operationalisation of semiotic noticing as a theoretical and methodological tool to support researchers interested in fine-grained semiotic analyses of students' solutions. We analysed in depth the enactment of SIK by explicating our semiotic noticing and examined these to identify invariants.

A limitation of the present study concerns the validation of its findings, which, following Mason (2021), cannot be reduced to external criteria alone but requires personal validation through metaphoric resonance with experience and metonymic recognition that enables informed future action. Accordingly, we do not claim that semiotic noticing is necessarily enacted following the scheme of use presented here. Rather, consistent with an instrumental perspective, we assume that the same theoretical artefact may support multiple possible schemes of use and, therefore, multiple enactments of semiotic noticing. As a future direction, we intend to involve other scholars in workshops by inviting scrutiny and feedback on the usability and usefulness of the seven-step scheme presented here. Through such engagement, the scheme may evolve from a theoretical artefact into an instrument for research practice.

References

Asenova, M., Del Zozzo, A., & Santi, G. (2023). Unfolding Teachers' Interpretative Knowledge into Semiotic Interpretative Knowledge to Understand and Improve Mathematical Learning in an Inclusive Perspective. *Education Sciences, 13*(1), 65.

Asenova, M., Del Zozzo, A., & Garzetti, M. (2025). Investigating the effectiveness of a training model for developing Semiotic Interpretative Knowledge in mathematics education. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of PME* (Vol. 1, pp. 67–74). PME.

D'Amore, B. (2003). La complexité de la noétique en mathématiques ou les raisons de la dévolution manquée [The Complexity of Noetics in Mathematics, or the Reasons for a Failed Devolution]. *For the Learning of Mathematics, 23*(1), 47–51.

Duval, R. (1993). Registres de représentations sémiotique et fonctionnement cognitif de la pensée [Registers of semiotic representations and cognitive functioning of thought]. *Annales de Didactique et de Sciences Cognitives, 5*(1), 37– 65.

Duval, R. (2017). *Understanding the Mathematical Way of Thinking: The Registers of Semiotic Representations*. Springer.

Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 41*(2), 169–202.

Mason, J. (2002). *Researching your own practice: The discipline of noticing*. Routledge Falmer.

Mason, J. (2021). Learning about noticing, by, and through, noticing. *ZDM Mathematics Education 53*(1), 231–243. https://doi.org/10.1007/s11858-020-01192-4

Presmeg, N., Radford, L., Roth, W. M., & Kadunz, G. (2016). Semiotics in Mathematics Education. *ICME-13 Topical Surveys*. Springer Open.

Rabardel, P., & Béguin, P. (2005). Instrument mediated activity: from subject development to anthropocentric design. *Theoretical Issues in Ergonomics Science, 6*(5), 429–461.

Ribeiro, M., Mellone, M., & Jakobsen, A. (2016). Interpreting students' nonstandard reasoning: insights for mathematics teacher education. *For The learning of mathematics, 36*(2), 8–13.