YOUNGER CHILDREN'S ENACTING ETHICS WITH MATHEMATICS

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This paper contributes to the ethical turn in mathematics education by illustrating the mutual development of mathematics and ethics in early childhood. Using a design research approach, we implemented an authentic mathematically involved activity in which six young children distributed ten oranges for actual consumption. The findings show that children's ethics were enacted through embodied mathematical trial and error and were inseparable from mathematical activity. The study also indicates that questionable applications of mathematics already emerge in early childhood. Developing generalisable hypotheses remains a task for future research.

INTRODUCTION

An ethical turn has gained attention in mathematics education research (Ernest, 2024). As Skovsmose (2021) argues, the application of mathematics can itself constitute a crisis (e.g., climate change). Developing ethical applications of mathematics has become an important task in mathematics education research. Against this background, Hedefalk and Sumpter (2025) demonstrated that ethics and mathematics are intertwined in young children's reasoning. They provided a valuable first step in a largely underexplored field. However, the children in their study engaged only within a fictional scenario, in which magnets were treated as biscuits.

The purpose of this paper is to illustrate the complexity of how young children enact ethics. Specifically, drawing on data from a larger project, this paper selectively reports on one group of children who enacted ethics through a complex process during a sharing activity. In this sense, the paper does not aim to confirm general claims but instead constitutes an exploratory study of a particular case (Foster, 2023).

THEORETICAL FRAMEWORK

This section outlines the respective frameworks used to identify ethics and mathematics in young children's activity. Throughout this paper, labels in square brackets (e.g., [Others]) denote ethics or mathematics identified within the frameworks.

A framework for identifying ethical dimensions and origins of harm

Hauge et al. (2024) propose a framework that integrates Boylan's (2016) ethical dimensions with Ernest's (2019) discussion of the origins of harm in mathematics. It

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conceptualises ethics as a matter of attending to and balancing multiple, potentially competing concerns. Although this framework was originally developed to analyse the ethical judgements that secondary mathematics teachers make when considering topics to address in their teaching, it may be useful for analysing children's ethical reasoning.

This framework addresses four interrelated ethical dimensions (Boylan, 2016): [Others] being with others, emphasising responsibility for and engagement with the other through dialogical relationships; [Societal/Cultural] the societal and cultural dimension, involving the responsibility towards social justice and democratic engagement; [Ecology] the ecological dimension, which attends to the roles of mathematics in current environmental crises; and [Self] the dimension related to the self, focusing on nurturing self-appreciation and personal integrity.

This framework also introduce five origins of harm (Ernest, 2019): [Curriculum-gap] situations in which mathematics that students need is not chosen as part of the curriculum; [Math-harm] cases where mathematics has negative impacts on learners; [Questionable-use] questionable applications of mathematics, including harmful products or instances of math-washing; [Hidden-harm] hidden harmful applications of mathematics that shape worldviews and actions in negative ways; and [Context-harm] situations in which the issue itself has negative impacts, independent of the mathematics involved.

A framework for identifying mathematics as the science of patterns

Devlin (1994) conceptualises mathematics as "the science of patterns." According to him, patterns in mathematics can be broadly classified into six categories: [Numbers] the patterned activity of organising situations through the use of numbers, including counting; [Thinking] the patterned processes in which individuals explain their own thinking or try to understand others' thinking; [Change] changes that occur over time and through processes of trial and error; [Shape] commonalities and differences observed in the shapes and arrangements of objects; [Regularity] structural features observed in repetition and evenness; and [Position] patterned judgments aimed at maintaining the coherence of activities by using spatial relations. From this perspective, children's behaviours are regarded as mathematical even if they do not intentionally display mathematical behaviours.

METHODOLOGY

This study is situated within a design research framework (Bakker, 2018). The task was collaboratively designed with preschool teachers. As an exploratory study, we do not aim to test general hypotheses about how young children make ethical or mathematical judgements. Instead, drawing on Hedefalk and Sumpter (2025) and Nakawa (2024), the overall design of the study is guided by the working hypothesis that a sharing activity is likely to elicit both ethical and mathematical judgements among young children. This research design enables the present study to identify a phenomenon worthy of further investigation. The research procedures are as follows.

In December 2025, the activity was implemented with a cohort of 47 children aged 4 to 6 years enrolled in a certified centre for early childhood education and care (*nintei kodomoen*) located in an urban area of Kumamoto City in the Kyushu region of Japan. The focal analysis reported in this paper concerns one group of six 5- to 6-year-old children engaged in a sharing task involving ten oranges. The activity lasted approximately 30 minutes and was video recorded. Field notes were taken during and after the activity. The teacher was interviewed about what she noticed during its implementation.

Within the Japanese early childhood education system, holistic learning through play is emphasised, and no formal mathematics curriculum is prescribed. Children are expected to acquire mathematical ideas informally through their daily experiences. Given this context, the participating centre places particular emphasis on dialogic engagement in children's everyday interactions. As a result, lively and sustained discussions frequently emerge among the children.

After approval from the relevant ethics review committee, written consent for the study's implementation was obtained from both the institution and the children's guardians. Children's participation was also guided by their willingness to take part.

The sharing task was designed to provide children with an open-ended situation in which they could make collective decisions about distribution (Nakawa, 2024). Children were encouraged to discuss, propose, and revise ideas through interaction with peers and the teacher, as well as through reflection on the outcomes of distribution.

The task flow comprised four main steps. 1) The teacher asked the children to decide how to share the oranges in a way they considered good for everyone, so that they would eat them together. The children were then asked to show how they had shared the oranges and to explain their reasons. 2) When some oranges remained, the teacher encouraged the children to find a way to distribute all of them. 3) If the children proposed giving the remaining oranges to the teacher, she stated that she was already full and suggested that the oranges might otherwise be thrown away, then waited for their reactions. 4) Once the children agreed that they had reached a good way of sharing, they ate the oranges as distributed. Throughout the activity, the teacher refrained from proposing solutions and created opportunities for collective negotiation.

This paper does not rely on operationalised analytic frameworks. The authors reviewed the video data and collaboratively identified episodes that could be justified by the analytic frameworks. To make effective use of Devlin's framework, the analysis included children's actions and resulting outcomes in addition to their utterances. Episodes for which their reasoning did not initially align were then discussed until a shared interpretation was reached.

RESULT

This section reports on the sharing activity of one group of children and highlights a complex relationship between ethical and mathematical reasoning. After an overview

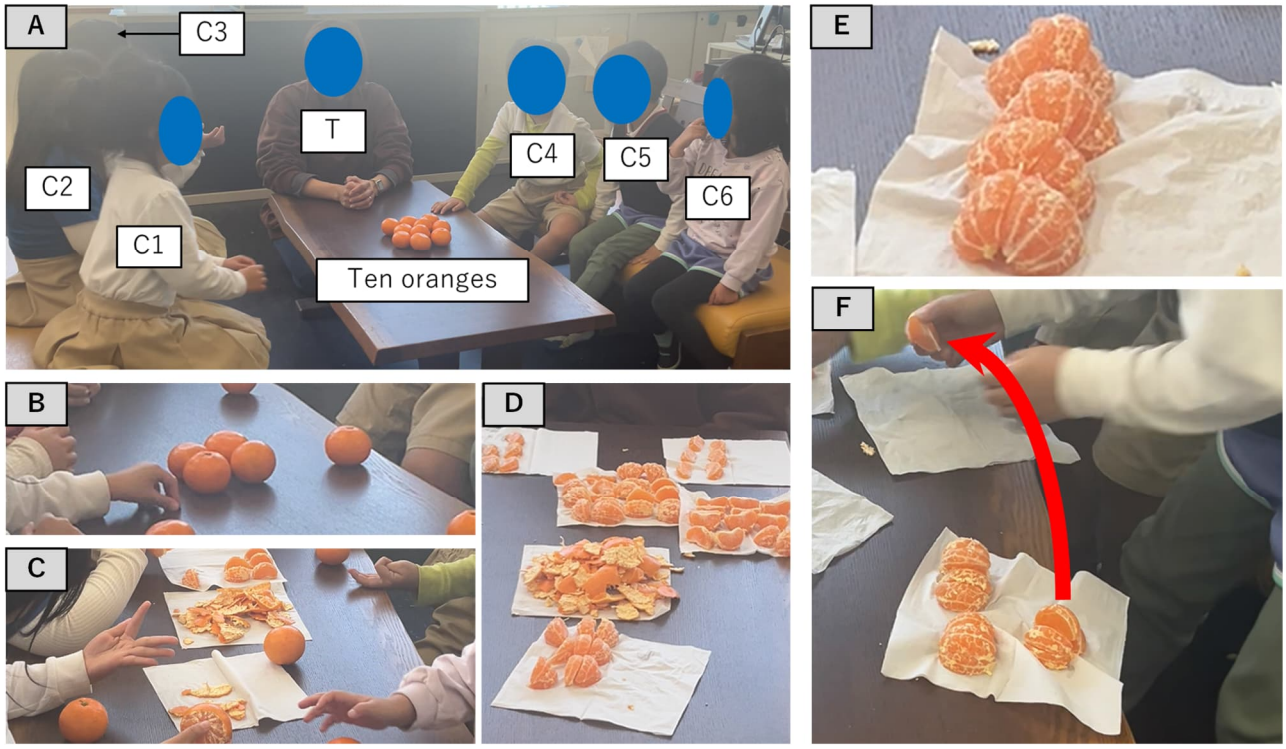
of the episode, the section presents three scenes of particular interest. For reasons of space, brief acknowledgement utterances were omitted from the transcripts.

An overview of the episode

Six children (C1–C6) were asked to distribute ten oranges in a way they considered good for everyone (Figure 1A). They first counted the oranges, confirmed there were ten, and agreed that "equal" sharing meant each child taking one whole orange. They then noticed that four oranges remained (Figure 1B). To address the remainder, the children proposed peeling the four oranges and sharing the segments equally (see Figure 1C). During this process, C6 withdrew from the activity, stating that she did not like oranges, and the activity continued with five children. Although ten oranges would have allowed two whole oranges per child after C6's withdrawal, none of the children proposed this option. Instead, the group continued to treat the task as a remainder problem to be solved at the segment level.

While peeling the four remaining oranges, some children also began peeling the oranges they had already secured. As a result, the plan shifted again. The children decided to peel all ten oranges (Figure 1D), place the segments on the table, and redistribute them one by one. However, coordinating many individual segments proved difficult. They also recognised that the quick arrangement of segments was difficult. At this point, C1 proposed reorganising the segments into dome-shaped half-orange clusters and using these clusters as the unit of distribution (Figure 1E). The group

Figure 1: Overview of the episode
(A: start of the activity; B: four oranges remaining; C: peeling the oranges; D: all oranges peeled; E: dome-shaped half-orange clusters; F: C5 returning one segment)



adopted this new unit and produced nineteen clusters, with some segments left over. This resulted in three clusters per child and one extra cluster.

When the teacher suggested discarding the extra cluster, C5 immediately responded, *"Mottainai!"* ("What a waste!" in Japanese). This response prompted the group to rethink how to redistribute it. The extra cluster was split back into nine segments. C5 then removed one segment from his own dome-shaped half-orange cluster and returned it (Figure 1F). This brought the total to ten segments. The children then took two segments each and agreed that they had found a good way to share.

Excerpt A: Reorganising segments into dome-shaped clusters

Excerpt A depicts a moment when the children peeled all the oranges and struggled to distribute the many segments. At this point, C1 proposed forming dome-shaped half-orange clusters as the unit of distribution.

196	C5:	1, 2, 3, … (omitted) …, 78, 79, 90, no, 80. 81. (Counting each segment individually)
199	C1:	Half each. Half each, about this much.
201	C1:	If there's any left, we'll share it all.
202	T:	In halves?
203	C1:	About this much (pointing to a half-dome orange)

In this scene, two ethical dimensions and four mathematical patterns can be identified. First, the children showed consideration for others [Others] by treating equal distribution as a good way of sharing for everyone. Although this short episode does not allow a definitive conclusion, it may also reflect a broader cultural assumption that sharing is typically equal [Societal/Cultural]. From the mathematical perspective, C5 demonstrated a pattern of counting [Numbers]. It extended to the relatively large number of 81. In addition, C1 displayed a pattern of thinking related to grouping by proposing the formation of dome-shaped half-orange clusters [Thinking], as well as a pattern of reasoning that treated the absence of remainders as evidence of equal distribution [Thinking]. As shown in Figure 1E, the children also improved the distribution's visibility by arranging the clusters sequentially [Position].

Excerpt B: "Throw it away" and "*Mottainai!*" (What a waste!)

Excerpt B depicts a moment when the children judged that they could not distribute the remaining orange segments equally and attempted to resolve the situation by giving the leftover segments to the teacher. As planned during the task design, the teacher told the children that she was already full and that any oranges she received would be thrown away. In response, C5 stated that this would be a waste, and C1 remarked that the oranges might still be shareable. The children then returned to discussing how the remaining segments could be redistributed.

273 T: Right then, I've got it back now, but I'm full, so... This means I'll have to throw this away too, is that alright?

274 C5: Mottainai! (What a waste!)

277 T: What a waste! Right then, I'll give it back. I'll give it back.

278 C1: I think we might be able to share it, so everyone gets one.

In this scene, three ethical dimensions and two mathematical patterns can be identified. First, the children considered the teacher's social position [Societal/Cultural], treating it as acceptable for an authority figure to receive a different number of orange segments than the children. This consideration was related to concern for others [Others], as it was intended to maintain fairness among the children themselves. At the same time, the value of not wasting food was also made salient [Ecology]. From the mathematical perspective, the children recognised that the number of orange segments could not be evenly divided among five children [Numbers]. C1 also expressed that the oranges might still be shareable by distributing one segment to each child [Numbers].

Excerpt C: From nine to ten—eliminating the remainder

Excerpt C depicts a moment when C5 removed one segment from his own half-orange cluster and returned it to the group to facilitate equal distribution (Figure 1F).

291 C4: There are nine segments: 2, 4, 6, 8, 9.

292 T: You have nine segments? Right, what shall we do with these nine segments?

293 C3: Two each for everyone...

295 C4: Then someone might end up with a leftover.

297 C1: Eh, but this orange's size is bigger than the others, you see. This is a big segment.

298 C5: Well then, let me treat my cluster, then do it like this, and make it ten (He takes one back from his own pile of oranges, which he surreptitiously had grabbed. Then, 10 orange segments were there to share two for each child)

Two ethical dimensions are particularly salient in this scene. C1 sought to maintain fairness by attending to differences in the sizes of the orange segments [Societal/Cultural]. C5 returned a segment that he had previously taken amid the confusion, thereby attempting to achieve personal integrity [Self]. It is also noteworthy that when dome-shaped clusters were adopted as the unit of distribution, C5 initially constructed his own cluster with one more segment than the others. This action indicates one possible origin of harm [Questionable-use]. From the mathematical perspective, a variety of patterns can be observed: For example, C4's counting by twos [Numbers], C3's proposal to distribute the segments in pairs [Numbers], C1's shift in focus towards differences in segment size [Shape].\

DISCUSSION AND CONCLUSION

Compared with the activity based on a fictional scenario (see Hedefalk and Sumpter, 2025), the activity in an authentic context shows both similarities and differences. In both studies, ethics and mathematics were closely intertwined in young children's reasoning. However, a key difference lies in the activity's purpose. In the present study, the children were distributing oranges to eat. This practical goal appeared to prompt frequent shifts between ethical and mathematical reasoning. When an attempted distribution failed to achieve equality, the children changed their mathematical views and their strategies (e.g., peeling the oranges, forming segment clusters, placing them sequentially, and accounting for differences in size).

Some ethical concerns (e.g., fairness or reducing food waste) seemed to motivate mathematical reasoning. By contrast, ineffective mathematical approaches prompted children to revise their approaches while maintaining their ethical values. This dynamic interplay highlights how ethical and mathematical reasoning were mutually responsive.

An interesting finding for understanding children's ethics in mathematical activity is that questionable applications of mathematics appear to emerge as early as preschool [Questionable-use]. For example, C5 constructed his dome-shaped cluster with more orange segments than the others. Similarly, even if unintentionally, the children initially focused only on numerical aspects, and no attention was paid to differences in size except for C1. Had volume been considered earlier, the children might have completed the distribution more quickly without breaking all the oranges into individual segments. This case may therefore be understood as an instance of disadvantage arising from the selection of inappropriate mathematics [Math-harm]. It may also reflect a tendency in Japanese early childhood informal activities to privilege number, thereby constituting a curricular gap [Curriculum-gap].

These findings suggest the complex relationship between ethics and mathematics in early childhood, though this paper only focused on a single group. The children's enactment of ethics emerged only through sustained trial and error. Thus, children's ethics were not abstractly defined prior to practice but were shaped by their perceptions and impressions of embodied outcomes. This observation may embody the insight that ethics should be purposively constructed (Dahnke, 2001). Mathematics played a significant role in enabling this embodied enactment of their ethical orientations.

Notably, when the group size decreased to five, the children did not simply redistribute the ten oranges evenly among themselves. Instead, they focused on how to handle the remainder. This may reflect their developmental tendency to use mathematics in ad hoc ways. However, in adulthood, such ad hoc approaches can no longer be regarded as deliberative ethical judgements. These observations point to the importance of educational activities that attend simultaneously to ethics and mathematics.

In conclusion, this paper has illustrated how young children's ethics are enacted with mathematics in an authentic sharing activity. Although this design study does not yield

generalisable findings, it offers rich insights into the mutual development of mathematics and ethics within the ethical turn in mathematics education. A key task for future research is to develop generalisable hypotheses about the relationship between ethics and mathematics in early childhood by accumulating further cases. Also, given the scarcity of equity-related studies from Japan (Davis, 2025), attention to the origins of harm may advance equity in mathematics education.

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