TOWARDS AN ONTOLOGY OF ECOLOGICAL PRACTICES: TEACHING MATHEMATICS AS "NOT KNOWING TOGETHER"

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We situate this theoretical report within the growing swell of interest in socio-ecological and "ecological practices" of teaching mathematics. We take ecological practices to imply a faithfulness to ethics and relationality, and we draw insights from biological ecologies for our work in mathematics education. We argue that ecological practices of teaching mathematics imply an attention to ontology, or the nature of knowing, and, in particular, an ethical stance of "not knowing together". What we mean by this is, in part, a teacher holding open a space of communication in the classroom which allows for the emergence of the new and a supporting of ambiguity. We illustrate what "not knowing together" might look like through a re-looking of some historical classroom data and draw out implications for mathematics education.

INTRODUCTION

This is a theoretical research report, in which we draw on some historical empirical data in an illustrative manner. Our aim is to develop insight into what it means to adopt ecological practices in teaching mathematics, a phrase we explain below, and to draw out broader implications for mathematics education. Our theorising draws on past research presented at PME around socio-ecological approaches to mathematics education (Coles, 2023; Valero, 2023), where key advances have been articulated.

In the next section, we set out some current thinking around ecological practices, relating these to mathematics education. We then offer some context to our theorising, which is mathematics teaching in England (ecological practices cannot be divorced from context). We focus on one dimension of ecological practices, which is the ontological, as we are interested in the nature of knowing and being, in teaching mathematics. We end by illustrating possibilities for ecological practices in England, drawing on illustrative empirical data, and pointing to further implications, including for mathematics teacher education and for further research.

ECOLOGICAL PRACTICES OF MATHEMATICS EDUCATION

Ideas around ecological, or socio-ecological practices of mathematics education are gaining increasing attention, for example as evidenced by an ICMI Study (le Roux et al., 2025) and journal special issues (e.g., Makramalla et al., 2025). There are many strands of thinking within this growing body of research and important theoretical roots in a range of areas, including critical mathematics education, new materialism, Indigenous ways of knowing, socio-political research and more (Makramalla et al., 2025). We take the word "ecological" to imply an expansion of concerns for social

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justice, to include the living world, humans and non-humans as well as non-living material. We also draw on the biological meaning of ecology, to point to the entanglement of the systems of relations which comprise the sites of education.

We draw on Stengers (2005) to take a "practice" to indicate that there are particular ways in which we: generate meaning; educate newcomers; discipline attention; and define problems worth having. These conditions are *local*, not universal (Stengers, 2005). We ask, what is it we need to be *faithful to*, in our practices of mathematics education? For us what makes a practice an ecological practice is that what we are faithful to is ethics and relationality. We do not seek to define what ethics to adopt, nor how to be relational, rather we are proposing that an ecological practice involves a pause and an attention to ethical relations. The instinct to pause is captured well by Stengers, in describing how her own politics does not seek to find or define what is common or true, but rather to hesitate before imposing (Stengers, 2020). Boylan (2016) wrote about the notion of our ecological selves, where we accept ourselves as *not* the bounded individuals which psychology might have us believe, but rather as embedded, entangled, relational beings, inseparable from the contexts in which we act. Our ethical considerations therefore need to extend beyond the human, to include the span of those material relations in which we are engaged.

We hear an ecological practice as a call on us, as mathematics educators, to protect the diversity of ways of making sense in the world, something mathematics education has been particularly poor at doing in the past (Valero, 2023). We believe insights from biological ecologies extend into the social and cultural realm. Biological ecologies are multiple, diverse, and coexisting and practices can be destroyed when disrupted, e.g., industrial agriculture wiping out Indigenous land practices. One of the key ideas driving ecological practices is to break down the separation between nature and culture (de Freitas & Sinclair, 2014; Coles, 2023). For example, we might point to a parallel between the disruption of land practices, and the way neoliberal audit culture is undermining university scholarship in some countries. Ecologies are not harmonised, rather they involve conflict, negotiation, and difference. Hence, in thinking about teaching in terms of ecological practices, we are not wanting to engage in questions such as what is "best practice", as though teaching can be separated from teachers.

Ecological practices, being ethically motivated, are necessarily reflexive and likely (in our case) to turn their gaze to the practices of mathematics education itself. And so, while we are focused on teaching mathematics, we consider briefly our theorising in terms of ecological practices more generally, as part of the conclusion to this report.

In a review of work on research in mathematics education which involves consideration of sustainable futures (and hence work highly relevant to ecological concerns and practices), Makramalla et al. (2025) pointed to a commonality that such work considers "the nature of knowledges" (p.548), and, in particular, acknowledging their multiplicity. We take this prompt to be focused on ontology, and it is ontologies we take up in the next section.

ECOLOGICAL ONTOLOGIES

In this section, we take the ideas above about the broad notion of ecological practices, and we apply them to mathematics teaching, weaving in the context of our concerns in this report, which is secondary schooling in England. We begin with further thoughts on ethics. We are both mathematics teachers and teacher educators and have spent most of our working lives involved with school education. It is therefore with some fear that we acknowledge education can involve harmful practices (Valero, 2023) as this includes work we have done. Valero (2023) pointed to some of the ways in which education creates a "subject" (in students) who, in mathematics, is expected to develop and make use of qualities such as rationalism, abstraction, neutrality. And, for Valero (2023), these are just the qualities of "modern" life which distance us from the living world to the extent that we are destroying the very habitats which are essential for life.

Valero (2023) points to how learning can change the way in which we learn, an idea central to Bateson's (2000) thought. Bateson (2000) warned about what he labelled potential "errors" in our worldviews, including errors in what we take to be the nature of knowing. He promoted the idea of "double description", i.e., focusing attention on relationships and describing them from the perspective of each object being related. For Bateson, double description allows access to novel forms of knowledge, just as two eyes allow depth perception, something unavailable to a single eye.

We want to explore one particular facet of the nature of knowing in the mathematics classroom and this is the stance of "not knowing". The reason for our focus on not knowing is partly that this seems to be rarely recognised and partly, that a stance of "not knowing" seems especially relevant to the ecological practices we envisage. We deliberately move between "not knowing" as a stance of a teacher in relation to students and a stance of students in relation to mathematics, in the spirit of double description.

An ontology of "not knowing"

Taking on the ideas from the previous section, if we accept that the ecologies of mathematics education are multiple, diverse, and not harmonised, what does this imply for ontology, for the nature of knowing, that could be mobilised in classrooms in England? Radford (2025) notes that "educational reflections have shown limited interest in tackling the *ontological* question of the *nature* of knowledge in general, and mathematical knowledge in particular" (p.71). It is in this sense of the word we take up the question of the ontological, which is the aim of our theorizing in this report. And here we return to Stengers' (2020) call for hesitation. If knowing is multiple and diverse, it seems to us that the appropriate stance of the teacher (in relation to others' knowledge, for example) is one of pausing, of listening. Grosz (2025) wrote about psychoanalysis, that it is "a particular form of not knowing" and involves two people "not knowing together" (p. 6). We believe the same is also true of important elements of mathematics teaching. We elaborate further on what such ecological ontologies could mean, drawing on Stengers' (2005) ideas around ecological practices, but first

we place our proposals in the context of two other central perspectives of mathematics education, which we take to be constructivist and socio-cultural.

From a constructivist perspective, teaching proceeds from the assumption that each learner needs to reconstruct, in their own minds, what is to be learnt (Simon, 1995). Meaning comes from internal sense making and, e.g., the development of personal schemas which align with those of more knowledgeable experts. Newcomers need to engage with experts in testing out the alignment of their schema. Attention is disciplined towards one's own sense-making, and the definition of problems is a personal matter. From a more socio-cultural perspective, teaching proceeds from the assumption that to learn mathematics means to participate in a socio-cultural practice (Radford, 2025). Meaning comes from that participation, with others. Newcomers need to engage in socio-cultural practices, often in ritualistic ways (Sfard, 2008), until those practices become meaningful in their own right. Attention is disciplined in ways that have become established through past practices, and the problems of a discipline are typically held by those elders and experts of that practice.

In both constructivist and socio-cultural approaches to teaching, there is knowledge which is known and which newcomers need to take on (either through mental construction or through practice). In contrast, what would practices look like in which teaching was taken as an encounter with the unknown, in which the teacher was oriented towards a stance of recognising the limits of knowing (about students and about the students' knowing of mathematics)? In such a stance, meaning arises through community and is always only provisional. Any sense, as a teacher, that we might think we have made, about our students, is subject to ongoing revision. Newcomers (if these are interpreted as being students) would be encouraged into the space of "not knowing together" as meaning emerges (always provisionally). Attention (for a mathematics teacher) is disciplined to tolerate ambiguity and to support others in tolerating ambiguity; problems relate to just how much ambiguity can be supported.

A teacher's stance of "not knowing" recalls Bion's advice to analysts, to enter therapeutic sessions "without memory or desire" (1970, p. 34). The point is not that we are suggesting mathematics teachers abandon lesson planning. The point is about not being attached to those plans, nor to expecting students to act only in ways they have acted in the past. There is a radical openness to new possibilities. Nor are we suggesting this is a helpful stance for mathematics teachers, at all times, when teaching. However, we are suggesting that a stance of "not knowing" is vital for considering and keeping open possibilities for *who* knows, for *how* knowing happens, and for *what* is and can be known, in the mathematics classroom.

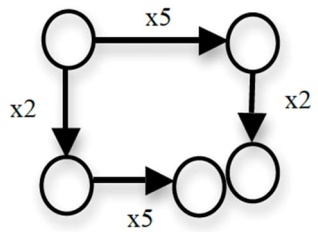
In the next section we draw on some historical data which we re-analyse, in order to illustrate one example of an ontology of not knowing in the mathematics classroom. The context of this data is a secondary mathematics classroom in England, with children aged 11-12 years (which in England is the start of secondary schooling).

ILLUSTRATING AN ECOLOGICAL ONTOLOGY OF "NOT KNOWING"

Below we present a teaching sequence to illustrate one example of what we mean by the ideas developed above. In other words, we are not presenting empirical findings but rather drawing on some historical data in order to continue our theorising about what we see as an ontology of ecological practices of mathematics teaching.

The data was collected in 2008, as part of Alf's doctoral research, in Tracy's classroom. The original data was video recorded and ethical permission gained through the University of Bristol. A transcript was made at the time. The lesson was recorded towards at the end of the academic year; we have chosen this recording as we assume, by this time of the year, the classroom culture and nature of knowing (i.e., ontology) was well established. We offer some snapshots from classroom dialogue, which begins following Tracy presenting Figure 1 on an electronic whiteboard. Numbers go "in" to the top left-hand circle and you work through "both ways" to get two output numbers in the bottom right circles. The transcription conventions are: […] indicates turns of the dialogue missing, in order to meet space requirements for this report, // indicates overlapping speech, (n) is a pause of n seconds, (.) is a pause of less than 1 second, [text] is a transcriber comment, S denotes a student speaking. We intersperse short sections of transcript with reflections on "not knowing".

Figure 1: Image of a "Both Ways" number machine



Tracy	[…] on the board there (2) we've got what I am going to call a number machine (.) right (1) so you put numbers in and you get numbers out (2) someone give me a number (.) S1
S1	seventy-four
Tracy	seventy-four (.) ohh
S2	can it be a decimal
Tracy	well it could be any number I suppose but we are going to work with seventy-four (.) someone tell me what's going to go in there [pointing to the top right-hand circle, i.e., 74 × 5] (4) what goes in that box (7) S3
[…]	
S7	three hundred and seventy
Tracy	three hundred and seventy (.) I'll go with that (.) I have no idea (.) okay (.)

When Tracy says, “I have no idea”, we interpret her commenting on whether 74 multiplied by 5 is 370. Here is an explicit articulation of “not knowing”. We believe it is also genuine, i.e., this is not Tracy knowing the answer but wanting the students to work it out, we interpret the situation here as Tracy’s attention being on the process of working with the class to set up how “Both ways” number machines work. At this point, it actually does not matter to Tracy if the answer is correct or not.

Having worked around the number machine in both directions, the two numbers in the bottom right are written as 740. As Tracy writes the second 740, some students speak:

S8	is that meant to happen miss
S9	yeah it is
S10	no
S11	oh yeah miss
[…]	
Tracy	um (4) any comments (.) any comments (.) yeah
S10	cos it can’t be seven hundred and forty which is the bottom one (.) because seventy-four times five is three hundred and seventy and then (1) no (1) no ignore me
S11	it (.) the two answers in the little circles no matter what you start up there will always be the same (.) because if you start there no matter what (.) if it’s times five times two it’s like times ten
Tracy	so if I’ve got something (.) times five (.) and times two (.) I’ll always get the same as times two times five

Tracy adopts a neutral tone in asking the class “any comments”, with a four second pause, which we take as significant, in terms of offering all students in the class the opportunity to look and to notice. Here we observe a student (S10) articulating an idea and, through the articulation, seemingly changing their mind. We interpret this sequence as an example of “not knowing together”. In the final line, Tracy re-interprets S11’s comment, with no explicit evaluation (of right or wrong). We observe expressions of agreement, disagreement and not knowing being held and tolerated.

In the final section of transcript, Tracy has gone to a new screen on the whiteboard, which has a number machine like Figure 1, but instead of ×5 and ×2, with ×5 and +2.

Tracy	okay I’ve changed the number machine (.) you may not have noticed (.) okay can we have any thoughts at the moment (1) about what’s going to happen in these two circles (1) any thoughts (.) S14 yeah go on
S14	might be (.) still going to be the same
Tracy	same (1) anything else (1) any other comments
S15	I think they’re going to be different because they’re a different order um (4)
[…]	

S16	I think it's going to be the same as well (.) because both numbers are times five and added by two (.) so it's kind of the same thing you're doing
S17	I reckon they're going to be different (.) because if you start with one again (.) if you times by five and plus two it's going to be seven (.) and if you add two first it'll be three and then you times five which is fifteen (.) so plusing on two first will make it a bigger number
Tracy	okay everybody can you draw that at the back of your book (.) don't worry if it's a mess (.) just draw circles squares and two more circles (.) put those in and I want you to choose your own starting number (4) okay you've got two minutes to do that (.) two minutes and no more

Tracy had clearly planned the sequence of Figure 1, followed by a number machine where ×2 is altered to +2. The change is subtle and prompts immediate responses of what will be the same or different to Figure 1. This task sequence was "known" in advance, but what we observe Tracy doing is holding open a space of "not knowing" for students, in not responding to S15, S16 or S17, in their articulation of predictions. Again, we observe the operation of "not knowing together". Tracy cannot have known how students would respond, nor if there would be agreement about this new scenario. She tolerates the unknown and, in doing so, encourages students to tolerate what is currently unknown to them about the number machine, prompting further work.

DISCUSSION

In this report we have argued, in an English secondary school context, that an important element of ecological practices in a mathematics classroom concerns the nature of knowing; such that what is being worked on is "held", in a suspended state, a state we have characterised as "not knowing together". We have argued that the nature of knowing being ambiguous and never fully resolved into certainty is an ontological commitment that stems from a teacher's ethical stance in the classroom.

We want to recognise that the way we have argued and illustrated this view has not touched directly on any environmental contexts (as might have been expected of an ecological practice). The context of England is important here. The current curriculum mitigates against the possibility of cross-curricular work, or broader societal issues becoming present regularly in mathematics lessons. So, when we have been elaborating ecological practices, we have focused on one dimension, ontology, that would be significant were more contextualised, socio-ecological issues to be addressed and that is equally significant in a pure mathematics context.

We offer the theorising in this report in a tentative manner. Ecological practices are context-dependent and so resonances from the ideas we have presented may or may not be found in others' teaching practices. As authors, we support each other in a stance of "not knowing" in our research, as an important state of being, in allowing new ideas to arise. We also recognise, in our practices as mathematics teacher educators, a similar stance of "not knowing", in working with prospective teachers (see Helliwell &

Schwarts, 2023) and in a future publication we aim to elaborate on the parallels between mathematics teaching and mathematics teacher education.

References

Bateson, G. (2000). *Steps to an ecology of mind*. University of Chicago Press.

Bion, W. (1970). *Attention and interpretation*. Tavistock Publications.

Boylan, M. (2016). Ethical dimensions of mathematics education. *Educational Studies in Mathematics*, *92*(3), 395–409. https://doi.org/10.1007/s10649-015-9678-z

Coles, A. (2023). Teaching in the new climatic regime: Steps to a socio-ecology of mathematics education. In M. Ayalon, B. Koichu, R. Leikin, L. Rubel, & M. T. (Eds.), *Proc. 46th Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 1, pp. 17-33). PME.

de Freitas, E., & Sinclair, N. (2014). *Mathematics and the Body: Material Entanglements in the Classroom*. Cambridge University Press.

Grosz, S. (2025). *Love's labour*. Chatto & Windus.

Helliwell, T., & Schwarts, G. (2023). Mathematics teacher educators in an unknowable world: Teaching mathematics for climate justice. In M. Ayalon, B. Koichu, R. Leikin, L. Rubel, & M. T. (Eds.), *Proc. 46th Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 3, pp. 35-42). PME.

le Roux, K., Coles, A., Solares-Rojas, A., Bose, A., Vistro-Yu, C. P., Valero, P., Sinclair, N., Makramalla, M., Gutiérrez, R., Geiger, V., & Borba, M. (Eds.). (2025). *Proc. 27th ICMI Study Conf: Mathematics Education and the Socio-Ecological*. MATHTED & ICMI.

Makramalla, M., Coles, A., le Roux, K., Wagner, D. (2025). Mathematics education for sustainable futures: a strengths-based survey of the field to invite further research action. *Educational Studies in Mathematics*, *119*, 535–556. https://doi.org/10.1007/s10649-025-10389-x

Radford, L. (2025). What does culture have to do with knowledge? A contribution to solving the riddle of the ontological constitution of knowledge from a sociocultural perspective. *Hiroshima Journal of Mathematics Education*, *18*, 71–88.

Sfard, A. (2008). *Thinking as communicating: Human development, the growth of discourses, and mathematizing*. Cambridge University Press.

Simon, M. A. (1995). Reconstructing Mathematics Pedagogy from a Constructivist Perspective. *Journal for Research in Mathematics Education*, *26*(2), 114–145.

Stengers, I. (2020). The earth won't let itself be watched. In B. Latour & P. Weibel (Eds.), *Critical zones: The science and politics of landing on earth* (pp. 228–235). The MIT Press.

Stengers, I. (2005). Introductory notes on an ecology of practices. *Cultural Studies Review*, *11*(1), 183–196. https://doi.org/10.5130/csr.v11i1.3459

Valero, P. (2023). Mathematical subjectivation: Death sentence or chance for a terrestrial life? In M. Ayalon, B. Koichu, R. Leikin, L. Rubel, & M. T. (Eds.), *Proc. 46th Conf. of the Int. Group for the Psychology of Mathematics Education* (Vol. 1, pp. 53-68). PME.