# SEMANTIC CONTAMINATION OF EXPLAINING: THE CASE OF "SETSUMEI" IN JAPANESE PRIMARY SCHOOL MATHEMATICS TEXTBOOKS

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*This study investigates how the meaning of explaining becomes semantically contaminated through its curricular use in Japanese primary school mathematics. Focusing on the Japanese word setsumei, the analysis examines mathematics textbooks from Grades 1 to 6 as an operational curriculum. Drawing on the notion of semantic contamination, explanation tasks were analysed in terms of objects of explaining and types of expected responses (that, how, why). The results show that, while the word setsumei remains unchanged, its normative meaning is reorganised through repeated use: procedural and justificatory forms of explaining are foregrounded from the outset, whereas other everyday possibilities are backgrounded. These findings highlight how curricular language shapes what counts as explanation in school mathematics.*

## INTRODUCTION

In mathematics education, explaining has been positioned as an important mathematical activity through which learners construct and share mathematical meaning. Bishop (1988) identified explaining as one of six key universal activities, characterising it as a social practice fundamental to learning mathematics as a cultural activity rather than merely as a pedagogical technique.

Classroom-based research has further shown that what counts as a mathematically acceptable explanation is not predetermined but is shaped by socio-mathematical norms (Yackel & Cobb, 1996). Explanations in classrooms may range from procedural to justificatory and reflective forms, indicating that explaining fulfils multiple functions and cannot be reduced to a single practice (Esmonde, 2009). Accordingly, even studies that focus on explaining highlight that what counts as an explanation varies considerably across contexts and analytic perspectives (Ingram, 2019). Explanation and explaining have also been discussed in close relation to proof and proving (e.g., Hanna, 2000). While explaining is seen as contributing to mathematical structure and proof (Bishop, 1988), proof itself has been characterised as a specialised form of explanation (Balacheff, 2010), suggesting that the meaning of explanation may be reorganised within mathematical practices.

While prior research has illuminated explaining as a mathematical activity and norm-governed practice, far less attention has been paid to explaining as a *word* used within school mathematics. Explanation belongs to everyday discourse and is introduced into mathematics classrooms without formal definition. As students progress through



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school mathematics, the same word comes to be used in ways that demand mathematically specific forms of reasoning and justification, particularly in relation to proof and proving, without typically being explicitly redefined in a formal sense. Instead, the word continues to be used as if its meaning were self-evident, while the expectations attached to it gradually change. How this semantic reorganisation takes place has remained largely implicit, although it directly concerns what students come to understand as requiring explanation in mathematics.

An important lens for addressing this issue is Pimm's (1987) notion of *semantic contamination*, which describes how mathematical terms are reorganised within mathematical practices while retaining traces of their everyday meanings. This notion is particularly suited to analysing mathematical terms that remain lexically unchanged while being normatively reorganised within mathematical practices. Semantic contamination does not involve incorrect word use, nor does it presuppose a shift from everyday to mathematical meanings. Rather, it refers to the phenomenon whereby, within a specific discourse, some meaning possibilities available in everyday use are foregrounded while others are backgrounded. Mejía-Ramos and Inglis (2011) applied this notion to the words *proof* and *prove*. This perspective resonates with research showing that mathematical terms such as *infinity* are reorganised in relation to everyday language (Kim et al., 2012). Given the close conceptual relationship between explaining, justification, and proof, extending this perspective to explanation is particularly relevant. However, the ways in which explaining is selectively reorganised as a normative mathematical demand through curricular discourse have not yet been examined.

The present study addresses this gap by investigating how the meaning of explaining becomes semantically contaminated through its curricular use in Japanese primary school mathematics. Specifically, this study focuses on the Japanese word *setsumei* (説明), translated as explanation or explaining, and analyses mathematics textbooks to trace how its meaning is reorganised through repeated use across grades. In the Japanese context, textbooks function as a substantive operational curriculum and provide stable linguistic cues indicating what kinds of mathematical activities are normatively expected of students. This focus on the use of the word *setsumei* in textbook tasks therefore provides an empirical basis for examining semantic contamination as a curricular phenomenon. Accordingly, this study addresses the following research question: *How does the meaning of the word setsumei become semantically contaminated through its repeated curricular use across grades in Japanese primary school mathematics?*

## METHOD

### Data source

The data for this study consist of Japanese primary school mathematics textbooks. In Japan, mathematics textbooks are developed in accordance with the *National Curriculum Standard* and its *Teaching Guide*, authorised by the Ministry of Education,

and are legally required to be used by teachers. As a result, they function as a common instructional resource nationwide and play a central role in shaping classroom practices and learners' expectations regarding mathematical activity (Shinno & Mizoguchi, 2021). Because Japanese primary school teachers are generalists who teach all subjects, textbooks are relied upon in mathematics teaching. While textbooks are often regarded as representing a "potentially implemented curriculum" (Valverde et al., 2002), in the Japanese context they function as a substantive operational curriculum. From this perspective, textbooks function to position particular mathematical activities—explaining—as normatively expected at different grade levels. They therefore constitute an appropriate data source for examining how meanings attached to the word *setsumei* are reorganised through curricular use across school mathematics.

This study approaches semantic contamination empirically by examining how the word *setsumei* becomes systematically associated with particular objects of explaining and types of expected response through its repeated use in mathematics textbooks. The analysis focused on textbook tasks that explicitly required students to engage in explaining activity using the word *setsumei*, excluding implicit requests for explanation that did not employ the term itself. To capture changes across grade levels, textbooks from Grades 1 to 6 were analysed. Taking into account their prevalence and nationwide adoption, this study focused on textbooks published by Tokyo Shoseki.

## Framework

To examine how the meaning of *setsumei* is reorganised through curricular use, explanation tasks were analysed by distinguishing between the object of explaining and the type of expected response. This analytical distinction makes it possible to trace how the meaning of *setsumei* is functionally organised within the curriculum, without evaluating the quality of explanations themselves. While the object of explaining literally indicates what students are asked to explain, the expected response reflects the normative function that the explanation is meant to fulfil. Semantic contamination is not observable at the literal level of objects alone, but emerges through changes in how particular objects of explaining become systematically associated with different expected responses. Accordingly, semantic contamination is not interpreted here as the emergence of an entirely new meaning of explanation within mathematical discourse, but rather as the articulation and foregrounding of particular meaning possibilities already available in everyday usage.

In everyday usage, *setsumei* does not correspond to a single discursive function, but may involve stating, describing, or giving reasons, often in overlapping ways. In mathematics education, explaining may also concern not only what is being described, but also how a procedure is carried out or why a mathematical relationship holds (Bishop, 1988; Mejía-Ramos et al., 2019). From the perspective of semantic contamination, the issue is therefore not the emergence of entirely new objects of explaining, but how particular objects become foregrounded and stabilised in relation to specific types of expected responses.

Building on prior research showing that explanations fulfil multiple discursive functions (Esmonde, 2009; Ingram, 2019), this study distinguishes three types of expected responses: *that*, *how*, and *why*. Explanation tasks were coded as *that*-type when students were expected to indicate results or states, as *how*-type when they were expected to describe procedures or methods, and as *why*-type when they were expected to articulate structures, relationships, or underlying principles. This distinction does not evaluate the sophistication of explanations, nor does it assume a developmental progression from that- to why-type responses. Rather, it captures the type of implicit question to which an explanation is normatively expected to respond within the curriculum. The object of explaining and the expected response type were coded independently.

Following this classification, explanation tasks were compared across grade levels to identify systematic tendencies in how the meanings attached to *setsumei* were reorganised over the course of primary school mathematics. Quantification was used to make such tendencies in curricular language visible across grades. While semantic contamination has been discussed primarily as a theoretical or interpretive concept in prior research (e.g., Mejía-Ramos & Inglis, 2011; Pimm, 1987), the present study operationalises this notion through an analysis of curricular language across grade levels.

## RESULTS

The overall results of the analysis are presented in Table 1. No explanation tasks were found in Grade 1. Explanation tasks appeared from Grade 2 and showed a relatively consistent level from Grade 3 onwards. From Grade 3 to 6, although the total number of explanation tasks varied slightly, their overall frequency within the textbooks remained relatively stable. In addition, the proportion of explanation tasks relative to the overall number of textbook pages showed little variation across these grades.

Table 1: Distribution of explanation tasks by grade and response type (Note. T, H, and W denote *that*-type, *how*-type, and *why*-type responses, respectively.)

| Grade | Idea | | | Way | | | Reason | | | Others | | | Sum | | | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | T | H | W | T | H | W | T | H | W | T | H | W | T | H | W | |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 1 | 7 | 2 | 0 | 4 | 0 | 0 | 0 | 10 | 2 | 1 | 2 | 3 | 12 | 14 | 29 |
| 3 | 2 | 13 | 6 | 0 | 23 | 0 | 0 | 0 | 18 | 2 | 1 | 2 | 4 | 37 | 26 | 67 |
| 4 | 0 | 16 | 5 | 0 | 27 | 1 | 0 | 0 | 28 | 2 | 0 | 8 | 2 | 43 | 42 | 87 |
| 5 | 0 | 12 | 12 | 0 | 18 | 1 | 0 | 0 | 20 | 0 | 0 | 3 | 0 | 30 | 36 | 66 |
| 6 | 1 | 12 | 12 | 0 | 13 | 6 | 0 | 0 | 13 | 2 | 0 | 6 | 3 | 25 | 37 | 65 |

Across all grades, the most frequent objects of explaining were *idea* (考え, *kangae*), way (方法・仕方, *hōhō* / *shikata*), and reason (わけ・理由, *wake* / *riyū*). Clear differences were observed in how these objects were associated with expected response types. The strong association between explaining a *way* and *how*-type responses, and the fact that this association is consistently observed across grades, may not be surprising. However, in Grade 6, *why*-type responses also appeared in a limited number of tasks focusing on explaining a *way*.

Tasks focusing on explaining an *idea* showed greater variation across grade levels. In the lower grades, *how*-type responses were dominant, indicating that explaining an *idea* was primarily associated with describing procedures or methods. In the upper grades, *why*-type responses increasingly appeared alongside *how*-type responses, suggesting a reorganisation of the expected form of explanation associated with the same object.

*That*-type responses were infrequent and remained peripheral across grades, whereas *how*-type and *why*-type responses constituted the majority of explanation tasks. From Grade 3 onwards, when the overall volume of explanation tasks stabilised, the proportion of *why*-type responses increased gradually, while *how*-type responses showed a corresponding relative decrease.

Among the three response types, the *why*-type had the highest overall frequency. For this reason, a more detailed analysis of *why*-type explanation tasks was conducted. Subcategories were constructed bottom-up from the data, resulting in four categories: *procedure*, in which procedures functioned as grounds for justification; *judgement*, which involved reasons supporting judgements of correctness, selection, comparison, or classification; *structure*, which focused on mathematical relationships, properties, or structures; and *error*, which involved identifying the location or nature of errors in others' work (see Table 2). These observations concern the relative composition of *why*-type responses within each grade and focus on proportional patterns within *why*-type responses, highlighting differences in their internal structure across grades.

Table 2: Internal composition of *why*-type explanation tasks

| Grade | Procedure | Judgement | Structure | Error |
|---|---|---|---|---|
| 2 | 2 (14.3%) | 6 (42.9%) | 3 (21.4%) | 3 (21.4%) |
| 3 | 2 (7.7%) | 13 (50.0%) | 7 (26.9%) | 4 (15.4%) |
| 4 | 9 (21.4%) | 9 (21.4%) | 13 (31.0%) | 11 (26.2%) |
| 5 | 9 (25.0%) | 12 (33.3%) | 13 (36.1%) | 2 (5.6%) |
| 6 | 19 (51.4%) | 7 (18.9%) | 9 (24.3%) | 2 (5.4%) |

In the lower grades, *judgement*-related *why*-type responses were relatively prominent compared to other grades. In the upper grades, *procedure*-related *why*-type responses increased markedly, accounting for more than half of all *why*-type responses in Grade

6. In contrast, *structure*-related explanations appeared consistently across grades, showing relatively little variation. *Error*-related explanations appeared in all grades but remained relatively infrequent, accounting for only a small proportion of *why*-type responses throughout the grades.

## DISCUSSION

This study examined how the meaning of the word *setsumei* (説明) becomes semantically contaminated through its curricular use across grades in Japanese primary school mathematics. Rather than treating explanation as a general communicative activity or an individual cognitive practice, the analysis focused on how the word *setsumei* is used within textbooks in relation to particular objects of explaining and normative explanatory demands. The findings suggest that semantic contamination does not manifest as a simple expansion or refinement of meaning, but as an implicit reorganisation of what counts as explaining in school mathematics.

Across all grades in which explanation tasks appeared, *that*-type responses remained peripheral, while *how*-type and *why*-type responses constituted the core of normative expectations associated with explaining. This tendency resonates with prior research that conceptualises explaining as a normatively regulated practice shaped by socio-mathematical norms (Yackel & Cobb, 1996), suggesting that such norms are also observable at the curriculum level. From the perspective of semantic contamination (Pimm, 1987), this finding is significant not because everyday meanings of explanation are replaced by mathematical ones, but because they are selectively reorganised through use. A central characteristic of this process is that meanings change without corresponding changes in terminology. As in the case of proof and proving (Mejía-Ramos & Inglis, 2011), what is reorganised is not the word itself, but what counts as an adequate explanation. This reorganisation is not explicitly marked in the curriculum: textbooks do not signal that *setsumei* now refers to a different kind of activity. From the learner's perspective, the same word continues to be used, while the norms for an adequate explanation change. In this sense, semantic contamination operates through the selective foregrounding of procedural and justificatory forms of explaining, while other everyday possibilities are backgrounded.

Importantly, this specialised use of *setsumei* is evident from the point at which explanation tasks first appear in the curriculum. From Grade 2 onwards, explanation is already tied primarily to *how*- and *why*-type responses. This finding challenges a developmental interpretation in which explanation is understood as gradually becoming more mathematical as students progress through school. Rather than reflecting developmental growth in students' explanatory competence, the observed tendencies point to a reorganisation of normative expectations introduced from the outset through curricular language. In this sense, explanation does not progressively evolve into a mathematical practice; it is normatively positioned as such through its repeated use in mathematics textbooks.

The analysis of objects associated with explanation tasks further clarifies how semantic contamination operates at the curricular level. Explanation tasks were linked to a limited set of objects—*idea*, *way*, and *reason*—while other potential objects of explaining remained marginal. Moreover, the association between stated objects and response types was systematic. This tendency does not indicate a growing distance from everyday uses of explanation. Rather, it shows how particular explanatory possibilities are selectively stabilised within the curriculum. By repeatedly coupling *setsumei* with particular objects and responses, the curriculum foregrounds specific meanings of explanation and backgrounds others, thereby establishing normative expectations for what counts as explanation.

More broadly, these findings contribute to research on mathematical language and curriculum by highlighting how everyday words acquire specialised disciplinary meanings through institutional use. As in the case of proof and proving (Mejía-Ramos & Inglis, 2011), semantic contamination in explaining should not be understood as a problem to be corrected, but as a constitutive mechanism through which mathematical practices are shaped. In the Japanese context, this mechanism is closely tied to textbooks, which function as a substantive operational curriculum. Although textbooks do not explicitly define explanation or describe how its meaning changes across grades, they play a central role in stabilising normative expectations for explaining through repeated use.

## CONCLUSION

This study investigated how the meaning of the word *setsumei* (説明) becomes semantically contaminated through its curricular use across grades in Japanese primary school mathematics. In response to the research question, the findings show that semantic contamination operates through the repeated and systematic use of *setsumei* in textbook tasks, which associates the same word with different normative explanatory demands. While the word itself remains unchanged, what counts as explaining is reorganised through curricular use.

By empirically tracing semantic contamination at the level of curricular language, this study extends prior research on explanation, justification, and proof in mathematics education. It demonstrates that explanation is not merely an everyday term imported into school mathematics, but a curricular language practice through which disciplinary meanings are articulated and normatively regulated. Attending to such processes is essential for understanding how students come to recognise what should be explained and what forms of justification are valued in mathematics.

Several limitations of this study should be acknowledged. First, the analysis focused on explanation tasks that explicitly used the word *setsumei* and did not include implicit requests for explanation. Accordingly, the findings illuminate how the meaning of *setsumei* is shaped through its explicit curricular use rather than capturing the full range of explanatory practices in Japanese primary school mathematics. Second, this study analysed textbooks as curricular documents and did not examine how explanation tasks

are enacted in classroom practice. Future research combining curriculum analysis with classroom discourse analysis would be valuable for examining how semantic contamination is taken up, negotiated, or transformed in practice.

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**References**

Balacheff, N. (2010). Bridging knowing and proving in mathematics: A didactical perspective. In G. Hanna et al. (Eds.), *Explanation and proof in mathematics: Philosophical and educational perspectives* (pp. 115-135). Springer.

Bishop, A. (1988). Mathematics education in its cultural context. *Educational Studies in Mathematics, 19*(2), 179-191.

Esmonde, I. (2009). Explanations in mathematics classrooms: A discourse analysis. *Canadian Journal of Science, Mathematics and Technology Education, 9*(2), 86-99.

Hanna, G. (2000). Proof, explanation and exploration: An overview. *Educational Studies in Mathematics, 44*(1), 5-23.

Kim, D.-J., Ferrini-Mundy, J., & Sfard, A. (2012). How does language impact the learning of mathematics? Comparison of English and Korean speaking university students' discourses on infinity. *International Journal of Educational Research, 51-52*, 86-108.

Ingram, J., Andrews, N. & Pitt, A. (2019). When students offer explanations without the teacher explicitly asking them to. *Educational Studies in Mathematics, 101*(1), 51–66.

Mejía-Ramos, J. P., Alcock, L., Lew, K., Rago, P., Sangwin, C., & Inglis, M. (2019). Using corpus linguistics to investigate mathematical explanation. In E. Fischer & M. Curtis (Eds.), *Methodological advances in experimental philosophy* (pp. 239-264). Bloomsbury.

Mejía-Ramos, J. P., & Inglis, M. (2011). Semantic contamination and mathematical proof: can a non-proof prove? *The Journal of Mathematical Behavior, 30*(1), 19-29.

Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms*. Routledge & Kegan Paul.

Shinno, Y., & Mizoguchi, T. (2021). Theoretical approaches to teachers' lesson designs involving the adaptation of mathematics textbooks: Two cases from *kyouzai kenkyuu* in Japan. *ZDM Mathematics Education, 53*(6), 1387–1402.

Valverde, G. A. et al. (2002). *According to the book: Using TIMSS to investigate the translation of policy into practice through the world of textbooks*. Springer.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458-477.