KNOWLEDGE CATEGORIES FOR SUPPORTING TEACHER LEARNING: EXPLORING FACILITATORS' FOCUS

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The quality of professional development (PD) essentially depends on facilitators and their knowledge. However, research shows that facilitators primarily draw on general pedagogical knowledge (GPK) and less on content-specific aspects for teacher learning. To further investigate this phenomenon, we examined how 54 facilitators reacted to a fictional PD-situation, examining the knowledge categories they used and the focus they pursued within these categories. The findings indicate a strong focus on classroom content without reference to the PD-level, indicating a struggle between the roles of facilitator and teacher. We also found that many facilitators used GPK, especially with a focus on general moderation. Therefore, content-specific support for teacher learning should be an integral part of facilitators' qualification.

INTRODUCTION

In the context of professional development (PD) programs for mathematics teachers, facilitators play a pivotal role. The efficacy of these programs and the quality of teacher learning depend on facilitators' contributions (e.g., Borko et al., 2014). Often, facilitators are teachers who provide PD courses and are not specifically prepared for the demanding role of facilitating teacher learning (Borko et al., 2014; Pöhler-Friedrich et al., 2024). Over the past two decades, research has focused on identifying the essential components of facilitator expertise (Borko et al., 2014; Prediger et al., 2022). Several frameworks have emerged that describe the content-specific knowledge facilitators need in order to lead mathematics PD (e.g., Prediger et al., 2022; Borko et al., 2014). These frameworks draw on the knowledge components emphasized by Shulman (1986) for teacher knowledge. Among these components is pedagogical content knowledge at the PD level, i.e., how facilitators can help teachers understand and implement content in their lessons. However, the literature shows that, although generic aspects of facilitation are already well described (e.g., discussion leading and establishing a learning culture), providing content-specific support for teachers, especially regarding complex mathematical content, remains a growing field of research (Bialy et al., 2024).

In this study, we examine the relevant knowledge categories that facilitators drew upon when responding to a content-specific fictional PD situation, involving a classroom task on unit conversion and a teacher assisting a student. Specifically, we focus on the aspects, facilitators bring in to explain the answers within the knowledge categories.

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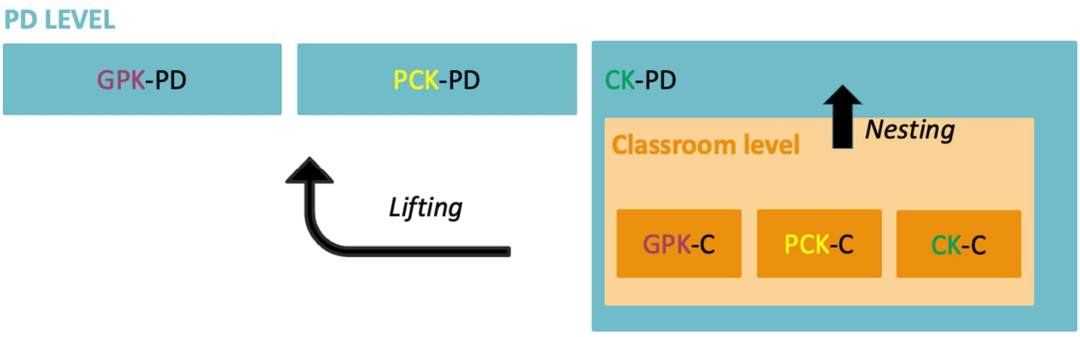
THEORETICAL FRAMEWORK

Numerous studies have demonstrated that facilitators require knowledge at the classroom and professional development levels, which are intertwined (Pöhler-Friedrich et al, 2024). Some authors have coined the term "mathematical knowledge for PD" to describe this specific type of knowledge (Borko et al., 2014), while others have adapted Shulman's (1986) influential classroom-level work for the PD level (Prediger et al., 2022). Based on the latter, three main categories can be distinguished at the PD level (Figure 1). First, facilitators must possess *generic pedagogical knowledge at the professional development level (GPK-PD)*, which encompasses fundamental aspects of teacher learning that are not content-specific. Second, they require *pedagogical content knowledge at the professional development level (PCK-PD)*, which encompasses aspects of content-specific teacher learning. More specifically, PCK-PD can be described as the "ability to engage teachers in purposeful activities and conversations about those mathematical concepts and relationships and to help teachers gain a better understanding of how students are likely to approach related tasks" (Jacobs et al., 2017, p. 3). Finally, facilitators must have a thorough understanding of the content that teachers will acquire during the PD program. According to Prediger et al. (2022), this *content knowledge at the professional development level* (CK-PD) includes the knowledge categories necessary at the classroom level: mathematical *content knowledge* (CK-C), *pedagogical content knowledge* (PCK-C), and *generic pedagogical content knowledge* (GPK-C).

Figure 1: Facilitator knowledge categories (Prediger et al., 2022)



For example, facilitators of a PD course on unit conversion, must have a comprehensive understanding of the relevant content at the classroom level. As Kilpatrick et al. (2001) indicated, it is important to cultivate both procedural skills and conceptual understanding. Thus, to successfully execute unit conversion, students need to develop an understanding of the underlying bundle structure of multiplicative thinking, as Peters and Prediger (2025) stated. In addition to the aforementioned aspects of CK-PD, facilitators must be able to explain the importance of this component of conceptual understanding to teachers (PCK-PD) and in the process foster a conducive learning environment (GPK-PD).

Previous findings by Bialy et al. (2024) have demonstrated that facilitators often prioritize GPK elements when discussing a specific PD situation. This aligns with the observation that facilitator professional development (FPD) courses often prioritize these general GPK elements. However, this results in insufficient coverage of important content-related preparation aspects, as noted by Prediger et al. (2022). Another possible explanation is that facilitators navigate multiple identities and sometimes prioritize their teacher identity. This could explain a focus on CK-PD (Schwarts et al., 2025). Schwarts et al. (2025) also found that novice facilitators tend to avoid confrontations with teachers, which could result in merely focusing on GPK-PD.

PRESENT STUDY AND RESEARCH QUESTIONS

The phenomenon of the various knowledge facets at play when facilitators moderate PD discussions has been well-documented. However, little is known about facilitators' specific perspectives on the content and generic aspects of facilitation. In our study, we examine the knowledge categories that facilitators apply when asked to support teacher learning in a specific PD situation. Additionally, we aim to investigate the applied knowledge categories and the focused aspects by the facilitators within these categories. Doing this we concentrate on the mathematics topic of unit conversion, an area that has been overlooked in previous research on mathematical learning (Cheeseman et al., 2017). The subsequent research questions are thus pursued in the study.

RQ1: Which knowledge categories (GPK-PD, PCK-PD and CK-PD) did the facilitators primarily use when responding to a fictional PD situation regarding supporting teacher learning in the context of unit conversion?

RQ2: What focus did the facilitators pursue within the applied categories?

METHODOLOGY

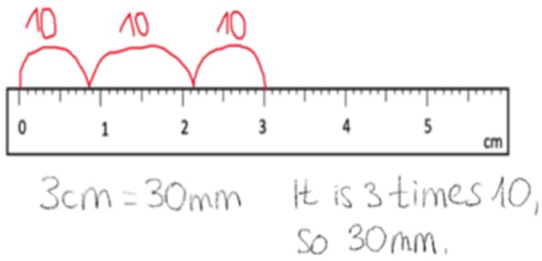
Participants, context, and data collection

The data for this study were collected as part of QuaMath (Prediger & Selter, 2024), a ten-year PD program that began in Germany in September 2023. The program aims to improve the quality of mathematics instruction in the long term. Fifty-four primary mathematics teachers took part in a PD program designed to qualify them as facilitators (FPD), and participated in the current study (N=54). On average, the facilitators had around 18 years' experience of teaching and five years' experience of leading PD for teachers.

After completing the one-year basic module of the FPD program, the facilitators selected the "Sizes and Measurement, Data and Chance" content module. Following the half-year qualification, facilitators will deliver PD courses for mathematics teachers on this topic, integrating teaching experiments and structured reflection phases for the participating teachers. As with all QuaMath PD courses for teachers and facilitators,

there is a specific focus on helping teachers foster their students' conceptual understanding. In our study, the facilitators were asked to respond to a fictional PD situation concerning the support of teacher learning in developing the aforementioned focus on understanding (Figure 2).

Figure 2: Vignette showing a fictional PD situation to react as a facilitator

Student solution: 	**PD situation:** In your PD course, you are discussing the student's solution with a focus on understanding the means of unit conversion. A teacher reacts as follows: "I would tell the child to count the millimeter markings on the ruler to prove whether there are always ten."
Task for facilitators: How would you react as a facilitator to the teacher's comment to support his or her learning?	

The facilitators had previously worked on a similar task regarding the conceptual understanding of multiplication in other FPD courses. Therefore, the facilitators were required to apply their existing knowledge to the new content of unit conversion. In the vignette, the facilitators were given an incorrect student solution for demonstrating a unit conversion from centimeters to millimeters using a picture of a ruler. Then, the teacher asked the student to count the marked millimeter markings on the ruler and prove that there are always ten. Thus, the teacher's response does not support the student's conceptual learning. It fails to explain why the representation is inappropriate and does not engage with the underlying concept. Instead, the teacher focuses on procedural aspects, overlooking opportunities to improve the student's conceptual understanding. The facilitators were asked to react to the teacher's statement considering that the student should understand what unit conversion means (Figure 2).

Data analysis

A qualitative content analysis was conducted to analyze the facilitators' answers. To answer RQ1, we drew deductively on different categories of facilitator knowledge for the coding system (Prediger et al., 2022). This enabled us to categorize GPK, PCK, and CK at the PD level and identify the primary knowledge category used by the facilitators. For RQ2, we divided the three knowledge categories into subcategories, arriving at them inductively from the data, revealing what was relevant from the facilitators' perspective. For GPK-PD, which focuses on non-content-specific teacher learning, the following subcategories were identified: *focus on moderation*: Facilitators elaborated on moderation strategies without mentioning specific content aspects. Consequently, they did not encourage teachers to consider their students' conceptual understanding; *focus on general learning*: Facilitators emphasized the importance of

fostering the learning process in general, without explicitly referencing the mathematical content of the task; and *focus on atmosphere*: Facilitators focused on creating a positive environment in their PD session.

In contrast PCK-PD emphasizes content-specific teacher learning. Categories were subdivided according to whether they contained aspects *supportive of content-specific teacher learning* or *referenced irrelevant aspects*. Aspects that supported content-specific teacher learning focused on fostering children's conceptual understanding. Those that paid attention to teaching arithmetic procedures or diagnosing conceptual understanding without questioning whether the child truly understood the concept of unit conversion due to an inappropriate drawing were categorized as irrelevant.

The subcategories for CK-PD have an explicit focus on student learning and distinguish between answers that provide insight into content knowledge with a *focus on general student learning* (e.g. active learning), responses that *support content-specific student learning* and those that *refer to irrelevant aspects*.

If a facilitator's answer addressed multiple aspects, it could also be assigned to several knowledge categories or subcategories. Twenty percent of the facilitator's reactions were coded by two members of the research team ($\kappa = 0.89$). After the qualitative analysis, the absolute frequencies and percentages for each category were determined.

RESULTS

With respect to RQ1, a total of 61 codes were assigned to facilitator responses. Most answers fell into the CK-PD category (44.2 %), in which facilitators focused explicitly on student learning. The second most common category was GPK-PD (34.4 %), with facilitators not emphasizing content-specific learning. Only 21.4 % of the reactions were assigned to the PCK-PD category, which indicates an explicit focus on supporting teacher learning with classroom-level content. With respect to RQ2, we were able to identify different subcategories in the facilitator reactions to the fictional PD scene, aiming at supporting teacher learning. Table 1 shows the absolute frequencies and percentages of each category.

Table 1: Absolute frequencies and percentages of the knowledge categories and subcategories used by the facilitators

knowledge categories	**subcategories**	**frequencies**	**percentages**
GPK-PD:	**in total**	**21**	**34.4 %**
no focus on content-specific teacher learning	focus on moderation	13	
	focus on general learning	4	
	focus on atmosphere	4	
PCK-PD:	**in total**	**13**	**21.4 %**

focus on content-specific teacher learning	supportive of content-specific teacher learning	12	
	references to irrelevant aspects	1	
CK-PD:	**in total**	**27**	**44.2 %**
explicit focus on student learning	focus on general learning	4	
	supportive of content-specific student learning	15	
	references to irrelevant aspects	8	

Of the thirteen responses coded as PCK-PD (21,4 %), twelve were found to be *supportive of content-specific teacher learning*. When facilitators referred to PCK-PD, we gained evidence that they substantially *supported content-specific teacher learning*, as the following answer indicates:

> I would respond that counting is a viable option for smaller numbers, but not an effective strategy for larger ones. I would ask the other teachers if they could suggest any strategies other than counting. If they could not think of any, I would refer to the connection with multiplication. If necessary, I would mention group language as a keyword.

In this answer, the facilitator addresses the relevant PCK elements by referring to the multiplicative structure of unit conversion and linking conceptual understanding to language-responsive teaching. The facilitator encourages teachers to think about the mathematical concept of unit conversion and supports their learning by helping them focus on conceptual understanding. Only one response in the PCK-PD category contained *references to irrelevant aspects* because it focused on teaching arithmetic procedures.

For GPK-PD (34.4 %) three subcategories were identified in the data. Thirteen of twenty-one answers were categorized as *focus on moderation*, as they addressed how the facilitator would further moderate the discussion regarding the teacher's statement, without explicitly mentioning supporting teacher learning to build up conceptual understanding in their lessons: "I would put that out there for discussion." The second subcategory consisted of reactions with a *focus on general learning*. The following statement is indicative of this subcategory: "Tip for the teacher: How can you better support the child in coming to this realization on their own?" There were four of these reactions in total. They emphasized the importance of an active learning process for the child, such as discovering one's own mistakes when responding to the teacher's statement. This reaction does not delve into conceptual understanding or mentions the mathematical content of unit conversion. Finally, four facilitators' answers were coded as a *focus on atmosphere* such as: "Appreciate and carry on." They responded with

appreciation for the teachers' comments in order to create a positive learning environment. However, like all answers in the GPK-PD category, they did not elaborate on unit conversion and the required conceptual understanding.

Regarding CK-PD (44.2 %), three subcategories emerged. Four responses from facilitators showed a *focus on general learning* of the student, for instance: "What do the other children think about this solution?". Fifteen other reactions were *supportive of content-specific student learning* by elaborating on relevant aspects to foster the student's conceptual understanding, as shown by the following example:

> […] The child should understand that 10 millimeters is always 1 centimeter. After the first arc, they should recognize without counting, but through cognitive activation, that after the second arc it is 20 mm.

However, we also found *references to irrelevant aspects* such as: "Perhaps the child has difficulty counting." Eight facilitator responses were coded in this subcategory, emphasizing aspects that did not help students to understand unit conversion conceptually.

DISCUSSION AND CONCLUSION

In summary, this study provided valuable insights into the knowledge categories most frequently used by facilitators when dealing with a fictional PD situation. Most responses were allocated to the CK-PD category, which focuses on student learning without referencing the PD level. This may indicate a struggle with the two identities, as pointed out by Schwarts et al. (2025). Additionally, many responses highlighted GPK-PD, with emphasis on how to moderate the PD session rather than on content-specific teacher learning. This reveals the same struggle reported by Pöhler-Friedrich et al. (2024) and Bialy et al. (2024). However, we also found that facilitators mostly concentrated on general moderation aspects. In contrast, fewer responses fell into the PCK-PD category, which explicitly considers supporting teacher learning. However, when the facilitators answered with reference to PCK-PD, they almost always supported content-specific teacher learning.

It is noteworthy, however, that even after completing the one-year basic module of the FPD program, the facilitators in our study placed the least emphasis on supporting content-specific teacher learning and adequately fostering students' conceptual understanding. These findings may be explained by the context of the facilitator task, as unit conversion is a rather neglected content in research of mathematics learning (Cheeseman et al., 2017). This confirms Prediger et al.'s (2022) request to align FPD courses with a content-specific focus: Understanding the subject matter content is essential for supporting teacher learning, and ensuring that facilitators focus on both teacher and student learning is crucial for them to fulfill their intended role. Therefore, our findings can contribute to improving FPD courses and better preparing facilitators for teacher PD sessions.

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