‘MEASURING IS NOT A VALID ARGUMENT IN MATHEMATICS.’ REALLY? DO STUDENTS RECOGNIZE THE CONTEXT-SPECIFICITY OF SOCIOMATHEMATICAL NORMS?

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Mathematics is encountered both as a deductive discipline, where proofs are central, and as an applied discipline, where reasoning is used to model and solve real-world problems. While arguments based on examples, measurements, or drawings are generally invalid in proofs, they are often legitimate when mathematics is applied in context. This study investigates whether high school students can distinguish between these normative frameworks and apply appropriate sociomathematical norms flexibly. Data from 436 Grade 11 students show that students’ knowledge of proof-related norms does not systematically predict their evaluation of applied mathematical context. However, the results suggest that some students are familiar with proof-related norms but seem to transfer them inappropriately to applied contexts.

INTRODUCTION

The validity of a mathematical proof is a matter of acceptance within a mathematical community (Manin, 2010). Nevertheless, there exist certain norms that an argumentation must satisfy in order to be recognized as a valid mathematical proof. For example, deductive reasoning is central in mathematics – consequently, empirical evidence, such as arguments based solely on examples or measurements, is generally considered invalid in the context of mathematical proof (Dawkins & Weber, 2017). As these norms may differ in their specific manifestations across mathematical communities and contexts, they are often referred to as sociomathematical norms (Yackel & Cobb, 1996). Research has repeatedly shown that students experience difficulties in identifying sociomathematical norms related to proof and consistently applying them in their own proving activities. In particular, students often display limited knowledge about what constitutes a valid proof and struggle to validate proofs appropriately (Komatsu et al., 2017). One possible reason for these difficulties lies in an inconsistency within school mathematics: sociomathematical norms that apply to deductive proof do not necessarily apply to mathematical applications and modelling tasks (Blum & Niss, 1991). When solving mathematical modelling problems in applied contexts, forms of reasoning that would be unacceptable in proofs – such as empirical-inductive arguments based on the use of examples, perceptual arguments based on drawings, or arguments based on measurements – are often legitimate (and even necessary) because the norms of the applied context apply. This raises the question of

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whether students are able to distinguish between these different normative frameworks and to flexibly switch between them depending on the mathematical context.

THEORETICAL BACKGROUND

Mathematics as a Deductive and an Applied Discipline

In school, students encounter mathematics both as a deductive and proving discipline and as an applied discipline used for modelling and solve real-world problems (Hansen, 2008). A central challenge for mathematics education is to provide students with access to both perspectives in order to foster an authentic understanding of the nature of mathematics (Blum & Niss, 1991; Stylianides, 2007). However, empirical findings indicate that many students struggle to characterize mathematics as a science and to reflect on its epistemological foundations. Some students even report that they have never explicitly considered what mathematics fundamentally is as a discipline (Fesser & Rach, 2022).

Difficulties become particularly apparent in the domain of proof, which represents one of the main practices through which mathematics is experienced as a deductive science. Prior research show that students struggle with proving, for example by having difficulty validating proofs or distinguishing between valid and invalid arguments (e.g., Buchbinder & Zaslavsky, 2019; Healy & Hoyles, 2000; Komatsu et al., 2017). Even beyond concrete proving tasks, students' general knowledge of proof-related norms has been found to be only moderate (Sporn et al., 2022). At the same time, research on mathematical modelling and applications demonstrates that students also experience difficulties in applied contexts, such as identifying relevant variables, making assumptions, or interpreting results (Blum & Borromeo Ferri, 2009). Importantly, students are not only required to engage with both aspects of mathematics separately, but also to navigate transitions between deductive and applied situations.

Sociomathematical Norms in School

When working on mathematical tasks, students are expected to adhere to different norms that depend on the underlying discourse and purpose of the activity. These norms are referred to as sociomathematical norms, as they regulate what counts as acceptable mathematical activity in specific contexts (Yackel & Cobb, 1996). Yackel and Cobb describe sociomathematical norms as normative aspects specifically related to mathematics, such as implicit understandings of what constitutes an acceptable explanation or the means by which technology can support a mathematical explanation.

Of particular relevance here is the fact that sociomathematical norms differ between deductive proof tasks and applied modelling tasks. This difference becomes evident when considering proof-related norms such as described in Heinze and Reiss (2003). Arguments based on examples illustrate this contrast particularly clearly: while inductive reasoning from examples is not acceptable as proof, examples are commonly used in applied mathematics to explore situations, to estimate quantities, and to formulate assumptions that underpin a model (Ärlebäck, 2009). Similarly, perceptual

arguments based on visual inspection, drawings, or even measurements are invalid in proofs, yet they represent common and accepted strategies in modelling processes, for example to explore relationships or obtain approximate values (e.g., Rellensmann et al., 2017).

A challenge for students, therefore, lies in recognizing which sociomathematical norms are relevant in a given situation and in successfully switching between them. Evidence for such difficulties is provided by Buchbinder und Zaslavsky (2019), who report inconsistencies in students' understanding of the role of examples in proof. When switching between norms does not succeed, different types of errors may occur. Students may fail to follow sociomathematical norms of applications because they are unfamiliar with them or because they inappropriately transfer proof-related norms to applied contexts. In other cases, norms that are commonly found in applied contexts, such as using examples or visual inspection, may be incorrectly transferred to proving, leading to violations of proof-related norms. To our knowledge, there are currently hardly any studies that have explicitly and systematically addressed students' transfer of norms between proof-related and application-related mathematical situations.

RESEARCH QUESTIONS

This exploratory study aims to investigate the extent to which students can navigate between sociomathematical norms of mathematics as both a deductive and an applied discipline. The study examined first students' knowledge of sociomathematical norms relating to proof, and second students' evaluations of specific applied mathematical solutions, which trigger the potential transfer of proof-related norms to applied contexts. The study addresses the following research questions:

- **RQ1:** What knowledge do students have about sociomathematical norms of mathematical proof, such as the invalidity of i) empirical arguments, ii) arguments based on measurements, and iii) perceptual arguments based on drawings?
- **RQ2:** How is students' knowledge of sociomathematical norms of proof related to their evaluations of solutions in applied mathematical contexts?

METHOD

In an exploratory study conducted as part of the project *MiO – Mathematik in der Oberstufe*, data were collected from N = 436 students in Grade 11 at academic-track secondary schools (Gymnasium). The data were gathered at the beginning of the school year 2025/2026 via an online questionnaire completed under teacher supervision. The focus was on three sociomathematical norms related to i) empirical-inductive arguments, ii) arguments based on measurements, and iii) perceptual arguments based on visualization or drawings. These arguments are generally not valid in mathematical proofs, but are commonly used in specific application situations.

To investigate students' knowledge of these norms concerning proof (RQ1), students were asked to evaluate statements addressing each of the three norms (e.g., "Perceptual

information obtained from drawings and measurement is not allowed to be used as an argument in a mathematical proof.”). To address RQ2, students were additionally presented three solutions to applied mathematical tasks which are acceptable in secondary-level mathematics. For each of the three proof-related norms, there was one applied mathematical task whose given solution ignored this norm (which was acceptable in this case). All solutions were presented alongside a fictional dialogue between two students, in which one student rejected the solution by citing one of the three proof-related norms, while the other argued that the solution was acceptable in the present applied context (see Figure 1 for an example).

Figure 1: Sample item for evaluating a fictional dialogue about norms when solving an applied mathematical task (here: perceptual argument based on drawings).

Mira and David are working on the following task in their mathematics class:
Determine the length of the ladder. Provide a mathematical justification for your answer.

David’s reasoning:
It looks like a right triangle, so we can apply the Pythagorean theorem here: $\sqrt{4^2 + 1^2} = c$
The ladder is $\sqrt{17}$ meters, which is approximately 4.12 meters according to the calculator.
Mira says: Your reasoning probably wouldn’t be accepted by Mr. Miller. Just because it looks like a right triangle doesn’t mean there really is a right angle. And the Pythagorean theorem only applies to right triangles.
David says: Yes, but here we can assume the right angle. It works in this case.
To what extent do you agree with David?

Students were asked to evaluate the three proof-related statements and the three dialogues using a six-point Likert scale (1 = Not true at all to 6 = Totally true). In addition, students were asked to give reasons for their ratings for each solution to the applied mathematical tasks (not all students provided such reasoning, and the analysis of these reasons is not yet complete, so only initial impressions are mentioned below).

Analyses

For data analyses, the students’ responses were dichotomized. Regarding knowledge of the three proof-related sociomathematical norms, responses on the six-point Likert scale were coded as indicating *no knowledge* (Likert scale 1-3) or *knowledge* (Likert scale 4-6) of each norm. Similarly, students’ evaluations of the applied solutions presented in the dialogues were dichotomized. Responses indicating disagreement (Likert scale 1-3) were coded as *negative rating* of the applied solution, reflecting a rejection of the acceptable solution in this context, whereas responses indicating agreement (Likert scale 4-6) were coded as *positive rating* of the applied solution.

To describe students' knowledge of sociomathematical norms of proof, we conducted descriptive analyses of the knowledge items (RQ1). To address RQ2, contingency tables were used to descriptively examine the relationship between students' knowledge of proof-related sociomathematical norms and their evaluations of applied solutions. In addition, logistic regression analyses were conducted to test whether knowledge of proof-related norms predicted students' evaluations in applied contexts.

RESULTS

The results for RQ1 indicate that less than 40% of the students correctly identified the sociomathematical norms related to proof for each of the three norms considered. In particular, only 21.5% recognized that arguments based on drawings are invalid in proofs. With 38.6% and 37.8%, respectively, the assessment is somewhat better for empirical arguments and arguments based on measurements. Overall, however, these findings suggest that students in Grade 11 have a substantial lack of knowledge regarding proof-related norms.

Table 1 presents the relative frequencies of students' classifications with respect to their knowledge of proof-related sociomathematical norms and their evaluations of the corresponding solutions to applied mathematical tasks.

Table 1: Relative frequencies of students' evaluations of applied solutions by knowledge of proof-related sociomathematical norms.

	Empirical arguments		Arguments based on measurements		Perceptual arguments based on drawings	
	No knowledge	Knowledge	No knowledge	Knowledge	No knowledge	Knowledge
Negative rating	18.4%	13.8%	31.5%	21.5%	39.5%	10.5%
Positive rating	41.5%	26.2%	28.2%	18.2%	38.3%	11.7%

Note: Due to occasional missing responses, the sum of percentages in each column "Knowledge" slightly deviates from the corresponding percentage reported for RQ1.

To investigate the relationship between students' knowledge of sociomathematical norms of proof and their evaluations of applied solutions (RQ2), separate logistic regression models were estimated for the three norms. Across all models, students' knowledge of the proof-related norm was not found to be a significant predictor of their evaluations of applied mathematical solutions. Specifically, knowledge of the invalidity of empirical arguments in proofs does not significantly predict students' evaluations in applied contexts ($b = -0.17$, $p = .46$). Similarly, knowledge that arguments based on measurement are invalid in proofs is not a significant predictor of

students' evaluations (b = -0.07, p = .73). Finally, no significant predictive effect is found for knowledge of the invalidity of perceptual arguments based on drawings (b = 0.14, p = .60). Overall, the results indicate that possessing or lacking knowledge of the examined proof-related sociomathematical norms is not predictive of students' evaluations of corresponding applied solutions. This is reflected by first impressions when analysing the reasons students provided for their positive or negative ratings. Some students disagreed with the solution to the application task and referred to the violation of the respective proof-related norm (e.g., "You need evidence of the 90-degree angle."). Others agreed that the solution is valid and argued that separate norms apply in the application context (e.g., "It is not a mathematical problem but a real-life question that can be assessed based on experience.").

DISCUSSION

Regarding RQ1, the results indicate that many students have limited knowledge of the sociomathematical norms related to proof. This finding is consistent with previous studies (Healy & Hoyles, 2000; Komatsu et al., 2017; Sporn et al., 2022). In order for students to develop an authentic understanding of mathematics that covers both deductive and applied perspectives on mathematics, it seems necessary for proof-related norms to be explicitly addressed in instruction (Sporn et al., 2023).

For RQ2, the results in Table 1 highlight several groups of students that are interesting for further research. Some students who do not know the proof-related norms still evaluate solutions of the applied tasks appropriately. This suggest that they may rely on everyday reasoning rather than disciplinary norms. These students need opportunities to learn the proof-related norms and it would be important to check whether they are using them correctly (Buchbinder & Zaslavsky, 2019). Other students are familiar with proof-related norms but seem to transfer them inappropriately to applied contexts. These cases are of particular interest, as they highlight potential instances of the inappropriate transfer of norms from deductive proof contexts to applied mathematical situations. These students may require instructional support to foster reflection on the contextual validity of different forms of reasoning, as well as the distinction between mathematics as a deductive and an applied science. Finally, a substantial group of students neither know proof-related norms nor evaluate applied solutions in a way that would be acceptable in this applied context. This suggests a broader need for instructional approaches that explicitly address and contrast both sets of norms.

At the same time, the results of the logistic regression analyses indicate that students' knowledge of proof-related sociomathematical norms does not systematically predict their evaluations in applied mathematical contexts. It appears that knowledge of a proof-related norm is not related to the evaluation of solutions to applied mathematical tasks that violate this norm. However, this does not negate the possibility of inappropriate norm transfer in other cases, as evidenced by the contingency analyses (see Table 1).

Overall, the findings suggest the importance of explicitly addressing sociomathematical norms and, in particular, supporting students in reflecting on the difference between deductive and applied perspectives on mathematics. The results further suggest that sociomathematical norms cannot be treated as a uniform set: rather, each norm appears to require separate consideration and explicit differentiation with respect to its meaning in deductive and applied contexts.

Limitation & Outlook

One limitation of the present study is that the dialogue-based items may be considered artificial, as the focused norm was explicitly triggered in each case. This measure chosen here to confront the students with the conflict of norms may have influenced the students' responses. In addition, there was only one application task for each norm.

To gain deeper insight into potential processes of norm transfer and contextual justifications, the qualitative data of students' reasons will be analysed. Future studies should investigate students' behaviour across both proving and applied mathematical activities. In particular, it would be valuable to examine whether students who rely on examples, visual inspection, or measuring in applied contexts appropriately abstain from using these strategies in proving situations - or whether they inappropriately transfer norms that are commonly used in applications to proofs. Such research would contribute to a more comprehensive understanding of how individual sociomathematical norms are transferred, differentiated, or conflated between deductive and applied perspectives on mathematics.

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