MATHEMATICAL DEBATE AS A SITE OF COLLECTIVE EMBODIMENT

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In this theoretical study, starting from the observation that AI systems can successfully solve mathematical problems without understanding them, we argue that this contrast forces a re-examination of the specificity of human mathematical sense making. We review influential accounts of embodied cognition, starting with Lakoff and Núñez's proposal, and we contrast this perspective with Radford's Theory of Objectification. Building on the latter, we focus on mathematical debate as a paradigmatic form of collective embodiment. We articulate four intertwined dimensions—objectification, subjectification, cultural-historical reenactment, and dialectical tension—showing how sense making becomes ethically and politically charged.

INTRODUCTION

Speaking of sensing and making sense of mathematics, one might say that mathematics has an unfair reputation for always making sense. In classrooms, however, it is often sensed as doing the opposite: confusing, resisting interpretation, and provoking the uncomfortable feeling that something important is happening just beyond reach. A year ago, while working in a classroom on a former project of our research group (Matteucci et al., 2024a), we happened to hear a student jokingly tell the teacher that he did not understand mathematics very well, but that this was not a big problem, since there was someone who understood it for him. He was referring to the elephant in the room: artificial intelligence. Since current AI Large Language Models (LLMs) are able to tackle and solve mathematical problems far better than the vast majority of human beings, including International Mathematical Olympiad winners (Castelvecchi, 2025), we may rightly ask whether they are capable of making sense of mathematics. The obvious answer is no, because this presupposes participation in biologically grounded, autopoietic processes of enlanguaged and embodied cognition and norm-governed coordination (Gahrn-Andersen, 2025), and, as allopoietic non-living systems without a sensing body, LLMs lack this grounding. And yet, precisely because we know that AI is not capable of making sense of mathematics, it becomes all the more important to understand what that means for us humans, for the very reason that we can no longer hide behind the fig leaf of answers that would equally apply to AI. Should embodiment be foregrounded in this explanation, and if so, how? Moreover, as educators, it is crucial to understand the

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specificity of human action and understanding in contrast to machines, because it is there that the future of education, whatever it may be, will be decided.

LITERATURE REVIEW ON EMBODIMENT

Before explaining our theoretical stance, illustrating some of the theoretical constructs used in our past research to address issues relevant to this paper, and formulating specific claims that directly address the issues described in the introduction, we review, in this section, one of the main references in the literature on embodied cognition, focusing on what sensing and making sense of mathematics mean within that perspective. We then review some aspects of Radford's Theory of Objectification, again focusing on the meaning of sensing mathematics and making sense of it through this particular lens.

Embodied Cognition

In the *Encyclopedia of Mathematics Education* we can find the following definition:

> Embodied cognition is a subdomain of cognitive psychology that focuses on the interaction between an individual and the environment (social, environmental, instructional). It moves beyond the traditional distinctions between mind and body in the sense that actions or interactions embody projections of the mind and vice versa. (Sriraman & Wu, 2020, p. 266)

A paradigmatic example of embodied cognition applied to mathematics education, illustrated later in the same encyclopedic entry, is the one by Lakoff and Núñez, as exemplified in their book *Where Mathematics comes from: How the Embodied Mind brings Mathematics into being* (Lakoff & Núñez, 2000). According to these authors, one can claim the embodied character of mathematics because it is a human cognitive product rooted in brain, body, and everyday interaction with the physical world, not a set of mind-independent Platonic truths. Mathematical meaning resides in human ideas rather than in symbols, and much of this meaning operates at the level of the cognitive unconscious. Humans possess minimal innate numerical capacities, which are extended into advanced mathematics through general cognitive mechanisms such as conceptual metaphors and blends. Mathematical concepts are grounded in everyday experience via grounding metaphors (e.g., object collection, measuring, motion) and interconnected across domains through linking metaphors (e.g., numbers as points on a line). Therefore, for Lakoff and Núñez, making sense of mathematics involves a shift from rote symbolic manipulation to conceptual understanding, grounded in human cognition rather than in symbols themselves. Mathematical meaning is located in the aforementioned embodied cognitive structures, often operating at an unconscious level, which connect abstract concepts to sensorimotor experience through metaphorical extensions. Understanding therefore requires moving beyond formalism toward analysis of mathematical ideas, clarifying what theorems mean and why they hold. This perspective emphasizes that calculation and formal proof do not guarantee understanding unless the underlying conceptual

relationships are grasped. Finally, demystifying mathematics as a human cognitive construction, rather than a transcendent and disembodied entity, renders it more accessible and intellectually meaningful for students.

If mathematics depends on the biological physicality of human beings, which has remained largely unchanged for millennia, the question arises of how its historical development can be explained. Lakoff and Núñez argue that mathematics evolves not because of changes in the brain, but because stable embodied cognitive resources are repeatedly reconfigured, extended, and stabilized through metaphor, symbolism, and social practice. From this perspective, and *contra* Thomas Kuhn (2012), mathematical change is not understood as a paradigm shift between incommensurable worlds, but rather as a constrained reorganization of enduring embodied resources.

While Lakoff and Núñez explain how historical change is cognitively possible, they do not fully explain how it becomes socially binding. They claim that change occurs when culturally specific ideas from outside mathematics are applied to mathematical domains as special cases (Lakoff & Núñez, 2000, p. 356), but they do not explain how and why these ideas are collectively stabilized, institutionally ratified, and eventually enforced as normative constraints within mathematical practice. In their view, mathematics becomes stable once it is firmly established within a community of mathematicians who draw out the consequences of shared metaphorical models. However, they primarily view this through the lens of individual cognitive mechanisms like inference-preserving conceptual metaphors.

The Theory of Objectification

Radford's Theory of Objectification, hereinafter referred to as TO, is "a teaching and learning theory inspired by dialectical materialism, Freire's concept of education, and Vygotsky's school of thought" (Radford, 2021a, p. 35), characterized as an "attempt to understand teaching and learning not as the result of the individuals' own deeds [...] but as cultural-historical situated processes of knowing and becoming" (Radford, 2021a, p. 34). TO shares, with Lakoff and Núñez's embodied cognition approach, a non-Platonic, non-formalistic stance on mathematical objects: mathematics is not a collection of abstract truths or symbols found in a textbook; instead, mathematics is a sensuous entity that comes into existence only through human activity. Consequently, in TO sense making is not an individual mental construction but a collective process through which mathematical objects and ways of reasoning become progressively objectified in collective activity while simultaneously those who participate in this objectification in turn become subjects shaped by the norms, values, and modes of reasoning of mathematical culture.

Here TO departs radically from Lakoff and Núñez's stance: while the latter's theory—as stated above—do not fully explains how mathematics becomes socially binding, according to Radford mathematics is binding because it is embedded in Semiotic Systems of Cultural Signification (SSCS) that are "dynamic systems that

originate in the practical and sensuous activity of the individuals" (Radford, 2021a, p. 186), which have a normative function and convey specific political and ethical views on how one should establish truth. SSCS offer to human beings, *qua* subjects constituted in joint labour, a spectrum of socially recognized positions allowing them to position themselves (to position their "selves", so to speak) and be positioned by others. "But the most important point is perhaps not to see the self […] already given […]. The self and its space are built and rebuilt constantly out of the ideas conveyed by the SSCS (ideas that are themselves constructed and reconstructed all the time too)." (Radford, 2021a, p. 188). These selves are not only consubstantial with their own bodies, they are consubstantial with their own culture: the embodiment of the self extends to the material and ideational tools, signs and systems provided by the society. According to TO, embodiment is not just neural activation or a private bodily or mental experience, embodiment is the material presence of historically constituted forms of human activity. Ultimately, society is the extended body of the self, and the individual body is the local instantiation of society. Thus, embodiment is not limited to one's biological body: the self is embodied in the historically objectified forms of social practice which function as a distributed, material, and normative social body.

THEORETICAL FRAMEWORK AND CLAIMS

Our research group has consistently operated within the framework of Cultural-Historical Activity Theory (CHAT) since its inception. Although CHAT is widely established in academic research, it lacks a fully consolidated theoretical identity and functions largely as an umbrella term (Levant et al., 2024). Within this landscape, our primary theoretical orientation is Radford's (2021a) Theory of Objectification, while the key contributions of Yrjö Engeström and Andy Blunden have been instrumental in enabling a rigorous treatment of the concept of contradiction and in articulating the core principles of CHAT in relation to its historical lineage along the Hegel–Marx–Vygotsky axis. Against this theoretical background, our research group has investigated dialogical classroom practices as a privileged means for foregrounding and problematizing the use of AI in mathematics education (Matteucci et al., 2024b) focusing in particular on forms of mathematical discussion (Bartolini Bussi, 1996) and mathematical debate (Matteucci & Tortoriello, 2023) as pedagogical devices capable of eliciting students' reflective engagement with mathematical knowledge and its mediating tools (Matteucci et al, 2025a).

Debate differs from mathematical discussion in two fundamental ways. First, it is a regulated dialogical interaction in which speakers do not act individually but as part of one of two teams, one for, one against an assigned motion. Second, the perspective advanced by debaters is not determined by their personal worldview but is assigned by a third party (Freeley & Steinberg, 2013). Although this feature may initially appear overly restrictive, it in fact enables the interaction to be raised from a personal level to one that is more properly dialectical. It is important to note that, while what we just described is applicable as well to the so-called competitive debate, that is the

form of debate as typically (but not exclusively) practiced in schools, the mathematical debate our group researched and devised, differs from it in important ways. Competitive debate is typically framed as a tool for developing soft skills and it is not aimed at increasing competence in a specific subject, with teachers acting primarily as referees and judges, while in mathematical debate not only there are explicit disciplinary goals, but teachers act also as mediators. Here, mediation is about theoretical constructs that are used in mathematics at the meta-level, using debate itself as a tool. The focus is not only on the metatheory, but also on the communitarian ethic typical of TO since students, through debating the historical and social emergence of mathematical objects, can indirectly engage with the science-technology contradiction. Although not a primary contradiction like that between labour and capital, this tension represents one of the few leverageable contradictions in the mathematics classroom capable of fostering social awareness as a prerequisite for social change (Matteucci et al., 2025b).

Within this theoretical framework, as made explicit in the comparison presented in the previous section, sense making is grounded in embodiment understood as objectification in the meaning of TO, rather than as processes occurring exclusively in the mind and body of the individual. In this perspective, we claim that mathematical debate, as a disciplinary pedagogical device, constitutes a paradigmatic example of this type of embodiment. Moreover, returning to the question about sense making in the AI era and its relation to embodiment, we maintain that this kind of sense making is important also because it is a kind of sense making that is not reproducible by the AI itself.

DEBATE AS COLLECTIVE EMBODIMENT

Here, we will show that mathematical debate constitutes an example of embodiment in the meaning of TO on four counts: sense making as objectification, sense making as subjectification, sense making as cultural history, sense making as dialectics.

1. In TO sense making is objectification, not interpretation. It is not constructing meaning in the mind of an individual, but a process through which mathematical objects and ways of reasoning become objectivated in joint labour. Likewise, in mathematical debate sense making is ontological before epistemological: it concerns what comes to exist for the learner in collective activity, not what the learner individually thinks about something already given.

2. In TO sense making is subjectification, this is why objectification is never neutral and has also ethical implications. Likewise, in mathematical debate the objectification of disciplinary knowledge entails that students, insofar as they are required to adopt a position, are *ipso facto* engaged in a process of becoming a certain kind of subject through which they assume epistemological responsibility.

3. In TO sense making is cultural and historical. Likewise, in mathematical debate sense making is historical because it reenacts trajectories of cultural mathematical

praxis, with this reenacting more to be intended as an open-ended confrontation than a stage play with a foregone conclusion.

4. In TO sense making is a dialectical movement, not linear acquisition. Likewise, in mathematical debate sense making unfolds through tensions and the consequent reconfigurations of meaning. And this confrontation of incompatible positions, whose resolution does not eliminate contradiction but reorganizes it at a higher level of coherence, is the essence of dialectics.

In subsuming embodied cognition *à la* Lakoff and Núñez under TO, we deliberately shifted the focus from the individual body to the social body. However, how can we be certain that, in doing so, we are not once again adopting an overly abstract stance where there is no space for embodiment? Could it not simply be objected that all of this amounts to nothing more than the claim that the self is social? According to Eval'd Il'enkov, one of the key philosophers informing the theoretical foundations of CHAT, the "ideal" exists materially not just *as* matter but *in* matter as the form of human activity (Levant & Oittinen, 2013), a paradigmatic example of this embodiment of the ideal, used and discussed by Il'enkov himself, is the commodity value, as analysed in *Das Kapital* (Marx, 1987). So the answer is: no, this is still embodiment, not abstraction, because society itself is not abstract, it is materially embodied.

The reference to Il'enkov's concept of the ideal is useful to us not only to characterize the sense making of TO as embodiment rather than as mental abstraction, but also to address the question of the specificity of sense making through debate as compared to what can be offered by AI. As Bakhurst wrote:

> So, on Ilyenkov's view, what Wilfrid Sellars pointedly called 'the logical space of reasons' is not situated in a purely mental domain; for the environment we inhabit as human agents is properly conceived as such a space. Our very 'external' environment, as a domain of meaning, offers us reasons for belief and action. It is cognitively and motivationally animating. (2020, p. 138)

This domain of meaning is linked to Radford's SSCS and it is the very same cultural-historical environment students do reenact through debate. In doing this, they are navigating the space of reasons positioning themselves, and being positioned in this space: this is not a formal logic exercise, but it is taking a stance while objectifying reasons. The AI cannot do this precisely because, given its nature as a statistical parrot, its output at best mimics "a pattern-based conformity to discursive practice, which nonetheless falls short of genuine participation in the norm-governed space of reasons" (Sambrotta, 2025).

CONCLUSIONS

The availability of AI systems that can generate mathematically correct solutions without participating in mathematical meaning forces a conceptual re-sorting of what sense making is. The first most obvious difference between LLMs and humans lies in

the fact that the latter are biological living beings capable of perceiving and interacting with the external world, whereas the former, besides not being living entities, have no capacity to interact with anything other than language or, more precisely, its formalisation. A recent paper (Floridi, 2025) argues, using Category Theory, that hallucinations in LLMs are not an evitable nuisance, but are the direct consequence of their lack of any experiential access to the real world, possessing only mediated access to information about the world through representations authored by humans. To use a sharp metaphor found in the aforementioned article, LLMs are like the chained prisoners in Plato's myth of the cave: they are condemned to see only shadows, not real things. But this difference is not merely biological: the comparison between the different conceptions of embodiment in Lakoff & Núñez and in Radford highlights that human mathematical sense making is also irreducibly normative, historical, and ethical: it unfolds in a space where reasons are owned, contested, attributed, and sanctioned, where "getting it right" is inseparable from what counts as getting it right and from who becomes the kind of subject who can answer for it.

Mathematical debate, in this framework, is not a technique for improving soft skills or winning an argument for the sake of it. We have shown that it is an activity where collective embodiment becomes visible because it organizes joint labour around claims, warrants, counterclaims, and shared criteria of acceptability. Moreover, the kind of sense making this collective embodiment can realize, not only is completely beyond the reach of the AI, but it is also unattainable for humans if they focus on individual interaction with AI itself. In a paper about Davydov, Luis Radford stated that within the creative historical movement of effective reality, objects come into being, and each emergence simultaneously discloses both a conceptual and a nonconceptual dimension and that "it is in the nature of the nonconceptual to resist translation into the symbolic. […] It resists encapsulation" (Radford, 2021b, p. 340), so this is the very thing that cannot be sensed, and cannot be made sense of, by the AI. The key risk in educational AI use is not just passive repetition, but the potential for the "AI persona" to manipulate users, unless met by a collaborative human group with strong cultural and argumentative skills. We think that debate is one of the activities that can help us resist encapsulation, and be collectively aware of it.

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