SENSE-MAKING VISIBILITY AND CONCEPTUAL SPECIFICITY IN PME PROCEEDINGS FROM 1978 TO 2025

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This study investigates the visibility and conceptualization of sense-making in PME proceedings (1978–2025). Proceedings were converted to searchable text as needed and searched for sense making and sense-making; papers were included when the term appeared outside references and paratext, yielding 277 papers (first occurrence: 1990). Visibility was coded by section (Title–Conclusion) using binary presence/absence indicators, and conceptual specificity was coded by type (e.g., implied definition). Most papers used the term in one section (mean = 1.43). Implied definitions predominated (N=158), sense-making appeared without a definition in 97 papers, while explicit definitions (N=15) and definitions with operationalization (N=8) were rare. Implied and explicit definitions were examined for their thematic content.

INTRODUCTION

While this year's conference theme addresses broader issues in sensing mathematics and in how students make sense of mathematics, this paper focuses on a related component within this larger theme, the construct of sense-making. This term has appeared in both hyphenated and non-hyphenated forms, but I use the hyphenated form hereafter to refer to both forms. This paper blends research on sense-making with a retrospective of broader themes in mathematics education.

BACKGROUND

Evolving Research in Mathematics Education

As a field ages, research within that field naturally turns inward. That is, how has research in that field changed over the years? How have the tools of investigation within that field's research changed over time? Over the last five decades, mathematics education research has shifted both in *what* it foregrounds and in *how* it explains learning and teaching. At the level of research foci, Inglis and Foster's (2018) large-scale analysis of *Educational Studies in Mathematics* and the *Journal for Research in Mathematics Education* (JRME) shows declining attention to some classical content areas (notably Euclidean geometry) alongside gradually increasing attention to school algebra and analysis, plus a steady rise in work on teachers' knowledge/beliefs, curriculum (especially reform), and the design of novel assessments. Constructivism attains prominence in the late 1980s–mid-1990s and then declines, while sociocultural theory and related emphases on interaction and discourse grow substantially, contributing to a more diverse theoretical landscape that also includes increased

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visibility of semiotics/embodied cognition, as well as didactical theories. The investigative tools have also changed. Early work in these journals relied heavily on randomized experimental designs (so much so that "experimental methods" dominated JRME terminology in the 1970s), but this approach declined sharply by the 2000s–2010s. As experimental designs receded, the field's investigative repertoire increasingly emphasized methods aligned with social and interpretive programs (e.g., classroom discourse and interaction analyses), and, in more recent meta-level work, computational techniques such as topic modeling have become viable tools for characterizing the field itself at scale.

Definitions in Mathematics Education Research

Across mathematics education, definitional work is a methodological requirement. When constructs are left implicit, studies can talk past one another, weakening construct validity and limiting what reviews and meta-syntheses can legitimately compare (Diego-Mantecón et al., 2019; Aquilina et al., 2025; Simon, 2017). This challenge is visible in several high-traffic constructs: students' mathematics-related beliefs are sometimes conceptualized so broadly that boundaries and sub-constructs blur (Diego-Mantecón et al.), and attitude is frequently measured without a clear working definition (Aquilina et al.). Similar definitional slippage appears in teacher noticing, where diverse conceptualizations have relied on intuitive and oversimplified models (Scheiner, 2021), and in sense-making, where the construct is used with different meanings, remains underspecified, and is at times invoked without an explicit theoretical frame (Scheiner & Pinto, 2022; Urquhart et al., 2025).

In the sense-making literature relevant to mathematics education, this terminology often functions as an appealing umbrella term whose meaning shifts with authors' theoretical commitments, creating persistent problems of definition and framing. In mathematics education, sense-making is widely used but has different meanings and is largely underspecified, making it difficult to determine what counts as sense-making across studies (Scheiner & Pinto, 2022). A further complication is that many studies use sense-making as a common-sense descriptor without situating it within prior sense-making scholarship or an explicit theoretical framework, thereby undermining interpretability and cross-study synthesis (Urquhart et al., 2025).

Scheiner and Pinto (2022) frame sense-making as students' efforts to understand a preexisting mathematical idea by refining and mobilizing their own conceptions so that those conceptions align with the target concept (i.e., the "fit" runs from concept toward conception). They note that prior literature commonly characterizes this work in two ways: one emphasizes drawing out meaning through operating on and coordinating representations to decide what matters mathematically and to distinguish an object from its representational forms; the other emphasizes constructing meaning by assigning interpretations to an object, considering the conceptions students bring to and activate within a situation. Scheiner and Pinto argue that some activity often grouped under sense-making is more accurately described as *meaning-making*. Students

generate or reshape the mathematical concept itself so that it coheres with their existing conceptions (a reverse direction of fit), including situations in which new interpretations are introduced as mathematical objects take shape for learners.

Research Questions

Given the content-independent nature of sense-making, it provides a useful lens for examining trends in mathematics education research, as reflected in PME Proceedings from 1978 through 2025, and for understanding how the term's definition has evolved. Consequently, this study is situated within the nascent subfield of mathematics education studies examining the nature of mathematics education research (e.g., Inglis & Foster, 2018). Specifically, this study was designed to answer two research questions. (1) What is the visibility of the term sense-making in papers accepted to the PME proceedings from 1978 through 2025? (2) What is the nature of how sense-making is conceptualized in papers accepted to the PME proceedings from 1978 through 2025?

FRAMEWORK

This framework operationalizes a focused content analysis of how the term sense-making and its variants are positioned and conceptualized in papers accepted at the annual PME meetings. It maps where the term appears across standard manuscript sections and characterizes whether and how authors define, implicitly define, simply mention, and/or operationalize sense-making. Together, these components treat visibility (distribution across sections) and conceptual specificity (definition/operationalization) as complementary indicators of how an article frames sense-making as a construct.

METHODS

Each of the PME proceedings from 1978 through 2025 was either in a form in which terms could be searched for or was converted into such a form. Each PDF of the proceedings of the annual meeting of the IGPME was examined for the presence of the terms "sense making" or "sense-making." If a paper contained at least one instance of either of these terms in any section (e.g., introduction) except the references, acknowledgements, and headers/footers, it was included in the analysis and converted into its own PDF document. For earlier proceedings, the document was first converted into searchable text before word searches were conducted.

To code for visibility, plural/possessive forms of sense-making were counted only when the sense-making terminology was present, thereby reducing false positives (e.g., excluding "sense-maker"). Each article was coded with a binary indicator (1/0) for whether the term appears at least once in each section: Title (T), Abstract (A), Introduction (I), Background/Literature Review (B), Framework (F), Methods (M), Results (R), Discussion (D), and Conclusion (C). Importantly, section-level coding was binary; multiple instances within a section did not increase credit, thereby emphasizing *distribution*. The frequency of sections in which sense-making appeared was tallied.

Additionally, the *mean section frequency* was calculated by summing the frequencies across all sections and dividing by the total number of papers.

Coding prioritized explicit article headings; when headings are unclear, the sentence within which sense-making appears is used to code its location. Section boundary rules address common genre variations: distinguishing Introduction from Background when combined; treating brief but clearly theoretical paragraphs as Framework even if embedded in the Introduction; counting occurrences in any methods subcomponent as Methods; and handling combined "Results and Discussion" sections by crediting Results when the term appears amid evidence-based reporting and crediting Discussion when interpretive/implicational work is present, coding both when inseparable. Conclusion coding was used when concluding language appeared within the discussion section.

To examine conceptual specificity, the unit of analysis shifts to the smallest meaningful segment (typically a sentence or paragraph). Papers were classified into four levels: mention-only (M), implied definition (I), explicit definition (E), and explicit definition plus operationalization (EO). In cases where sense-making appeared in the article but was not elaborated, it was coded as M. When an article lacked an explicit definition but used the term in ways that implied meaning, it was coded as I. Several sentences where the term is invoked or clearly implied were used to reconstruct a definition in alignment with the term's use in the article (implied definition). When a sentence was used to define sense-making, this was coded as E. If the explicit definition was followed by one or more sentences involving indicators of sense-making it was coded as EO. Reconstructed implied definitions and explicit definitions (E and EO codes) were examined for themes over time.

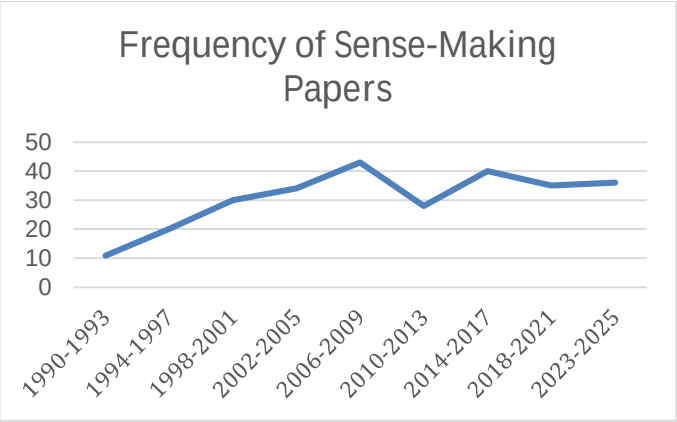
FINDINGS

Before answering the research questions, I present the quantitative data. The first instance of sense-making occurred in the 1990 proceedings. There were no instances of the terms in the proceedings for 1991 and 1997. A total of 277 articles included the term sense-making at least once. The frequency of sense-making appearances in the proceedings from 1990 through 2025 is shown in Figure 1 in four-year increments. The graph shows a steady increase in articles from 1990-1993 until 2006-2009. There was a decline in 2010-2013, a rebound in 2014-2017, and a relatively flat trend thereafter.

Visibility

The frequency of locations in the papers where sense-making appears is shown in Figure 2. Sense-making appeared most frequently within the introduction section of the papers. The second most frequent category was the framework and results. The least frequent category was title, with methods containing sense-making second least. The mean section frequency was 1.43, suggesting that sense-making appeared in fewer than two different sections across most sense-making papers. Specifically, in 195 of 277 articles, sense-making appears in a single section.

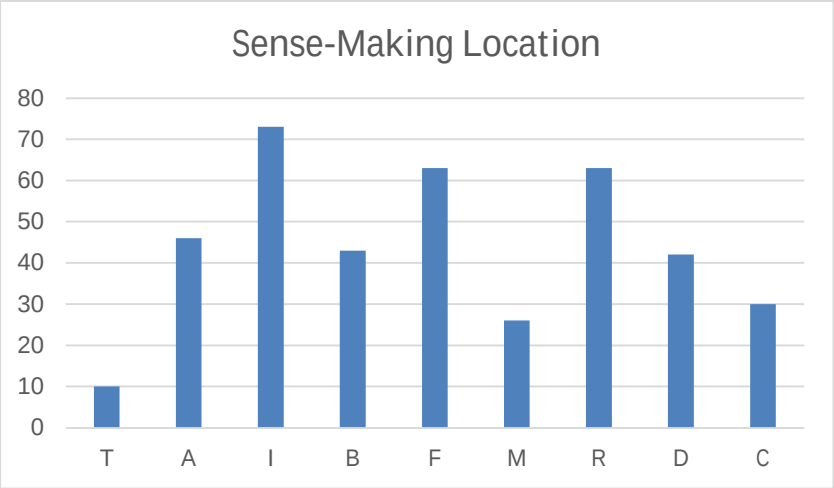
Figure 1: Frequency of Sense-Making Papers from 1990 through 2025



Conceptual Specificity

Over 50% (*N* = 158) of the sense-making papers contained an implied definition, while 35% (*N* = 97) of the papers mentioned sense-making but did not define it, nor was it possible to discern a definition from the surrounding text. Only 5.4% (*N* = 15) provided an explicit definition of sense-making, with the smallest percentage, 2.9% (*N* = 8), providing both an explicit definition and an operationalization of the construct. The number of explicit definitions stayed steady from the 1990s through the 2020s. While implied definitions were the most common type of definition in sense-making papers, their percentage decreased over time. Sense-making mentions without definitions increased over time, reaching their highest share during the 2020s. Explicit definitions of sense-making were rare over time. Explicit definitions, along with operationalizations, were absent from sense-making papers before 2013 but appeared intermittently thereafter.

Figure 2: Location of Sense-Making Across Papers



Implied Definition Themes Over Time

The implied definitions of sense-making were examined from the first incidence in 1990 until 2025. During this period, the dominant implied definition of sense-making

is constructing and/or building understanding, rather than simply following procedures. That theme appears in 76-81% of implied definitions each decade. During the 1990s, implied definitions often position sense-making as a social activity involving shared meaning and/or representations but rarely include language that functions as explicit indicators or evaluative criteria.

During the 2000s, there was an increased emphasis on process, plus a noticeable rise in indicator-like phrasing. Implied definitions during this decade begin to sound quasi-operational, as they are not explicit definitions but include language such as evidenced by, characterized as, or multi-part descriptions of what sense-making entails. For example, Glass and Maher (2004) link opportunities for students to express, revise, and share their reasoning with their sense-making. In many papers from this decade, sense-making is implied as a visible form of reasoning-in-action (often discourse-mediated, sometimes representationally mediated), even if the paper does not formally define it.

In the 2010s, the implied definitions more frequently frame sense-making as producing coherence (a connected, intelligible structure) intertwined with justification/validation practices. This is evident in the increase in coherent/coherence language from 1.6% to 12.5% in the 2000s. An example of this is seen in Hollebrands, Cayton, and Boehm's (2013) paper connecting sense-making to a type of pivotal teaching moment in which teacher actions extend/connect student ideas so that students develop coherent understanding of mathematical properties/theorems, evidenced by students articulating or coordinating properties and constructions (often supported by dynamic-geometry actions such as dragging/checking).

In the 2020s, there was a noticeable pivot towards language, access, and context in implied definitions of sense-making in PME proceedings papers. In this case, sense-making is disproportionately framed through language/communication access and sense-making in context, including real-world reasonableness checks. Quantitatively, this is reflected in the use of language, which was 19% in the 2020s compared to 3-4% in the 2000s and 2010s and 0% in the 1990s. Implied definitions involving modeling/real-world/context interpretation reached their highest percentage (24%) of any decade. For instance, in a paper written by Rios (2023), sense-making is treated as rich engagement with mathematical ideas during group interaction, supported by linguistic resources, such as students' home language.

Explicit Definition Themes Over Time

Across 23 papers from 1993–2024, explicit definitions of sense-making cluster into seven themes. First, sense-making is framed as meaning-making (Scheiner & Pinto, 2022) that renders situations intelligible and connects new learning to prior knowledge. Second, many definitions emphasize interpretation within existing cognitive frameworks, characterizing sense-making as construal or as fitting new information into schemata. Third, several entries define sense-making as attending to structure and relationships, sometimes naming sub-processes such as recognizing, comparing, explaining, and verifying.

A fourth theme ties sense-making to modeling under real-world constraints. Fifth, definitions increasingly describe sense-making as socially negotiated and discourse-based, including collective construction of shared understanding and, at times, multimodal indicators (gesture, bodily action). Sixth, sense-making is treated as a practice of reasoning involving exploration, conjecture, justification, evaluation, verification, and revision. Seventh, the construct is further operationalized for research through indicators, coding units, segmentation rules, and instruments.

Historically, early work (1993–2000) emphasizes individual interpretation; the mid-2000s expand into modeling and structured-reasoning trajectories; the early 2010s emphasize interactional and embodied accounts; and 2017–2024 increasingly decompose sense-making into sub-processes while strengthening methodological operationalization, including teacher- and curriculum-facing formulations.

DISCUSSION

Across PME proceedings (1990–2025), sense-making functions less as a stable construct and more as a field-level discourse marker whose uptake tracks broader shifts in mathematics education research. The growth in papers that mention sense-making, alongside varied implied definitions, aligns with changing research foci and theoretical orientations (Inglis & Foster, 2018). Implied definitions also reflect the field's increasing emphasis on learning as socially and discursively mediated (Inglis & Foster). Thus, sense-making can serve as a barometer of theoretical commitments—yet its breadth allows it to index multiple commitments simultaneously rather than to name a tightly bounded construct (Scheiner & Pinto, 2022).

Visibility patterns suggest that sense-making seldom carries methodological weight. It appears most often in introductions and rarely in methods, and most papers mention it in only one section. This distribution implies that sense-making is frequently used to orient or legitimate a study, signaling valued goals such as understanding, reasoning, or meaningful engagement, yet it is not translated into design decisions, analytic categories, or evidentiary warrants. When constructs circulate primarily as framing language, they can be rhetorically powerful while remaining analytically thin (Simon, 2017), particularly amid theoretical pluralism (Inglis & Foster, 2018).

Conceptual-specificity results raise concerns for construct validity and cross-study synthesis. Over half of papers rely on implied definitions, more than a third are mention-only, and explicit (especially operationalized) definitions remain rare and intermittent. This pattern echoes broader warnings that implicit constructs encourage studies to talk past one another, limiting defensible comparisons and meta-syntheses (Diego-Mantecón et al., 2019; Aquilina et al., 2025; Simon, 2017). The rise of mention-only usage in the 2020s suggests that sense-making is becoming a shared vernacular, even as meanings diversify and are sometimes invoked without explicit theoretical framing (Urquhart et al., 2025). A practical implication is that future PME work should specify which sense-making tradition is intended and, when claims depend on it, articulate data-linked indicators (Scheiner & Pinto, 2022).

One implication for PME authors, reviewers, and editors is to treat sense-making as a claim-bearing construct rather than a motivational slogan. When sense-making is central to an argument, a minimal reporting standard would include (a) an explicit definition or positioned working definition, (b) a rationale for why that definition fits the study's theoretical commitments, and (c) traceable indicators showing how the construct is evidenced in data. Even brief indicator statements would improve interpretability and facilitate synthesis. Conversely, if sense-making is used only as a broad aspiration, authors might specify that role and avoid overclaiming.

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