MATHEMATICS CO-TEACHING: SPECIALIZED KNOWLEDGE FOR TEACHING MATHEMATICS BY ELEMENTARY SCHOOL AND SPECIAL EDUCATION TEACHERS

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This research report examines how specialized knowledge is enacted by a mathematics teacher and a special education teacher in primary mathematics classrooms organized around co-teaching. The analysis draws on six lessons taught in first, third, and sixth grade. The findings indicate that instructional practices were predominantly structured according to the "one teaches, one assists" and "one teaches, one observes" models of co-teaching. Across lessons, the knowledge enacted in classroom interaction was shaped by the teacher who assumed instructional leadership. We conclude by discussing implications for teacher education and research on teachers' professional knowledge.

CO-TEACHING FOR TEACHING IN INCLUSIVE CONTEXTS

Co-teaching has been widely recognized as an instructional tool with considerable potential to support inclusive education; yet its impact is closely tied to how it is enacted and to the professional and institutional contexts in which it is embedded (Cook & Friend, 1995; Scrugg et al., 2007). Research on student learning suggests that, in mathematics, co-teaching can contribute to meaningful gains for students with special educational needs (SEN) and for their peers without SEN, particularly when it is sustained over time and positioned as a core mode of instruction rather than as an ancillary form of support (Lehane & Senior, 2020). These outcomes, however, are not an automatic consequence of having two teachers in the classroom. Instead, they are associated with forms of collaboration that involve joint planning, shared responsibility for instruction, and flexible enactments of co-teaching models (Gardesten, 2023).

Despite this potential, the literature consistently documents tensions between mathematics teachers and special education teachers when co-teaching is enacted. Studies with both pre-service and in-service teachers show that classroom practice is often dominated by a one-teaches, one-assists configuration, which tends to reproduce hierarchical views of expertise: mathematics teachers are positioned as content specialists, while special education teachers are confined to individualized or behavioral support roles (Harbour et al., 2022; Shin et al., 2016). At the same time, research highlights asymmetrical gaps in professional knowledge. Special education teachers (SETs) frequently report limited confidence in mathematical content and topic-specific instruction, whereas mathematics teachers (MTs) identify shortcomings

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in their knowledge of accessibility, adaptation, and inclusive teaching strategies (Shin et al., 2016). What remains insufficiently understood, however, is the nature of the specialized knowledge that each professional brings to co-teaching. In response, this study focuses on describing mathematics specialized knowledge enacted by MT–SET teacher dyads during co-teaching.

THEORETICAL PERSPECTIVE

Co-teaching

Co-teaching is usually understood as an instructional approach in which two teachers intentionally share responsibility for teaching a diverse group of students within the same classroom, to expand opportunities for access, participation, and learning (Cook & Friend, 1995; Graziano & Navarrete, 2012; Scrugg et al., 2007). Within the literature, co-teaching is typically operationalized through six distinct models:

- First, *one teaches, one assists* positions one teacher as the instructional lead, while the other circulates to provide targeted support, monitor student work, and respond to questions; this model is often adopted as an entry point when teacher pairs are beginning to collaborate.
- Second, *station teaching* involves dividing the content and organizing instruction across multiple stations or subspaces, with students rotating while teachers coordinate instruction and scaffolding at each station.
- Third, in *parallel teaching*, both teachers engage in joint planning and simultaneously teach the same content to two smaller groups, thereby increasing opportunities for participation and feedback.
- A fourth model, *alternative teaching*, entails one teacher working with a small group—for purposes such as pre-teaching, re-teaching, reinforcement, or enrichment—while the other leads the larger group. Its value lies in providing focused support without permanently separating students.
- Fifth, *team teaching* is characterized by shared leadership of the lesson, with fluid exchanges of instructional roles; this model requires high levels of trust, communication, and coordination between teachers.
- Finally, Graziano and Navarrete (2012) also describe the *one teaches, one observes* model, in which teachers make certain decisions collaboratively and jointly analyze the lesson after its completion. During instruction, however, one teacher delivers the lesson in its entirety, while the special education teacher is largely confined to an observational role, with limited substantive intervention.

Mathematics Teacher Specialized Knowledge (MTSK)

This study adopts the Mathematics Teacher's Specialized Knowledge (MTSK) framework developed by Carrillo et al. (2018), as it provides a robust lens for characterizing teachers' professional knowledge and for examining the types of knowledge and skills mobilized in classrooms when special education teachers teach

mathematics. According to Carrillo et al. (2018), "this model features a reconfiguration of mathematical knowledge, a reinterpretation of pedagogical content knowledge and a new way of conceptualising the notion of specialisation" (p. 240). The framework distinguishes two main knowledge domains: Mathematical Knowledge (MK) and Pedagogical Content Knowledge (PCK), each articulated through specific subdomains. MK comprises Knowledge of Topics (KoT), Knowledge of the Structure of Mathematics (KSM), and Knowledge of Practices in Mathematics (KPM). PCK, in turn, includes Knowledge of Mathematics Teaching (KMT), Knowledge of Features of Learning Mathematics (KFLM), and Knowledge of Mathematics Learning Standards (KMLS).

RESEARCH METHOD

This study adopts a qualitative, narrative research design (Cohen et al., 2018), as its primary aim is to characterize mathematical knowledge for teaching and examine how such knowledge is enacted by teachers engaged in co-teaching.

Context and participants

The study involved three teacher pairs, each consisting of an MT and a SET, from a regular Chilean school implementing a School Integration Program. This program is mandatory for all state-funded schools and is characterized by the presence of multidisciplinary teams that support students with SEN. These teams typically include SETs, occupational therapists, educational psychologists, among other professionals. However, the most salient feature is the participation of a special education teacher in Language and Mathematics classes.

The participating pairs of teachers worked in 1st, 3rd, and 6th grades of primary education; in this way, different educational stages were covered. Each class consisted of approximately 40 to 45 students. The content addressed during the lessons included plane and solid geometry (1st grade), division (3rd grade), and equations (6th grade).

All participants and their families provided informed consent, ensuring ethical treatment of the data. In addition, data management was supervised by an Ethics Committee (Ethical Approval No. 179/2024).

Data collection and analysis

We conducted video and audio recordings of six mathematics lessons (two per teaching pair), each lasting approximately one hour and twenty minutes. The recordings focused on teachers' interactions with students and the knowledge mobilized during instruction.

A content analysis (Kuckartz, 2019) was conducted on the lesson transcripts, which were segmented into episodes and subepisodes using a conversational criterion. Special attention was given to teachers' interventions with students. The units of analysis corresponded to units of recording (Krippendorff, 2019), as we coded specific content segments in which categories from the MTSK framework—namely MK and PCK and their respective subdomains—could be identified, as well as the model of co-teaching

enacted. To identify the categories mobilized by teachers during co-teaching, we first conducted a deductive content analysis. This involved identifying the specialized knowledge mobilized by each teacher across the six lessons. We then identified subepisodes in which both teachers interacted around the mathematical content being addressed (36 subepisodes), followed by identifying the inferred models of co-teaching within those subepisodes. Finally, an inductive content analysis was conducted to further examine the findings within each MTSK domain according to the type of co-teaching involved. Particularly, our analysis adopted a perspective centered on the object of interest—the MTSK and co-teaching—rather than on the participants themselves.

RESULTS

Although co-teaching by each teacher pair is intended to span the entire lesson, our analysis identified 24 episodes characterized by a one–teaches–one observes dynamic and 12 sub-episodes aligned with a one teaches–one-teaches-one-assists model. The following section outlines the patterns identified for each co-teaching type.

Mobilized Knowledge in One-Teaches–One-Observes Co-Teaching

This model of co-teaching occurs exclusively in the *MT teaches–SET observes* configuration. Three patterns were identified: (a) one teacher explains the lesson objective while the other intervenes on central aspects; (b) one provides task instructions while the other paraphrases; and (c) one focus on students' representations or strategies while the other directs students' attention.

From the perspective of specialized knowledge mobilized, the first pattern—one that explains the lesson objective while the other intervenes on central aspects—involves *KMLS* on the part of both teachers. The MT draws on knowledge of expected learning outcomes to frame the objective and prompt prior knowledge, while the SET draws on knowledge of students' conceptual development to reinforce correct responses. For example:

MT	I want to ask a question. Do you remember that last week the teacher said "solving divisions in ___ and ___ ways"?
Student	Sym… symb…
MT	Okay, ¿sym… bo…?
Student	¡Symbolic!
MT	Which of the two would be symbolic?
SET	Which one of the two?

In this sub-episode, the MT engages students in a discussion of prior knowledge related to the lesson objective, particularly regarding the types of representations previously addressed. Likewise, the interaction serves to recall the representations previously used to solve division problems, with the SET merely reiterating the MT's question. Similar

interactional moves appear in the second and third patterns, though with different instructional goals.

In the second pattern—*one provides task instructions while the other paraphrases*—the MT mobilizes *KoT* (e.g., definitions, properties, procedures) and *KMT* (e.g., tasks and examples). The SET's participation varies according to the nature of the intervention. In some sub-episodes, the SET mobilizes *KoT* to complement the MT's instruction. For example, when the MT asks *"What are we going to share now?"* and students struggle to respond, the SET draws on the problem context to highlight the divisor by reformulating the question as *"Let's see if that's it—between Ana and Silvia, right?"*. In other sub-episodes, the SET mobilizes KMT to refocus the task by rephrasing or narrowing the MT's questions.

MT	What is the green one called, the third?
SET	The last one…
Student	Square
MT	Correct. And the fourth one, Student? What is it called?
SET	What is it called, Student?
Student	Rectangle!

In the third pattern—one probes students' representations or strategies while the other focuses attention—the MT mobilizes *KoT* (e.g., definitions, properties, procedures) alongside *KMT* (e.g., tasks and examples). The SET, in turn, mobilizes *KMT* to direct students' attention by isolating specific steps or breaking the task into smaller units. For example, in one sub-episode:

MT	What question do you have? Or can a classmate explain what needs to be done?
Student	You have to cut out the shapes, paste them in the square, and put similar ones on each side.
SET	Okay… the shapes have to be cut out.
Student	Paste them in the squares.
SET	And paste them here, on these sides… what else needs to be done?
Student	Paste the similar ones together.
SET	Right, paste the similar ones on each side.

Here, the SET structures the task step by step to support students' understanding. Overall, the data suggest a complementary distribution of knowledge between teachers. While the MT mobilizes a broader range of knowledge, the SET mobilizes a distinct form of *KMT* aimed at enabling students to reach an answer, often by lowering the cognitive demand of the task.

Mobilized Knowledge in One-Teaches–One-Assists Co-Teaching

This form of co-teaching involves role switching, with both the MT and the SET assuming the role of lead teacher at different moments. Two implementation patterns were identified: *complementarity* and *combination*.

In the *complementarity* pattern, one teacher initiates an instructional action and the other extends it. For example, the MT introduces the task, while the SET carries it out with students:

MT	You are going to listen to the following statement and turn your card If you agree or disagree.
SET	Okay, very good. The first statement says: "Rectangles have all sides equal." Do you agree or disagree? Let's see—show your cards. I'll read it again so you can think: "Rectangles have all sides equal." I see some green and some red cards; the red ones—those who disagree—are winning. Agustín, why do you disagree?

In the *combination* pattern, one teacher leads the activity while the other contributes new perspectives through active intervention. In the following extract, the SET writes an equation symbolically, and the MT completes missing elements:

SET	Four, right? Because terms are separated by plus or minus signs.
	So we have one, two, three, and four. And what is the equal sign called?
Students	Equality.
MT	And we're missing something… the…
Students	Coefficients
MT	Coefficients—and the…
Students	Unknowns.

Regarding specialized knowledge mobilized, in the complementarity pattern, the leading teacher typically mobilizes *KoT* related to the mathematical idea or *KMT* related to the task. The other teacher complements this by mobilizing *KoT* (e.g., procedures or alternative representations), *KMT* to adapt the task, or *KMLS* by attending to students' conceptual level. For instance, when the MT asks students to represent the equation $2x + 1 = 7$ pictorially, emphasizing the terms, the SET intervenes by saying, *"Guys, and also, do you remember this thing that goes in between the two?"* (referring to the 2 and the x), a common source of difficulty in symbolic representations.

In the combination pattern, the leading teacher primarily mobilizes *KMT* related to task design or *KMLS* regarding students' conceptual development, while the other teacher mobilizes *KMT* to implement the task previously introduced.

DISCUSSION AND CONCLUSIONS

Because the effectiveness of co-teaching depends on multiple conditions—such as how it is implemented and teachers' professional resources, including their knowledge base

(Cook & Friend, 1995; Scruggs et al., 2007)—this study set out to examine the specialized knowledge mobilized by MT–SET pairs during co-teaching.

Consistent with previous research (Harbour et al., 2022; Shin et al., 2016), the most prevalent model observed was one teaches–one assists, typically led by the mathematics teacher. However, our findings also show role fluidity, with the SET assuming instructional leadership in some moments. By contrast, in the one–teaches–one observes model, the SET's participation was limited to reformulating the MT's contributions and did not involve assuming a leading role. While this may be partly explained by our focus on interactional episodes rather than full lessons, the findings highlight the relevance of examining how leadership shifts across co-teaching arrangements and the knowledge mobilized when it does.

A key contribution of this study concerns the *nature of the knowledge mobilized*. Across both co-teaching models, teachers drew on similar forms of knowledge (KoT, KMT, and KMLS). Nevertheless, the teacher complementing instruction typically introduced alternative representations or procedures. Moreover, assisting or observing teachers appeared to engage in greater cognitive work, as they interpreted the leading teacher's input and adapted or reformulated it to provide support. As a result, the knowledge mobilized by the assisting or observing teacher was contingent on that mobilized by the teacher leading instruction.

These findings underscore the importance of accounting for the variability of classroom situations in the preparation of both MTs and SETs. By documenting co-teaching models and the specialized knowledge mobilized in primary mathematics classrooms, this study offers insights that may inform teacher educators in both mathematics education and special education, particularly in courses addressing co-teaching practices.

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