TEACHING PATTERNS IN A REFORMED CURRICULUM. A CASE STUDY

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This study explores how an experienced practicing teacher, in a reform curriculum context, interpreted and responded to the expected learning outcomes regarding the use of a realistic context for pattern generalization. The data came from two professional development meetings and was analyzed according to the curriculum noticing framework. The results highlight the importance of generalization in teaching patterns, supported by visualization and manipulatives. The realistic context was not emphasized.

INTRODUCTION

In a centralized educational system, teaching decisions are framed by the official curriculum materials. Within the Greek educational context (prior to the reform) there is a single mathematics textbook that is exclusively used in all schools. This strongly guides teaching decisions, making it the most influential curriculum material and creating a textbook culture. A new mathematics curriculum has been introduced to the Greek secondary education which includes two official documents: a) The teacher's guide and b) a document (version 1) presenting in matrixes statements that express what students are expected to know, to do, and to understand in a measurable way called Expected Learning Outcomes (ELOs).

Content related to patterns and generalization is a major innovation introduced to the Greek educational context by the new curriculum. The 7th grade ELOs on patterns call for recognition, expansion, making a pattern, multiple representations, general term, and solving daily life problems (Table 1).

The dynamic relationship between teachers and curriculum materials which shapes both curriculum materials and teaching practices, has been framed by resource usage theories. These theories cannot explain the reasons for this shaping, while deeper research may focus on how and why this shaping occurs (Males et al., 2015).

Reviewing the official curriculum materials, teachers may imagine using some of its features in their future teaching (Mason, 2002). Professional noticing of children's mathematical thinking is described by Jacobs et al. (2010) as a set of interrelated skills: attending to students' strategies, interpreting their understanding, and deciding how to respond based on these interpretations. Attending involves recognizing the mathematical knowledge in students' strategies within specific contexts. Interpreting relies on reasoning about these strategies and the development of mathematical

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M.S. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: Research Reports*, pp. 1387-1394. PME.

thinking. Deciding refers to formulating responses based on interpretations of students' mathematical understanding. Focusing on teaching, Dietiker et al. (2015) oriented this concept towards the use of curriculum ideas in teaching and defined the Curriculum Noticing (CN) framework as having three phases of skills. Curriculum attending involves recognizing, searching, seeing, and perceiving. Curriculum interpreting involves making sense, digesting, questioning, comprehending, and connecting. Curriculum responding involves deciding, choosing, manipulating, sequencing, and adapting.

The CN concept, combined with a reformed Greek educational context where algebra teaching and learning experiences include patterns and the absence of a textbook, creates a unique research area. The focus of our research is the ways teachers use the new curriculum to teach patterns and generalization for the first time in lower secondary grades. Our research aims to answer the following research question (RQ1): *How did a practicing teacher, in a reform curriculum, interpret and respond to the expected learning outcome of using a realistic context for pattern generalization?*

LITERATURE REVIEW ON CURRICULAR NOTICING

Using CN as theoretical lens creates an active research area that exhibits many interesting results. Liakos et al. (2022) used CN to study the design practices of a teacher who used a new textbook and concluded that the attending skill of locating should be added. Males and Setniker (2019) studied four prospective teachers using CN and concluded that, while noticing, teachers may simultaneously attend, interpret, and make teaching decision.

Gümüş (2020) concluded that Preservice Teachers (PTs) engaged in designing and enacting pattern lessons had difficulty in making the general term of the pattern. A similar difficulty was also concluded by Lee and Lee (2025), who used CN to study 155 university students enrolled in an elementary teacher course to gain teaching certification. Furthermore, PTs seem to design pattern tasks mostly without a daily life context, even when they intend to use a daily life context (Gümüş, 2020).

Noticing is an open research area. For example, Van Es and Sherin (2021) refined the teachers' noticing framework and added the dimension of shaping, which describes teaching actions made to further enhance the interpretation of student thinking. Woller (2025) is currently studying what critical events preservice teachers attend to while watching videos of their own teaching. Some preliminary results indicate PTs interest in learning difficulties. Rotem and Aylon (2022) developed a theoretical construction to better frame the complexity of what is described as critical events. These events are classroom situations where students' mathematical thinking becomes apparent. The framework includes the dimension of who acted (teacher, student, both), what happened (math, pedagogy, both), and the teaching and learning dimension (affective, cognitive, social).

METHODOLOGY

Reform in the Greek context

Mathematical content is organized around key mathematical concepts such as structure (commonalities observed in the given terms) and generalization (expanding commonalities to all terms), referred to big mathematical ideas. Some of the mathematical practices proposed as ways of engaging with the big mathematical ideas are the use of manipulatives, using multiple representations (visualization), and engaging students with an unknown problematic situation (problem solving). Within the context of the Professional Development (PD) practice of lesson study, teachers collaborate to design math lessons, enact them, and learn by reflecting on what happened during those lessons.

Although the new curriculum does not distinguish between the addition strategies used to make a nearby term (arithmetic generalization) and the multiplication strategies to make a distant term and handle unknown quantities (algebraic generalization) (Demonty, 2018), such distinction can be considered a curriculum idea because the 7th grade ELOs on patterns gradually move from the expansion of a set of given terms (arithmetic generalization) described in ELO2 to making the general term (algebraic generalization) described in ELO5 (Table 1).

Context, participant, data, and theoretical framework of the study

We used CN as our theoretical framework. A practising mathematics teacher in lower secondary education, named JW, was the participant in our research. JW was enrolled in a PD focused on the use of the new Greek curriculum. The PD was in the form of lesson study. JW and nine other practicing teachers, along with two teacher educators, had a design meeting where JW's teaching designs were discussed and a reflection meeting where teachers focused on the enactment of these designs. JW designed a pattern generalization task with a daily life context, engaging students in the use of manipulatives and visualization. Teachers had access to the new curriculum materials and some pattern figures provided by the Institute of Educational Policy (IEP). These curriculum materials were almost the only sources JW had for designing and enacting the teaching and learning of patterns.

Data Analysis

We initially worked with the data from top to bottom and looked for prompts that fit into one of the three CN phases. We also coded those prompts from the ground up and, after many rounds of consensual coding (all authors had to agree), we divided the data into design episodes (DE) and reflection episodes (RE). Each episode is a segment of JW's prompts with the same conversational topic. Finally, we connected those episodes and answered RQ1. Some prompts belonged to multiple CN phases, so we categorized those prompts based on our overall focus on reflection.

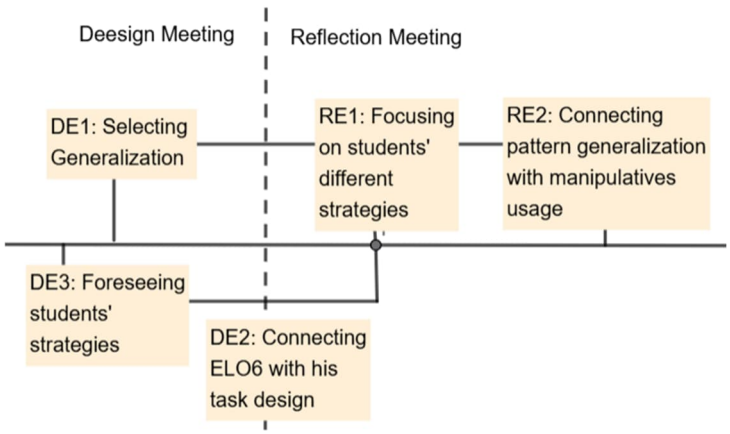
Table 1. DE1: Selecting ELOs on grade 7 patterns

Data	Coding
It's not a specific unit. We just want to make them understand the concept of patterns, the concept of continuity, the concept of generalization and proof simultaneously.	Attending = select generalization & proof
Let's see what this new is…It says [ELO1:] to recognize, compare, describe patterns and to express them as arithmetic patterns with natural numbers. That's the first one.	Interpreting = Describing ELO1
[ELO2:] To fil, expand and create arithmetic patterns with natural numbers	Interpreting = Describing ELO2
[ELO4:] To represent patterns in multiple ways like images [or geometric shapes, value matrixes and points on an axis system, and to move between these representations] …	Interpreting = Describing ELO4
[ELO5:] To investigate patterns that can be expressed in the form a_v (with a as the rational and v as the order of the term) and to formulate their general term verbally and symbolically.	Interpreting = Describing ELO5
[ELO6:] To solve problems that can be encountered in daily life using patterns.	Interpreting = Describing ELO6
The problems I have designed are kind of within daily life. It's not something difficult.	Interpreting = connects ELO6 with the tasks

RESULTS

We answer RQ1 by describing six themes that were raised from JW's interpretation of the ELOs on patterns. These themes are expanded throughout both meetings (design & reflection) (Table 2).

Table 2. The Themes



Interpreting ELO6: the daily life context

JW interpreted ELO6 (solving daily life problems) as unimportant. This can be concluded because, although he designed the task (Table 3) with a daily life context in alignment with ELO6, his focus on the context was limited to the problem's initial description, and he did not make any reference to this context in any of the task's questions. Furthermore, JW did not mention this context in the reflection meeting prompts.

Table 3. The task

The school board decided to move forward with the replacement of the school's backyard tiles. The shape of the backyard where the square tiles are going to be placed is in accordance with the 100th figure as shown in the following pattern.

A) How many squares you think there are in the 5th figure? Try to construct it by drawing or using puzzles.

B) How many squares you think there are in the 6th figure? Where do they come from?

C) How many squares you think there are in the 100th figure?

D) How many squares you think there are in the nth figure? Can you explain it?

Selecting Generalization

JW interpreted the ELOs on patterns as being built around the big idea of generalization. We concluded this because, in DE1 (Table 1), he selected generalization as the main mathematical content in teaching patterns. In DE1, he related all ELOs on patterns to generalization and connected them with the task he designed. In the reflection meeting, JW talked about generalization in both REs.

The Teaching Difficulty of Generalization

JW interpreted the ELOs on patterns as difficult to teach, since his students failed to generalize algebraically (RE1). However, teaching patterns can be supported by engaging students with the practices of visualization and the use of manipulatives (Table 4), as happened in RE1 when student teams drew the 5th, 6th, or even the 100th term to calculate the number of squares. JW also concluded that engaging students with the big idea of structure can support the teaching of arithmetic generalization but not the teaching of algebraic generalization (RE2).

> RE2: Here's another group…in each new column 2 more new squares are added so the sixth figure will be 36. They found 36, they found 25 but in the 100th term they got lost again. That was the critical point.

JW interpreted the teaching of the ELOs on patterns as feasible, since one team managed to make the general term of the pattern while working on the task (RE2).

RE2: This team [group 1] reached the 100*100…This team [group 2] didn't write anything, didn't justify anything the student just wrote 100*100 and made the 10000 and then following the same strategy made the general term. It was the only team out of 8 teams that made it. This team [group 3 made the 100th term]. They can do it! They just need practice.

Evaluating Students' work on pattern problems

When JW reflected on the teaching of ELOs on patterns had an evaluative way of thinking. We concluded this because JW attended the only team that made the general term and interpreted this as a correct answer (RE2). He did not focus on the way students solved the problem, and he even considered as positive the absence of an explanation for making the 100th term.

Foreseeing strategies and familiarity with manipulatives & visualization

JW interpreted the ELOs on patterns as being better enacted by engaging students with the practices of visualization and the use of manipulatives. We concluded this because, in DE1, he foresaw that students would be familiar with visualization.

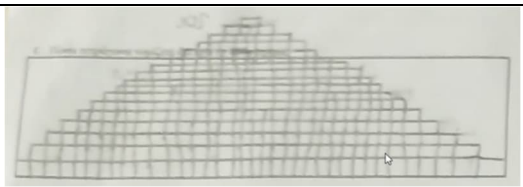
DE1: It is something they have worked with in the elementary classroom […] this with the pattern. […]. Working with squares is something doable and they like it. There are such tasks in extracurricular materials for the summer. If it is difficult for the students to handle it a test will tell us.

JW's forecast was verified because most students made the arithmetic generalization by counting the number of squares in each figure, and one team made the algebraic generalization (RE2).

Interpreting ELO5: a quadratic pattern

JW interpreted ELO5 as calling for the engagement of students with quadratic patterns despite ELO5's detailed description of linear patterns. We concluded this because JW designed the task with the mathematical content of the quadratic pattern n^2 (Table 3).

Table 4. Visualization and Manipulatives usage

Using post-it	The team that drew the 100th term
	

CONCLUSIONS AND DISCUSSION

The practicing teacher, JW, designed and enacted a lesson on pattern generalization based on the 7th grade ELOs on patterns. Throughout the two PD meetings (design & reflection), it was apparent that JW selected generalization as the main mathematical content when teaching patterns. Teaching generalization poses the challenge of

successfully engaging students in algebraic generalization. This difficulty was addressed by engaging students with the mathematical practices of using manipulatives and visualization. Although students managed to understand the structure of the pattern and made the arithmetic generalization, teaching the algebraic generalization remained a difficulty. The success of one team in generalizing algebraically was interpreted by JW as evidence that algebraic generalization can be taught successfully. These interpretations are in alignment with previous research that describes the difficulty of teaching and learning generalization (Gümüş, 2020; Lee & Lee, 2025).

JW interpreted curriculum materials related to daily life problems (ELO6) as being of low priority, and thus, he limited the incorporation of a realistic context into his teaching designs. This conclusion seems to be in alignment with previous research results (Gümüş, 2020).

Indicative of JW's interpretation of curriculum materials was the fact that he adopted an evaluative way of interpreting the mathematical experiences developed within the classroom. Evaluative is a term we adopted from the broader noticing literature (Van ES & Sherin, 2021). While reflecting on students' mathematical experiences as framed by his teaching designs, JW responded to students' work by simply characterizing it as correct or wrong without delving deeper into students' mathematical thinking. Being evaluative may be a hint of considering arithmetic generalization (ELO1, ELO2, ELO4) and the supportive mathematical practices of visualization and the use of manipulatives usage as less important than generalizing algebraically and stating the pattern's algebraic formula (ELO5).

References

Demonty, I., Vlassis, J., & Fagnant, A. (2018). Algebraic thinking, pattern activities and knowledge for teaching at the transition between primary and secondary school. *Educational Studies in Mathematics, 99,* 1-19. https://doi.org/10.1007/s10649-018-9820-9

Dietiker, L., Males, L. M., Amador, J. M., & Earnest, D. (2018). Research commentary: Curricular noticing: A framework to describe teachers' interactions with curriculum materials. *Journal for Research in Mathematics Education, 49*(5), 521-532. https://doi.org/10.5951/jresematheduc.49.5.0521

Jacobs, V. R., Lamb, L. L., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for research in mathematics education, 41*(2), 169-202. https://doi.org/10.5951/jresematheduc.41.2.0169

Lee, M. Y., & Lee, J. E. (2025). Investigating Pre-Service Teachers' Problem-Solving Ability and Their Curricular Noticing Ability Through Pedagogical Sequencing Activities. *International Journal of Science and Mathematics Education, 23*(5), 1211-1236. https://doi.org/10.1007/s10857-021-09520-5

Males, L. M., Earnest, D., Dietiker, L., & Amador, J. M. (2015). Examining K-12 Prospective Teachers' Curricular Noticing. *North American Chapter of the International Group for the Psychology of Mathematics Education.*

Males, L. M., & Setniker, A. (2019, April). An examination of prospective secondary teachers' curricular noticing when planning. In *Paper presentation at the annual meeting of the American Educational Research Association*.

Mason, J. (2002). *Researching your own practice: The discipline of noticing*. Routledge

Özyildirim Gümüş, F. (2022). Preservice elementary mathematics teachers' use of patterns and pattern problems when planning and implementing lessons. *International Journal of Mathematical Education in Science and Technology, 53*(8), 2152-2175. https://doi.org/10.1080/0020739X.2021.1952325

Rotem, S. H., & Ayalon, M. (2022). Building a model for characterizing critical events: Noticing classroom situations using multiple dimensions. *The Journal of Mathematical Behavior, 66,* 100947. https://doi.org/10.1016/j.jmathb.2022.100947

Yannis Liakos, Saba Gerami, Vilma Mesa, Thomas Judson & Yue Ma (2022) How an inquiry-oriented textbook shaped a calculus instructor's planning. *International Journal of Mathematical Education in Science and Technology, 53:1,* 131-150, https://doi.org/10.1080/0020739X.2021.1961171

van Es, E. A., & Sherin, M. G. (2021). *Expanding on prior conceptualizations of teacher noticing. ZDM–Mathematics Education, 53*(1), 17-27. https://doi.org/10.1007/s11858-020-01211-4

Wöller, S. (2025). Noticing of preservice primary school teachers in mathematics while watching own video vignettes. In Cornejo, C., Felmer, P., Gómez, D. M., Dartnell, P., Araya, P., Peri, A., & Randolph, V. (Eds), *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 411–418).