THINKING CLASSROOMS: CHARACTERIZING THIN SLICING

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This study examines practices that promote explorative participation within a secondary mathematics M.Ed. course, focusing on the Thinking Classrooms approach. It investigates how mathematics student-teachers applied the principle of thin slicing in lesson planning. Five lesson plans were analysed through two lenses: variation theory and the type of participation the tasks afford. The findings show that all task sequences aligned with principles of variation theory and exhibited strong bondedness. Substantiation was necessary but implicit, and flexibility was limited. Across all lessons, ritual-enabling tasks preceded exploration-requiring tasks, serving as a foundation for exploration. The study offers a framework for characterizing thin slicing and highlights instructional practices that promote explorative participation.

INTRODUCTION

The pedagogical approach known as *Thinking Classrooms*, developed by Peter Liljedahl (2021), seeks to enhance student engagement and foster opportunities for meaningful mathematical thinking and discovery. Within this framework, students work on mathematical tasks while standing in front of vertical whiteboards, enabling full visibility of their work to peers and the teacher. Lessons typically begin with a concise teacher explanation, after which students proceed through a sequence of pre-designed tasks.

The task sequence is constructed according to the principle of *thin slicing*, which is grounded in variation theory (Frazee et al., 2023). The purpose of thin slicing is to sustain a level of difficulty that is both challenging and attainable, aligned with students' zones of proximal development (Vygotzky, 1962), thereby supporting continuous learning in which success on one task naturally leads to the next, typically without requiring extensive teacher explanation. Liljedahl (2021) highlights two key principles of variation theory: learners notice change in relation to what remains constant, and only a single component should vary from one task to the next. This design has the potential to foster exploratory engagement and active participation.

Teaching that fosters *ritual participation* emphasizes the execution of familiar procedures and relies heavily on memorization and imitation. This mode of instruction remains the most prevalent in mathematics classrooms (Nachlieli & Tabach, 2019). In contrast, teaching that supports *explorative participation* provides students with opportunities to construct new narratives and procedures independently. Despite its potential benefits, such instruction is relatively uncommon (Brown, 2003). Research indicates that learning to teach in a way that provides students opportunities for

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explorative participation are complex, and require sustained time and support (Nachlieli & Heyd-Metzuyanim, 2022).

The aim of this paper is to characterize the thin slicing of a task sequence through two complementary lenses: variation theory and the opportunities to learn (OTLs) afforded to students.

We use the term *ritual-enabling OTLs* to describe teachers' actions that offer students tasks that could be successfully solved through the rigid application of previously learned procedures. In contrast, *exploration-requiring OTLs* refer to teachers' actions that present students with tasks that cannot be completed through ritual performance alone; rather, a successful completion of the task necessitates explorative participation (Nachlieli & Tabach, 2019).

The process by which learners proceed from ritual to explorative participation while learning mathematics, is named de-ritualization (Lavie et al., 2019). Students' participation along the continuum between ritual and exploration can be conceptualized through several characteristics of de-ritualization include: (1) *bondedness*, referring to the extent to which the input required for solving a task depends on the output of the preceding task; (2) *substantiation*, which concerns whether the task prompts students to justify their choices; and (3) *flexibility*, which examines whether the task can be solved in multiple ways by the learners.

The *Thinking Classrooms* pedagogical approach was incorporated into a course on teaching practices for secondary school mathematics teachers enrolled in an M.Ed. program in mathematics education at a teacher-education college, with the aim of supporting instructional practices that promote explorative participation.

Research questions (RQs):

1. What are the characteristics of the thin slicing evident in the task sequence created by practicing teachers?
2. What opportunities to learn do these tasks afford students?

METHODOLOGY

Eight practicing teachers participated in the teaching practices course (they will be referred to as student-teachers). As part of their learning, they were introduced to the principles of the *Thinking Classrooms* pedagogy, engaged in mathematical activities aligned with this framework, and designed mathematical tasks based on the principles of thin slicing. Their final assignment required them to design a lesson structured according to the Thinking Classrooms framework. In this assignment, the student-teachers were asked to articulate the mathematical ideas and processes that learners would engage with throughout the lesson and to provide a detailed rationale for the sequencing and thin slicing of tasks. The assignment was completed in small groups, resulting in five submissions. Each of the groups taught the lesson to the other student-

teachers. To address the first research question concerning the characteristics of thin slicing, we examined:

a. Whether the task sequences aligned with the principles of variation theory, with particular attention to what remained invariant and what varied from one task to the next.
b. *Bondedness* - the extent to which each task built on insights that emerged from the preceding task.
c. *Substantiation* - whether the task required students to justify or explain their reasoning.
d. *Flexibility* - the degree to which the task allowed for multiple solution paths or procedures

To address the second research question concerning the type of participation afforded by the tasks, we examined whether students were expected to develop an unfamiliar narrative or procedure. In such case, the task was classified as an exploration-requiring OTL. Otherwise, it was categorized as a ritual-enabling OTL.

FINDINGS

We illustrate the findings of this study through two lesson plans designed by two student-teacher groups. The first lesson plan focuses on quadratic inequalities (see Figure 1), and the second on perpendicular lines (see Figure 2). For each lesson, we first address RQ1 by characterizing the thin slicing of the task sequence, examining the variation across tasks as well as their bondedness, substantiation, and flexibility. We then address RQ2 by analyzing the opportunities to learn (OTLs) that the tasks afford students. Finally, we draw some general observations based on the five assignments handed by all student-teachers.

Lesson1

The goal of the lesson was for students to develop a procedure for solving quadratic inequalities. Lesson 1 begins with the student-teachers reminding students how to solve linear inequalities. Students are then sent to work in groups on Tasks 1 – 6.

Variation. Throughout the sequence, the use of graphs as a mediator for solving quadratic inequalities remains constant, while the specific quadratic function varies. In this sense, the sequence aligns with the principles of variation theory.

Bondedness. Bondedness is high: each task relies on the solution or insights developed in the preceding task.

Substantiation. Substantiation is partial. Tasks 4 – 6 call for generalization, which inherently requires justification, even if this is not explicitly requested.

Flexibility. Tasks 1 and 6 afford partial flexibility. Task 1 allows for drawing various lines and Task 6 also allows for several solution strategies.

In terms of opportunities to learn, Task 1 is ritual-enabling for students who have previously learned how to solve quadratic inequalities, but exploration-requiring for those who have not. Task 2 is ritual-enabling, serving primarily to verify the solution obtained in Task 1 and to determine whether a new procedure is needed. Task 3 (for students who were unsuccessful in Task 2) introduces a new mediating object, a graph, into the discourse of solving algebraic inequalities. Task 4 is ritual-enabling with potential for exploration: although it can be solved procedurally, it is designed to encourage students to connect graphical and symbolic mediators, effectively aligning the two discourses around key narratives such as "there is no solution" or "there is a solution for any value of x." Task 5 may require exploration, and Task 6 is clearly exploration-requiring, as no previously learned procedure can be directly applied.

Figure 1: Plan of Lesson 1

Lesson 1: quadratic inequalities

Task 1:

Solve the inequality $x^2 > 4$

Task 2:

a. Substitute the following values into the inequality given in task1 and check whether the result is a true statement:

   $x = 0$; $x = (-7)$; $x = 3$

b. Do your answers in part (a) match the solution set you found in Task 1?

   If yes, proceed to task 4

   If not, proceed to task 3.

Task 3:

Draw the graph of the function $f(x) = x^2$

Draw the graph of the function $g(x) = 4$

Based on the graphs, find the interval for which $f(x) > g(x)$.

Is this the solution to the inequality $x^2 > 4$?

Task 4:

Given the function $f(x) = x^2 + 6x + 9$. Use a drawing of a relevant graph to match each inequality to its solution:

$x^2 + 6x + 9 > 0$			Every number is a solution
$x^2 + 6x + 9 < 0$			$x = (-3)$
$x^2 + 6x + 9 \geq 0$			$x \neq (-3)$
$x^2 + 6x + 9 \leq 0$			The inequality has no solution

Task 5:

Solve the following inequality: $x^2 - 3x - 10 < -2 + 4x$

a. Solve graphically by comparing two functions

b. Solve graphically by finding the intervals where a function is positive or negative

Task 6:

Give an example of a quadratic inequality:

a. For which every value of x is a solution of the inequality

b. For which no value of x is a solution of the inequality

Lesson 2

The goal of this lesson (see Figure 2) was for students to discover the conditions under which two lines are perpendicular. That is, the goal is for students to develop a mathematical narrative. The student-teachers who designed the lesson chose not to include an introductory discussion to launch the activity.

Variation. The graphs of the two lines on the coordinate plane remain constant throughout the sequence of tasks. In each task, the diagram develops further, including the symbols used to represent mathematical objects. What changes is the specific aspect students are asked to articulate. In this way, the task sequence aligns with the principles of variation theory.

Bondedness. Bondedness is high: each task depends on the solution strategies or insights generated in the previous one.

Substantiation. Substantiation is only partial. Although the tasks inherently require justification, since expressing each segment across the tasks involves a process of generalization, this requirement is not explicitly stated. We therefore conclude that substantiation is necessary for completing the tasks, even though it is not directly requested.

Flexibility. Tasks 1 and 6 offer partial flexibility. Task 1 can be approached in several ways, and Task 6 similarly allows for multiple valid solutions.

In terms of opportunities to learn, Tasks 1 – 3 function as ritual-enabling activities: students draw graphs based on given conditions and express lines using procedures they have previously learned. These tasks lay the groundwork for Tasks 4 - 6, which are exploration-requiring and for which no pre-taught procedure can simply be applied.

It is important to note that the student-teachers did not articulate an explicit overarching goal for the lesson. Had the student-teachers informed students at the outset that they would be discovering a rule (or condition) for determining when two lines are perpendicular, their engagement could be interpreted as exploration-requiring.

This distinction resembles the difference between a person carving stones without knowing their purpose and a person carving stones as part of building a cathedral, the same action, but with a fundamentally different orientation and investment.

Figure 2: Plan of Lesson 2

Lesson 2: perpendicular lines

Task 1

- Draw a coordinate system and label the origin as O
- Draw an increasing linear function through the origin: $y = m_1x$
- Draw a decreasing linear function through the origin: $y = m_2x$

Define point A on the line $y = m_1x$

Express the segment OA in terms of m_1 and x.

Task 2

Define point B on the line $y = m_2x$ so that the x-value of point B is similar to the x-value of point A.

Express the segment OB in terms of m_2 and x.

Task 3

Connect a segment between points A and B

Express the segment AB in terms of m_1 and m_2

Task 4

Express the condition for the existence of a right-angled triangle AOB ($\angle AOB = 90°$) in terms of m_1 and m_2

Task 5

Formulate a theorem for the perpendicularity of lines, based on the condition you found in Task 4

Task 6

Give 4 examples of slopes that satisfy the condition $m_1 \cdot m_2 = (-1)$ which you found in Task 4.

General findings of all five lesson plans

All lesson plans included a discovery-oriented goal, whether the aim was for students to develop a procedure (as in Lesson 1) or to develop a narrative (as in Lesson 2). The tasks within each lesson plan were strongly bonded. Although substantiation was not explicitly requested, it was necessary for completing the tasks. In the geometry lesson plans, the central mathematical activity involved proving or formulating a theorem,

which inherently required substantiation. In the algebra lesson plans, the lessons culminated in generalization, again making substantiation essential.

Flexibility was largely absent from the tasks. When flexibility did appear, it was limited to choices such as selecting which graph to draw or providing examples, rather than offering genuinely different solution paths. The tasks generally steered students toward specific solutions. All lesson plans were consistent with the principles of variation theory.

Regarding the forms of participation invited, all lessons began with ritual-enabling tasks that served as a foundation for subsequent exploration-requiring opportunities.

DISCUSSION

This study examined how student-teachers design task sequences aligned with the *Thinking Classrooms* approach, with particular attention to the practice of *thin slicing* and the opportunities to learn afforded by these sequences. Our analysis focused on two research questions: (1) What characterizes the thin slicing evident in the task sequences created by student-teachers? and (2) What forms of participation do these sequences invite?

To address RQ1, we suggested four characteristics, from two lenses. One is variation theory (Frazee et al., 2023) and the other is de-ritualization (Lavie, et al., 2019). From variation theory we relate to what changes and what remains constant from one task to the next. From de-ritualization, we adopt three characteristics - bondedness, substantiation, and flexibility. As can be seen in the Findings section, we applied this conceptualization to characterize the thin slicing of lesson plans. One contribution of this paper is this innovative theoretical-methodological approach.

To address RQ2, we examined the types of participation afforded by the tasks. We found that in all the lesson plans, the tasks encouraged explorative participation. As for the sequence of OTLs that was afforded by the sequence of tasks throughout the lesson plan - all lessons began with ritual-enabling OTLs which served as a foundation for the explorations afforded by the last tasks of the lesson plan. This finding is consistent with findings from Nachlieli and Tabach (2019), where it was found that teachers provide ritual-enabling OTLS for four different reasons, one of which is to serve as foundation for exploration.

We also observed a similar pedagogical choice in all lesson plans, which aimed at enabling shared communication among all groups of students. The first tasks in each sequence included a requirement for drawing specific graphs or diagrams, as well as determining specific naming or symbolizing parts of the graphs and diagrams. The deliberate naming and symbolizing of the drawn objects were intended to support whole-class discussion, ensuring that all students referred to similar diagrams using a shared set of terms and symbols.

This study is limited in scope. The data consist of a small number of lesson plans designed in the context of a single course, and the analysis focuses on intended rather

than enacted instruction. Future research should examine how such task sequences unfold in classroom interaction and how teachers' in-the-moment decisions further shape opportunities for explorative participation.

Teaching practices aimed at encouraging explorative participation is complex (Nachlieli & Heyd-Metzuyanim, 2022). In light of this, the findings of the present study, which constitutes an initial investigation into the potential of teaching the *Thinking Classrooms* approach as an example of a set of practices designed to promote explorative participation, are encouraging.

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