KNOWLEDGE CO-CONSTRUCTION IN ONLINE INTERNATIONAL LEARNING: A STUDY OF COLLABORATIVE LINEAR ALGEBRA PROBLEM SOLVING

Alice Barana[1], Giulia Boetti[1], Marina Marchisio Conte[1], Adamaria Perrotta[2], Matteo Sacchet[1]

[1]Università degli Studi di Torino, [2]University College Dublin

This paper investigates collaborative problem solving in the context of an online international experience in higher education. The study aims to examine how students interact while working in mixed international groups on application-oriented linear algebra tasks. Data were collected through students' responses to individual reflective questionnaires focusing on their perceptions of collaborative learning. The responses were analysed using the adapted Interaction Analysis Model to identify phases of knowledge co-construction. The results show that all phases of the model emerged across all group tasks. Although some differences were observed among groups, the findings suggest that international online collaborative activities can effectively support collaboration in solving applied linear algebra problems.

INTRODUCTION

Collaborative learning has long been recognized as a powerful approach to support students' cognitive and social development (Gillies, 2016), especially in mathematics education. When students work together on mathematical tasks, they are encouraged to articulate their reasoning, compare solution strategies, and engage in discussion aimed at resolving differences in understanding, especially when the complexity cannot be overcome by an individual (Retnowati et al., 2017). Such interactions are particularly relevant for the development of problem-solving skills. International learning experiences add an additional layer of complexity and richness to collaborative problem solving. Working with peers from different educational systems and cultural backgrounds exposes students to diverse mathematical practices and communicative norms (Barana et al., 2025a).

In this paper, we investigate collaborative problem solving (CPS) in a Collaborative Online International Learning (COIL) experience named "Linear Algebra for Economics, Finance, Security and Defence Strategies" (LAEFDS). The COIL involved first-year undergraduate students from the University Interdepartmental School in Strategic Sciences (SUISS) of the University of Turin, Italy (UNITO), and the University College Dublin, Ireland (UCD), who worked in mixed international teams on applied linear algebra tasks. The study addresses the following research question: How do collaborative interactions manifest among students during problem-solving activities in Linear Algebra applied to Economics, Finance, Security, and Defence

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within an international experience? To answer this question, we analyse students' perceptions of their collaborative activity through the lens of the adapted Interaction Analysis Model (IAM) (Gunawardena at al., 1997; Barana et al., 2023a; Barana et al., 2023b), focusing on the presence of different phases across mathematical tasks.

THEORETICAL FRAMEWORK

This study is grounded in research on CPS in mathematics. According to this perspective, learning is conceived as a socially mediated process emerging through interaction, negotiation, and shared activity around mathematical tasks. Problem solving is a collective process in which meanings, strategies, and solutions are progressively constructed through dialogue and joint action (Krummheuer, 2007). Several studies highlighted how interaction among learners plays a central role in the development of problem-solving skills (Lin at al., 2015) and mathematical understanding (Felmer, 2023a). CPS can also enrich teacher-student and student-student interaction (Felmer, 2023b). From a socio-constructivist standpoint, mathematical knowledge develops through participation in practices that involve explaining one's reasoning, comparing solution strategies, and resolving inconsistencies between different approaches. In collaborative settings, these practices are made explicit and observable through interaction. Communication should be placed in the heart of mathematics education, transforming not only teaching strategies but also how we think about learning and what is being learned (Sfard, 2008). In collaborative environments, learning may be characterized as a circular, self-referential sequence of events, with students as protagonists; they participate in establishing the situations in which they learn (Cobb et al., 1992). Technologies and digital tools can also help this process and its analysis (Hannula, 2022). In international collaborative contexts, CPS is further enriched by the presence of diverse educational and cultural backgrounds. Differences in prior mathematical experiences and communicative norms can trigger moments of divergence and negotiation, which are known to be productive for learning when appropriately supported. Language is not a barrier, but an infinitely generative system that encapsulates and embodies a community's cultural experiences (Stahl, 2006).

The adapted Interaction Analysis Model (IAM)

To analyse how CPS unfolds through interaction, this study adopts the Interaction Analysis Model (IAM) (Gunawardena et al., 1997). The IAM conceptualizes collaborative knowledge construction as a progressive process articulated through five phases: (1) sharing and comparing of information; (2) discovering and exploring dissonance or inconsistency among ideas or statements; (3) negotiation of meaning/co-construction of knowledge; (4) testing and modification of proposed synthesis or co-construction; (5) agreement statement(s)/applications of newly constructed meaning. The IAM is particularly well-suited to the analysis of collaborative mathematical problem solving, as it captures both the epistemic dimension of mathematical activity (e.g., validation, generalization, application) and the interactional dimension (e.g.,

comparison of strategies, negotiation of meaning). Although originally developed to analyse online discussions among experts, the model has been successfully adapted in mathematics education research to study students' collaborative activity (Barana et al., 2023a; Barana et al., 2023b). From a mathematical perspective, the IAM phases can be meaningfully mapped onto typical stages of CPS. Phase 1 corresponds to the initial exchange of ideas, solution attempts, or interpretations of the task. Phase 2 captures moments where different strategies, results, or assumptions come into conflict. Phase 3 reflects a moment in which students work towards shared definitions, representations, or solution paths. Phase 4 aligns with mathematical validation processes, such as checking calculations, testing solutions against constraints, or exploring special cases. Finally, Phase 5 corresponds to the consolidation of shared understanding and the application of newly constructed meaning to reach a final solution or interpretation.

RESEARCH CONTEXT

The research has been carried out within the COIL project LAEFDS (Barana et al., 2025a), co-created and co-delivered by professors from UCD and UNITO. The COIL project was conceptually framed within the INVITE Erasmus+ project, which provided a structured Learning Design Framework for virtual and blended international collaboration (Barana et al., 2025b), grounded in the well-established ADDIE instructional design model (Analysis, Design, Development, Implementation, Evaluation), adapted to address the specific pedagogical and intercultural challenges of international online learning. The COIL project targeted first-year undergraduate students enrolled in the BSc in Economics and Finance and the BSc in Financial and Actuarial Studies at UCD, who attended a linear algebra module (MATH10120), and in the BSc in Strategic and Security Sciences at SUISS at UNITO, who attended a mathematics module (CPS0977), which includes Linear Algebra in its syllabus. Participation was voluntary and extracurricular. Planned for maximum 20 students per institution, 32 students ultimately enrolled, divided into 8 mixed international teams of four students, ensuring representation from both institutions and attention to gender balance. Group composition was central to fostering intercultural dialogue and CPS, applying linear algebra concepts to real-world problems in economics, finance, security, and defence. A scenario-based approach supported the activities: students acted as international "secret agents" collaborating across borders. The COIL activity was delivered in November 2024 and corresponded to a total workload of 10 hours. It was scheduled after students had completed the core linear algebra content in their respective degree programmes, so that the COIL could act as an application-oriented experience. Outside of four scheduled live online events, students worked autonomously within their teams, organizing meetings and dividing responsibilities according to their preferences and constraints to solve discipline-oriented tasks, elaborated into several sub-tasks. Task 1 addressed cryptography, requiring students to explore matrix-based encryption methods and interpret their relevance in strategic scenarios. Task 2 involved a one-sector macroeconomic model to explore policy problems. Students were required to analyse real-world parameters and interpret

mathematical results to justify specific policy interventions. Using actual Italian or Irish scenarios as a baseline, this task leveraged on and moved beyond the linear algebra theory, requiring students to interpret data and justify policy decisions, thereby engaging deeply with the analysis and evaluation stages of the learning process. Figure 1 and Figure 2 illustrate the main ideas of the tasks.

Figure 1: Example of matrix-based encryption used in Task 1

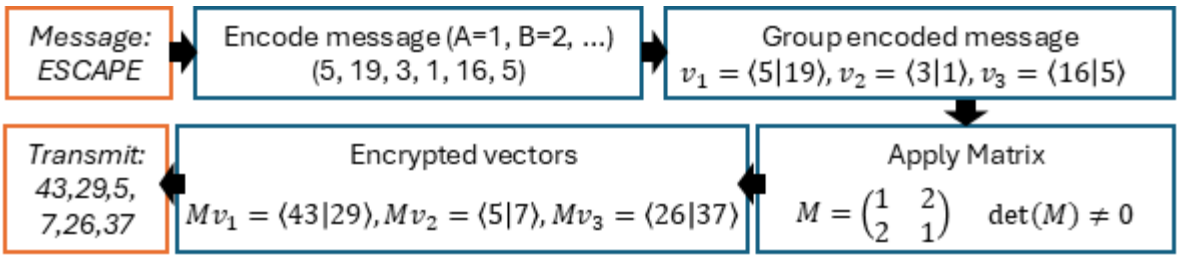
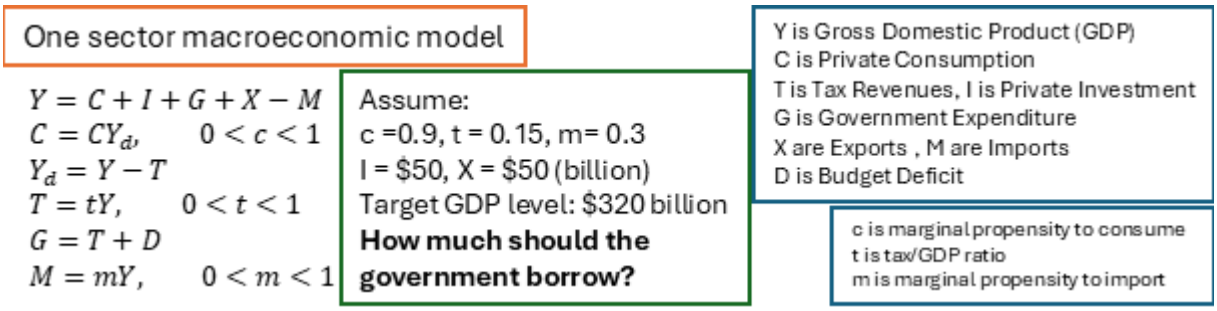


Figure 2: Example of One-target macroeconomic policy problem used in Task 2



These tasks engaged students in interpretation, justification, and interdisciplinary reasoning. At the group level, teams produced presentations for each task and a final presentation reflecting on their problem-solving process and teamwork.

RESEARCH METHODOLOGY

In the present study, the IAM in the adapted form (Barana et al., 2023a) is used to analyse students' perceptions of collaborative problem-solving activity. Adapted IAM phases are interpreted as perceived indicators of collaborative processes rather than direct evidence of interaction. Data were collected from the students, who individually completed initial and final surveys, reflective diaries after each task, and a final reflection focusing on learning outcomes, collaboration, and intercultural experience. The model directly informs the structure of the reflective diary questions, which mirror the adapted IAM phases and enable quantitative comparison of perceived collaborative processes across tasks. For each task, students had to answer the following questions.

(Q1) In relation to the working team activities, to what extent do you agree with the following statements: (P0) There was no interaction between team members during team activities, (P1) My team effectively shared and compared information during the team work, (P2) Divergent and dissonant ideas and concepts emerged within my team, (P3) My

team discussed to clarify terms, find agreements on divergent ideas, and propose compromises, (P4) My team effectively tested and modified the proposed ideas in building the solution, (P5) My team reached a common solution by trying to apply what was discussed

(Q2) How effective do you believe the collaboration was in: (1) solving the task, (2) supporting knowledge construction

Items of (Q1) could be marked with "Disagree", meaning that the phase was not performed, "Neutral", meaning uncertainty about the phase, or "Agree", meaning that the phase was performed. Items of (Q2) could be marked with "Not effective", "Somewhat effective", or "Very effective". Other questions of the diary requested students to justify their ratings in (Q1) and (Q2), describe the schedule of group activity and highlight difficulties. We collected responses from 28 of the 32 participating students, as some withdrew for personal reasons. Among these respondents, 9 (32%) were UNITO students, and 19 (68%) were UCD students. Data have been analysed using a mixed quantitative and qualitative approach, making use of descriptive statistics and hypothesis testing while pairing with students' open responses to explain the quantitative results.

RESULTS AND DISCUSSIONS

Table 1 shows the students' responses to questions related to phases, from (P0) to (P5), for the two tasks.

Table 1: Absolute values and frequencies of occurrence of the phases for Task 1.

	Task 1			Task 2		
Item	Disagree	Neutral	Agree	Disagree	Neutral	Agree
P0	23 (82%)	3 (11%)	2 (7%)	24 (86%)	4 (14%)	0 (0%)
P1	0 (0%)	2 (7%)	26 (93%)	0 (0%)	2 (7%)	26 (93%)
P2	8 (29%)	8 (29%)	12 (43%)	8 (29%)	10 (36%)	10 (36%)
P3	0 (0%)	4 (14%)	24 (86%)	0 (0%)	4 (14%)	24 (86%)
P4	1 (4%)	6 (21%)	21 (75%)	1 (4%)	5 (18%)	22 (79%)
P5	0 (0%)	5 (18%)	23 (82%)	0 (0%)	1 (4%)	27 (96%)
Total	32	28	108	33	26	109

From Table 1, we can notice that students mainly perceived the presence of phases of collaborative knowledge construction. All phases are represented for both tasks with strong percentages, and this is in line with other research (Barana et al., 2023a; Barana et al., 2023b; Gunawardena et al., 1997). This remark is validated by the fact that the median of both items in (Q2) about the effectiveness of collaboration is "Very effective". For both tasks, the least-agreed item is that related to phase 2 "discovering and exploring dissonance or inconsistency among ideas or statements". Lower

agreement on Phase 2 may suggest that, when present, cognitive conflict is not readily recognized by students as a productive component of collaboration. Moreover, words like divergent and dissonant may sound negative to students, even if they are part of a process for CPS, and they may tend to avoid them. The second least-agreed item is the one related to phase 4 "testing and modification of proposed synthesis or co-construction", a phase that in mathematical tasks may be more silent since it is done on numbers and perceived as part of a calculation more than testing. This suggests that students may not always be consciously aware of testing hypotheses and solutions, even when it plays a role in their CPS. Student A (who disagreed on having performed phases 2 and 4) wrote:

> For the most part we went through the activity as individuals comparing work rather than solving it together. This meant that collaborative activities did not have a large impact on the process.

In this case, the adopted group strategy consisted of more than one team member performing every sub-task and then negotiating and agreeing. This strategy has been detected in more than one response, like the one of Student B (who disagreed on having performed Phase 2), but this time with the emergence of dissonant results:

> […] doing the calculations simultaneously meant we could crosscheck our answers and investigate why answers weren't matching.

However, Student C mentions:

> We were able to work on the calculations individually and check our answers with the other members to ensure that we were correct. […] After trying a few different approaches, however, we were able to come to a conclusion.

This response shows that, even when skipping some phases (Student C disagreed on performing phase 2), the agreement and discussion still took place. In these responses emerge the sharing of individual solutions, that Student D matches with cognitive accountability for what was his/her contribution to the team:

> Working with people forced me to be surer of my answers as I would have to justify them to others.

Justification of reasoning and of resolution strategies to peers is emphasized by Student D as a core feature of collaborative mathematical problem solving.

Analysing differences in phase presence between the two tasks, we observe that the variations account for approximately 20% of responses: agreements for Task 1 tend to occur more frequently on Phases 0 and 2, whereas higher agreements for Task 2 tend to occur more frequently on Phases 2, 4, and 5. This difference may be due to the nature of the two tasks: Task 1 is more procedural, while Task 2 is more discursive. Task characteristics appear to mediate the emergence of collaborative phases, with interpretative demands acting as a trigger for higher-level interaction. We performed a Wilcoxon Signed-Rank Test at the 0.05 significance level to evaluate differences between the two tasks within each phase. The only two phases with significant

differences are Phase 2 and 4, which are also the least-agreed ones. For all four other phases, we fail to reject the null hypothesis of equality between the two groups. We can account for the absence of a significant difference by noting that some groups worked on both tasks in a single, longer session, which may have influenced the phase distinctions between the two tasks.

CONCLUSIONS

The results show that all phases of the IAM emerged in both tasks, indicating that students perceived to have been engaged in a range of collaborative processes, from sharing information to reaching agreement and applying newly constructed meaning. The comparison between tasks highlights that Task 2 elicited slightly richer evidence of divergent and dissonant ideas, as well as testing solutions. This finding suggests that tasks requiring interpretation, justification, and decision-making may be especially effective in promoting advanced forms of CPS. Conversely, more procedural tasks appear to have limited the explicit emergence of certain phases, even when collaboration was strong. These findings contribute to mathematics education research by providing empirical evidence on how CPS unfolds in an international COIL context and by highlighting the role of task design in fostering different collaborative processes. A comparison that could be investigated, given that the COIL setting is international, is the responses difference between UNITO and UCD students, but the pattern shown by these two groups is quite similar. Moreover, the present dataset does not allow for a fine-grained comparison, so, internationality is treated as a contextual condition shaping collaboration.

This study has some limitations. First, the occurrence of IAM phases is self-evaluated by students and therefore reflects their perceptions rather than direct observations of interaction, which may explain higher occurrence of some phases. Second, the perception of phase occurrence may vary among members of the same group, as collaborative processes are not always experienced or recognized in the same way by all participants. We plan to further develop the analysis to investigate how the international context contributes to CPS in both tasks.

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