THE INTERPLAY BETWEEN MATHEMATICS TEACHER'S IDENTITY AND THEIR TEACHING

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This research report examines differences in the classroom practices of two high school teachers using an identity perspective. Drawing on commognitive theory, the author qualitatively analyzes teaching practices in a Japanese high school, focusing on how teachers' identities and their interpretations of student identities shape classroom interactions. The findings suggest that differences between the two classes are influenced by how each teacher perceives their role and their students. An identity perspective is helpful for interpreting classroom practice, and teachers need to be aware of how their own identities shape how they perceive and respond to students.

INTRODUCTION

Since 2000, there has been increasing attention in the construct of identity in mathematics education (e.g., Graven & Heyd-Metzuyanim, 2019). One reason for this is that mathematical learning is not universal to all people but is influenced by social and cultural backgrounds and individual experiences (Sfard, 2008). Previous studies studying identity, however, have been noted for issues such as ambiguous definitions of identity and a lack of focus on mathematical content (e.g., Graven & Heyd-Metzuyanim, 2019).

This research report addresses to this gap by employing commognitive theory (Sfard, 2008), which posits not only thinking as communicating, but also provides a definition of identity. Numerous studies on identity using the commognitive theory have been conducted including: how teachers' perceptions of student's identities influence the teaching and learning processes (Heyd-Metzuyanim, 2013), and changes in the identities of teachers (Molfino & Ochoviet, 2019). However, previous research has not examined how teachers influence their class practices through their own identities. Do teachers' multiple identities influence one another and affect their educational activities? To clarify this question, this RR investigates whether class practices differ depending on the teacher since each teacher possesses distinct identities. This paper qualitatively analyses the transcript of both classroom interactions and interviews with two teachers in relation to their mathematics teaching.

THEORETICAL FRAMEWORK

Identity

Commognitive theory is employed in this paper for two reasons. First, commognitive theory offers the opportunity to analyse teachers' utterances, as it posits that thinking

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manifests as utterances. Second, in commognitive theory, "identity" is defined "as collections of stories about persons or, more specifically, as those narratives about individuals that are reifying, endorsable and significant" (Sfard & Prusak, 2005, p.16).

Sfard & Prusak (2005) used subscript notation to distinguish the identified person (A), author (B), and recipient (C), demonstrating multiple identities. For example, when an author (B) speaks about A to A themselves, it is ${}_{B}A_{A}$. In commognitive theory, who speaks an identifying narrative and to who it is addressed is crucial, as the same narrative about an individual can be interpreted differently depending on the audience. For example, when student A tells mathematics teacher B, "I am a student who likes mathematics," the context differs from telling the same story to student C, who struggles with mathematics. While teacher B may view student A favourably, student C may not and could become envious. These differing interpretations can affect collaboration between students in mathematics learning.

Mathematizing and subjectifying

In commognitive theory, learning and teaching mathematics emerge from the interplay between mathematizing and subjectifying (Sfard, 2008). Mathematizing refers to talking about mathematical objects (e.g., "2 + 3 = 5"), while subjectifying refers to talk about the people involved in learning. Simply put, subjectifying makes people the subject, as in statements such as "I don't understand this method" or "He nodded at a student" (Heyd-Metzuyanim, 2013). Subjectification involves not only people's actions but also their identities, understood as their properties (Sfard, 2008).

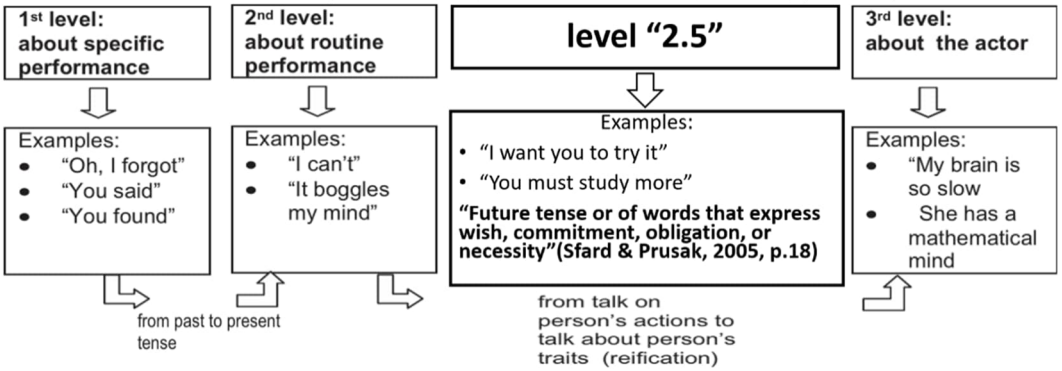
Heyd-Metzuyanim and Sfard (2012) introduced a classification of direct subjectifying utterances with three levels. The first level uses past tense and refers to specific actions (e.g., "She explained," "I said"). The second level uses present tense and describes frequent or typical performance (e.g., "I cannot do this," "She doesn't know how to multiply"). The third level includes forms of 'be' and 'have' and attributes inherent properties to a person (e.g., "He is mathematically gifted," "I am smart"), where identity appears most explicitly. However, lower-level subjectifying utterances can accumulate into a coherent identifying narrative when they are recurring and consistent (ibid., p. 131). For example, repeated statements such as "I can't solve this problem" may lead to an identifying narrative like "She is someone who does not think she can do mathematics."

Many utterances in class settings cannot be classified within the existing three levels, necessitating the establishment of a level positioned between 2nd and 3rd levels. For example, utterances employed by teachers during classes often strongly suggest teacher properties, even when not explicit identity expressions using the verbs 'be' or 'have'. These cannot be adequately addressed by the existing classification.

Kawamura (2025) expanded this classification by adding level 2.5, based on 'designated identities' (Fig. 1; next page). This is "stories believed to have the potential to become a part of one's actual identity" (Sfard & Prusak, 2005, p.18). Designated

identities can be recognized through the use of future tense; should, ought, have to, must, want, can, and cannot, and so on (ibid). For example, in sentences like “I want to be a nurse”. Designated identities are narratives directed toward different narrators and different audiences, constantly influencing one another and becoming incorporated into narratives about us and others. Level 2.5 includes utterances concerning actions the speaker requests of others, which Sfard & Prusak (2005) do not claim.

Fig 1: An Expanded classification of direct subjectifying utterances (The author modified Heyd-Metzuyanim & Sfard, 2012, p.130)



In Heyd-Metzuyanim and Sfard’s (2012) framework, a theoretical gap remains between action-oriented utterances (2nd level) and explicit identity statements (3rd level), as it cannot fully account for identity expressed through actions. Level 2.5, addresses this issue as well since it captures identity conveyed implicitly through action-focused utterances that do not use the verb ‘be’ or ‘have.’ For example, a teacher may not say “I am a mathematics teacher,” yet statements like “I want you to engage in trial and error” implicitly express their identity. Such utterances resemble 2nd level in prompting action but also align with 3rd level in expressing identities, while fitting neither category completely. Level 2.5 provides a theoretical space for these intermediate cases, enabling more nuanced interpretations of teachers’ utterances. Using the expanded classification described above, the research question asks how teachers’ identities influenced classroom practices.

METHOD

After nine years of compulsory education, nearly all students in Japan enter high school. High schools are categorised into academic, vocational, and other types. Classroom is organized with attention to gender ratio, electives, and relationships. Each classroom has a teacher who handles instruction, guidance and career counselling.

The teacher collaborators were teacher Ai and teacher Br, both with 11 years of experience, who taught a class (40 students) of first-year students (aged 15-16) at a Japanese public high school. Ai and Br were also homeroom teachers for the students in their mathematics classes. The mathematics content for this study was geometric measurement. The analysis focused on a scene where the class addressed the problem: “Let $0° < \theta < 180°$. $4\cos\theta + 2\sin\theta = \sqrt{2}$, find the value of $\tan\theta$”.

The author conducted semi-structured interviews with the two teachers to assess their identities. For example, questions such as “What kind of class do you think is a good class?” were asked, and follow-up questions were added based on their responses. The data of this paper also consisted of classroom recordings made from the back of the room focusing on teachers and students’ interactions. The transcripts of the interviews conducted with teachers were analysed using Fig. 1. Utterances employing future tense or words expressing wish, commitment, obligation, or necessity were classified as “2.5 level” utterances, while those expressing state using verbs like ‘be’ or ‘have’ been classified as 3rd level utterances.

DATA AND ANALYSIS

In the case of teacher Ai

Table 1 shows an overview of teacher Ai’s class. They are numbered for organization.

Table 1: Overview of teacher Ai’s class

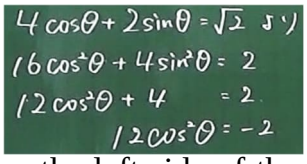
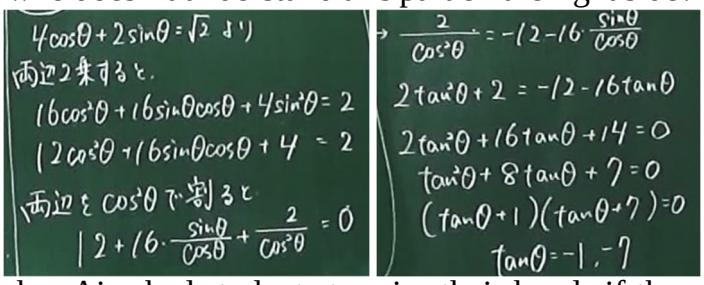
		Distinctive direct utterances and board writing
1	Ai1:	First, share your answers with those around you or recall what the problem was.
2	Ai2:	Quite a few people gave this kind of answer, so please discuss with those around you what you think about this answer. 
3	Ai3:	So, if you square this the same way, will it work? Try it yourself.
4	Ai4:	Is there anyone who doesn’t understand this part on the left side of the arrow? Is there anyone who doesn’t understand this part on the right side?
5	Ai5:	Following this line of reasoning, I got tanθ values of -1 and -7. Are both of these correct answers? 
6	Ai6:	(In the classroom, Teacher Ai asked students to raise their hands if they thought, “both are correct” and “-1 is correct.” Since some students raised their hands for “-1 is correct,” Teacher Ai had one student explain why tanθ = -1 is incorrect)
7	Ai7:	Why did this happen? Because when you square both sides, you actually need to check whether squaring is really okay.

Table 2 (next page) shows Ai’s utterances at 2.5 and 3rd level is underlined italics and identities. Teacher Ai addressed an incorrect solution by comparing an incorrect expansion for $(a+b)^2$ with a correct expansion of another example. In Ai3, Ai said “Try it yourself” suggesting students try, no matter whether it was right or wrong. Afterwards, teacher Ai proceeded with the incorrect approach and, upon arriving at the answer, said an utterance, “Are both of these correct answers?” (Ai5). Noting that tanθ

= -1 was unsuitable, teacher Ai herself revealed in Ai7 that the method of squaring both sides of the equation was not a correct method. Teacher Ai's class reflected Ai's identity, as seen in Ai's utterance "…always thinking about whether something can be gained even from incorrect answers" (Ai11), where Ai clearly wanted to develop the class using students' incorrect solutions. Teacher Ai addressed solutions that incorrectly applied the expansion formula for $(a+b)^2$. This appears to be accommodation for students struggling with mathematics, as indicated in utterance, "…struggle with mathematics" (Ai8). Furthermore, by outlining the solution process as in utterance, "So if you square this the same way…" (Ai3), teacher Ai seemed to encourage students like those in "…copy my answers…" (Ai9) to proceed with the solution themselves.

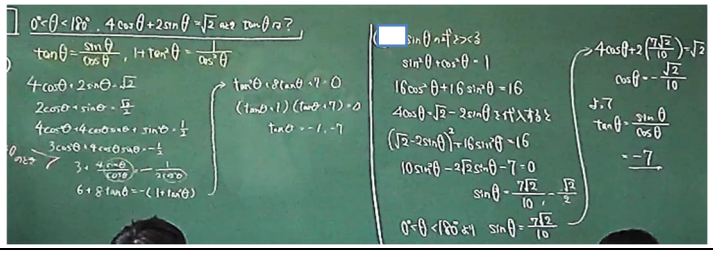
Table 2: Teacher Ai's utterances to the interview and identities

Teacher Ai's utterances in interview			Ai's identities
8	Ai8:	Somehow, there really <u>*are*</u> a lot of students who struggle with mathematics...	Utterances made by teacher Ai concerning students' identity ($_{Ai}S_R$) R: Researcher
9	Ai9:	There <u>*are*</u> many students who copy my answers without doing the calculations themselves.	
10	Ai10:	(I) <u>*want to*</u> let students express their own thoughts as much as possible...	Utterances made by teacher Ai concerning Ai's own identity ($_{Ai}Ai_R$) R: Researcher
11	Ai11:	(I) still <u>*want to*</u> collect the students' written answers, and (I'm) always thinking about whether something can be gained even from incorrect answers.	
12	Ai12:	I also <u>*want to*</u> do activities that explore how to think if you didn't know the proof...	

In the case of teacher Br

Table 3 shows an overview of teacher Br's class. They are numbered for organization.

Table 3: Overview of teacher Br's class

Distinctive direct utterances and board writing		
13	Br1:	As a general approach, what's your goal in transforming the equation? Don't just randomly change things and hope it works out—talk to the person next to you about how you should be thinking through this. 

14	Br2:	(Teacher Br moved between desks while guiding students) Hey, some of you have two answers written down, while others only have one. (Teacher Br had two students write their solutions on the board and confirmed that both solution processes contained no errors)
15	Br3:	The solution Coh (pseudonym) came up with had only one answer, while the solution Dan (pseudonym) came up with had two answers. Why is that?
16	Br4:	(Because the students couldn't explain a proper reason) The point is whether -1 can be the answer. Discuss it with your neighbours. (After having them discuss whether -1 could be the answer, teacher Br had the students explain that $\tan\theta = -1$ was not a valid solution)
17	Br5:	Where in this equation's process could the -1 have slipped in? (Teacher Br then set up a discussion)
18	Br6:	(Since the students couldn't come up with the idea that "both sides were squared) When you checked the result earlier, it came out as $-\sqrt{2}$ instead of $+\sqrt{2}$, right? What if you thought about that? You'd get ± 1. That's what I'm concerned about. (And teacher Br then set up further opportunity for discussion)

Table 4 shows Br's utterances at 2.5 and 3rd level is underlined italics and identities.

Table 4: Teacher Br's utterances to the interview and identities

Teacher Br's utterances in interview			Br's identities
19	Br7:	Many students in my classroom <u>*are*</u> the type who just memorize formulas, know the results, but don't understand why they get those results.	Utterances made by teacher Br concerning students' identity ($_{Br}S_R$) R: Researcher
20	Br8:	What I focused on were questions that make students think. (Omitted) I <u>*want to*</u> create questions that students can't immediately answer when asked but instead need to think about for a moment.	Utterances made by teacher Br concerning Br's own identity ($_{Br}Br_R$) R: Researcher
21	Br9:	I <u>*want to*</u> create classes that make students feel the necessity and think for themselves as much as possible.	

Unlike teacher Ai, who addressed students' incorrect solutions, teacher Br simultaneously addressed two student solutions: one that squared both sides of the equation, and another that combined it with $\sin^2\theta + \cos^2\theta = 1$. Then, Br said "The solution Coh came up with had only one answer, while the solution Dan came up with had two answers. Why is that?" (Br3) and had students consider why the solutions differ. Since students could not explain the reason, teacher Br broke the question down into parts, as in "…whether -1 can be the answer" (Br4), "…Where in this equation's process could the -1 have slipped in?" (Br5), and "What if you thought about that?..." (Br6), and had students discuss them. Teacher Br's class practice progressed through

teacher-led questioning, revealing Br's identity as someone who wants to develop classes based on questions that make students think, as expressed in utterance "I want to create questions…" (Br8). Teachers Ai and Br handled incorrect solutions similarly in some respects, however, their approaches differed: Ai gradually addressed them to leverage the approach behind the incorrect answers, while Br addressed them all at once to engage students in activities exploring why the answers differed.

Teacher Br selected two students' answers: one had two values for tanθ, the other had one. These two answers prompted students to think, "Why are the answers different? I want to consider this", suggesting the influence of the awareness described in utterance, "…classes that make students feel the necessity and think for themselves…" (Br9). Teacher Br appeared to have focused the class on the students mentioned in utterance "…don't understand why they get those results" (Br7). While teacher Ai proceeded with the class assuming students struggled with mathematics, teacher Br proceeded assuming all students could solve the problems.

DISCUSSION

Although both teachers covered the same topic, their class approaches differed. Teacher Ai's own identity was articulated in utterances Ai11 and Ai12: "(I) still *want to* collect the students' written answers" and "I also *want to* do activities that explore how to think if you didn't know the proof...". Teacher Ai's identity can be seen as focusing on students who struggle with mathematics or who do not attempt to solve problems themselves because it valued "making mistakes" and "trying even when unsure." In teacher Ai's class, rather than simply dismissing incorrect answers, the approach consistently treated mistakes as learning opportunities by referencing the underlying ideas or reflecting on the solution process. Through such classes, the message that "it is important to try without fear of making mistakes" can potentially be conveyed to students.

Teacher Br perceived the students in the classroom, in utterance Br7, through an identity characterized by "just memorize formulas, know the results, but don't understand why they get those results". Teacher Br's own identity was articulated in utterances Br8 and Br9: "What I focused on were questions that make students think" and "I *want to* create classes that make students feel the necessity and think for themselves". Teacher Br prioritised "questions" and "student needs" in Br's identity, Br focused on students who do not emphasize why the formula holds. Teacher Ai, teacher Br's perception of student identities and teacher's own identities exhibited an interplay, influencing the class's progress. Throughout this class, teacher Br consistently prioritized questions arising from the necessity to think, rather than merely post-hoc judgments. Through such teaching, the message that "understanding why one can think that way is more important than merely deriving the answer to a problem" can potentially be conveyed to students.

The differences between the two teachers' classes can be interpreted as being influenced by the differing their own identities and their differing interpretations of the identities relating to students. Applying level 2.5 aids in the interpretation of interactions between identities. It becomes apparent that lessons are shaped not merely by the teacher's personal preferences, but by how each teacher interprets students' identities and what they treat as mathematically valuable. To interpret classes, it may be necessary to interpret how goals, content, and teaching methods interplay with the teacher's identities.

By focusing on teachers' identities, and demonstrating the interplay among class, students' identities told by teacher, and teachers' own identities, this study adds a new perspective to existing research identity work in commognitive theory. The analysis progressed beyond merely describing differences between classes to enabling an interpretation of those differences. Specifically, it revealed the interactions between identities that directly shaped teachers' teaching decisions.

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