HOW DIFFERENCES BECOME GENERATIVE: DIALOGICAL LEARNING BETWEEN MATHEMATICIANS AND TEACHERS

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Discussions between mathematicians and secondary mathematics teachers are important for addressing shared concerns about mathematics teaching, yet productive dialogue across these professional boundaries is challenging. This study examines how mathematicians and secondary mathematics teachers engage in dialogical learning during a joint discussion of how students experience a university preparatory mathematics course. Drawing on the literature on boundary crossing, the analysis focuses on dialogical learning mechanisms to explore how differences between perspectives are articulated and engaged with in interaction. The findings show how processes of identification and reflection can support the re-framing of the problem under discussion, contributing to the emergence of a shared problem space.

INTRODUCTION

The field of mathematics education constitutes a meeting point between diverse professional communities, including education researchers, disciplinary researchers, teachers at various levels, teacher educators, and policymakers. While collaboration across these communities is widely acknowledged as important, research has shown that cross-community communication is often undermined by struggles over authority and power, as well as by epistemological and sociocultural differences (Fried, 2014). Over the years, many scholars and educators have emphasized the importance of communication and collaboration across professional communities in education (e.g., Corriveau, 2023; Pinto & Cooper, 2023). These challenges are particularly evident in the relationship between secondary mathematics teachers and mathematicians (Lai & Baldinger, 2021). Although both communities have a decisive influence on educational practice, they have relatively few opportunities for sustained interaction (Goos & Bennison, 2018). The importance of dialogue between these communities is further highlighted at the boundary between secondary school and university, in light of persistent challenges such as high dropout rates in the first years of university studies (Gueudet, 2023) and ongoing concerns regarding the relevance of university mathematics courses for prospective teachers (Wasserman et al., 2023). Nevertheless, collaborations explicitly aimed at fostering dialogue and integrating pedagogical and disciplinary expertise remain limited (Pinto & Cooper, 2023).

Previous studies of mathematician and SM teacher interactions suggest that joint discussions of teaching do not automatically lead to shared understandings or mutual learning. In earlier work, I analyzed a discussion in which teachers and mathematicians interpreted the same instructional situation in different ways, often responding

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primarily to members of their own community. This interaction included forms of "parallel" talk, in which participants addressed related issues from distinct professional perspectives without fully engaging with one another's reasoning (Goor et al., 2023). Building on this work, the present study shifts the analytic focus from identifying realized or unrealized opportunities for dialogical learning to examining how dialogical learning unfolds through interaction over the course of a discussion.

This study addresses this gap by examining a discussion from the fourth year of a long-term co-learning partnership involving secondary mathematics teachers and mathematicians. The timing of the discussion is significant, as it took place within an ongoing collaborative context rather than as an initial encounter between the two communities. While the partnership has primarily focused on issues of secondary mathematics teaching and learning, the discussion examined in this paper centered explicitly on the transition from secondary to university mathematics.

This shift in focus followed teachers' visits to a university preparatory mathematics course and their subsequent sharing of experiential reflections on students' difficulties during the transition. As a result, the object of the discussion moved from secondary classroom practice to students' experiences at the transition from secondary to university. inviting mathematicians to engage with questions raised from teachers' perspectives. Examining the discussion that emerged in this context offers a valuable opportunity to investigate how dialogical learning unfolds when participants engage with an issue that concerns both communities in different ways. Drawing on the literature on boundary crossing (Akkerman & Bakker, 2011), the study investigates how such dialogical learning mechanisms occur in interaction.

THEORETICAL FRAMEWORK

This study focuses on discussions between two communities: mathematicians and secondary mathematics (SM) teachers. From a boundary crossing perspective, such cross-community encounters not only introduce discontinuities but also open up opportunities for dialogical learning (Akkerman & Bakker, 2011). The challenge in such encounters is not to eliminate differences between communities, but to engage with them productively. While it is relatively easy to speculate on how differences and tensions may hinder mathematicians and teachers from sharing and negotiating ideas, it is far less clear what dialogical learning looks like in interaction, and what may encourage and support it. To conceptualize learning in such interactions, this study draws on the literature on boundary crossing, and in particular on the dialogical learning mechanisms proposed by Akkerman and Bakker (2011): identification, coordination, reflection, and transformation. These mechanisms describe different ways in which participants from two communities may engage with differences during interaction. Identification involves making differences between perspectives, practices, or forms of expertise explicit without attempting to resolve them. Coordination refers to efforts to establish and sustain interaction across boundaries by enabling continuity without necessarily transforming practices. It involves managing diversity, for

example, by translating terms that have different meaning in each context. Reflection entails examining one's own assumptions or practices in light of others' perspectives, often through perspective making and perspective taking. Transformation involves confrontation with differences that may give rise to the recognition of a shared problem space. While transformation is often described in the literature as involving sustained change in practices, hybridization, and crystallization, the present study focuses on its interactional manifestation - namely, how engagement with difference within a single discussion may reshape how the problem under discussion is understood, without presupposing longer-term changes in practice.

From this perspective, the productivity of cross-community discussions does not depend on achieving agreement or convergence, but on how differences are articulated, examined, and worked with over the course of interaction. The dialogical learning mechanisms thus provide a way to conceptualize productive dialogue as a process in which differences can function as resources for dialogical learning rather than as obstacles to interaction. Accordingly, the study addresses the following research question: In what ways do mathematicians and secondary mathematics teachers engage in dialogical learning when jointly discuss the transition to university mathematics?

METHODOLOGY

Data collection: The setting

Data were collected in a collaborative setting designed to support sustained dialogue between secondary mathematics (SM) teachers and mathematicians. The setting consisted of monthly meetings held over four years, focusing on issues of mathematics teaching and learning. The design of the meetings drew on boundary-crossing literature (Akkerman & Bakker, 2011) and aimed to create opportunities for participants from two different communities to engage with one another around issues they consider relevant to mathematics, its teaching, and learning. Within this context, participants explored concrete teaching-related issues using artifacts such as classroom episodes, assessment tasks, and materials from university preparatory courses. Discussions were facilitated by mathematics education researchers, who encouraged participation from all members and supported the articulation of differing perspectives.

The present paper focuses on a single discussion that took place during a meeting devoted to teachers' reflections following a visit to a university preparatory mathematics course. Teachers' impressions from the visit were first collected by one of the facilitators (F1, the author of this paper) and then presented to the broader group, which included six teachers (T1-T6) and three (research) mathematicians (M1-M3), as a basis for joint reflection. The discussion centered on students' experiences in the preparatory course, particularly their emotional engagement, sense of confidence, and participation in mathematical tasks.

Data analysis

The discussion analysed in this study was video-recorded and transcribed in full. The analysis focused on how participants from the two communities responded to one another while discussing the teachers' impressions from visiting a preparatory mathematics course at the university. The discussion was segmented into episodes based on shifts in topic and focus, which served as the units of analysis. Analysis was guided by the four dialogical learning mechanisms proposed by Akkerman and Bakker (2011): identification, coordination, reflection, and transformation. The mechanisms were used as analytic categories to describe different ways in which participants engaged with differences during the discussion, rather than as stages of a linear process, and more than one mechanism could be identified within the same episode.

Identification was coded when participants recognized or articulated differences between perspectives, practices, or forms of expertise, for example in relation to what counts as a convincing mathematical argument or how the goals of instruction are understood. Coordination referred to talk that allowed the discussion to continue across differences without addressing them directly; in this study, this mechanism played a more limited role, as the analytic focus was on moments in which differences were taken up for further examination rather than on maintaining the flow of discussion. Reflection was identified when participants examined their own assumptions or interpretations in relation to how others understood the issue under discussion, thereby reconsidering aspects of their own position without necessarily resolving differences. Transformation was identified when core differences were addressed rather than bypassed, leading participants to articulate a shared concern or to shift how the issue under discussion was understood. While transformation in literature is often associated with sustained changes in practice, in this study it was operationalized at the level of the discussion and did not presuppose longer-term outcomes. Across the analysis, the four mechanisms were distinguished by whether differences were made explicit (identification), left unexamined while enabling continuation of the discussion (coordination), examined in relation to alternative interpretations (reflection), or taken up in ways that altered how the problem was framed (transformation).

FINDINGS

The analysis examines how the discussion developed as mathematicians and secondary mathematics teachers talked together about students’ experiences in the transition from school to university mathematics. Rather than unfolding in clearly separated stages, different kinds of dialogical learning mechanisms appeared throughout the discussion and often overlapped. In particular, the discussion involved (a) making differences between school and university teaching cultures explicit (identification), (b) examining the considerations and assumptions underlying these differences (reflection), and (c) moving toward redefining the challenges of the transition and reconsidering possible ways of supporting different students (transformation).

Identification: making differences between school and university teaching explicit

The first part of the discussion brought to the foreground differences between school and university teaching cultures. In this phase, participants primarily articulated what is taken for granted in each context, thereby exposing contrasting expectations about students, instruction, and learning. Mathematicians described how students are expected to encounter unfamiliar tasks and to struggle with them as part of university mathematics, while teachers emphasized students' emotional experiences and the importance of a safe and familiar learning environment, as illustrated herein:

> M1:"The goal of the course is to expose them to things they have not seen before… there is no point in spoon-feeding them, because this is how university mathematics works."
>
> T1 "If this [the given task] were in high school, with a familiar teacher and a safe space, they would not be so frightened."

Moreover, the facilitator (F1) brought into the discussion a quote from the preparatory course that one of the teachers had reflected on, explicitly emphasizing how different it is from what students typically encounter in school. In response to students' complaints that the task was "harder than the tasks from the lectures," the tutor (in the preparatory course) stated that "the goal is that you don't succeed in solving the assignments by yourselves; we solve them together," which was framed as an antithesis to school practices, where teachers usually select tasks that students are expected to be able to solve independently. This intervention placed the contrast between the two instructional cultures at the center of the discussion.

Importantly, identification did not align neatly along community lines. Different mathematicians articulated different understandings of students' difficulties, with one emphasizing overconfidence and another emphasizing anxiety resulting from students having "heard frightening things about the course." These differences indicated that what counts as the central problem is not yet stabilized, even within the university perspective. At this stage, the discussion remained focused on making differences explicit rather than on evaluating or resolving them.

Reflection: examining assumptions and anticipated student responses

Alongside the articulation of differences, participants began to examine the assumptions and considerations underlying them. This reflective work took the form of explanations and justifications - why particular instructional decisions are made and how they are expected to affect students. A key focus of this phase was how students were likely to interpret instructional messages during the transition to university mathematics. One mathematician proposed expectation-setting as a rational way to normalize difficulty:

> M2: No, but it would be different if we told them, instead of telling them: "you are not expected to solve everything" – a reasonable situation is one in which there are some questions that you solve fairly quickly, there are some questions that take an hour or two,

> and so on, and maybe in the evening in the shower, and there are some questions that maybe the next day, … I mean, isn't this really a matter of expectation-setting?

Teachers responded by questioning not the intention behind this message, but how it is likely to be heard by students:

> T4: No. No. They will not believe you. Because there is always one person, that they know, who succeeds at all the questions. And what they will hear you saying is: I know that most of you are stupid […] so I am telling you that it's okay that you don't succeed at everything.

A second mathematician then situates the same message within existing university practices:

> M1: This sentence you quote is, in my opinion, a paraphrase of the sentence we tell all our student instructors to say in the tutoring sessions in the first year […] one of the problems is that we design one worksheet for everyone, so there are always more questions than the students can solve. So, we tell the instructors to tell them: don't stress if you don't finish it all, you are not expected to finish it all.

Together, these exchanges illustrate reflection as a process of examining the same instructional idea through different assumptions about students, learning, and communication. The discussion did not move toward agreement; instead, it exposed a persistent gap between how instructors intended messages to function and how teachers anticipated students would interpret them. Reflection here operated by keeping this tension visible, allowing participants to explore the implications of their assumptions without resolving them.

Transformation: redefining the problem of transition

Transformation became visible when the discussion moved beyond examining differences and their implications toward redefining the nature of the transition to university mathematics itself. At this point, one mathematician framed the challenge in terms of managing students' emotional trajectories during the transition, asking how to support students in moving from an initial sense of capability to an acceptance of sustained struggle as part of university mathematics:

> M1: The question is how to contain this. It is important that students feel capable, but most of them are going to struggle a great deal in their first year. So the question is whether we really have three weeks to move them from a state of "okay, I am capable, therefore I can" to a state of "I am going to struggle and I can still manage."

This framing still treated students' experiences as largely uniform. The discussion shifted when a teacher reframed earlier contradictions by explicitly introducing a distinction between different student populations with different needs at the transition:

> T4: But it is more complicated than that. One of the main reasons for the gap is that you and [M2] said contradictory things - and both of them are true, because there are two populations. There are students who need their sense of capability to be nurtured, and there are others - who are a very, very small group in high school but are proportionally more present in the first year - who arrive with an over-sense of capability.

Through this contribution, earlier disagreements were reorganized as referring to heterogeneous student experiences rather than to a single difficulty requiring a single response. The transition was thus redefined as a complex challenge involving different student trajectories.

A further instance of transformation occurred when a different mathematician reframed the role of rigor and uncertainty in the first university course, drawing an explicit comparison to the way examinations are prepared:

> M2: One thing I think we actually do well in this context is that when we prepare an exam, we go over it again and again with great care. But we don't apply the same level of meticulousness to the tasks in the first course. I think we should invest the same degree of careful preparation there, in order to reduce the uncertainty students experience.

Here, the challenge of the transition was no longer framed as a matter of students needing to cope better with uncertainty, nor as a call to lower disciplinary standards. Instead, rigor and precision - central values of university mathematics were repositioned in time: the responsibility for managing uncertainty was shifted toward instructional design in the early stages of university study. Taken together, these moments illustrate transformation as a change in how the problem of transition was defined. Rather than converging on specific instructional solutions, participants arrived at a shared understanding of what is at stake: supporting a transition marked by heterogeneous student experiences and by the need to deliberately structure when and how uncertainty is introduced.

DISCUSSION

The findings show how dialogical learning can take shape when mathematicians and secondary mathematics teachers engage with differences at the boundary between their practices. Rather than being resolved or eliminated, these differences played a central role in shaping the discussion. Through their explicit articulation and examination, participants moved from describing students' difficulties to rethinking what the transition to university mathematics entails for various students.

A key outcome of the discussion was the reshaping of a shared problem space. As differences between school and university teaching cultures were made explicit and examined, participants began to question simplified accounts of students' experiences. The recognition that students encounter the transition in different ways, reflecting distinct student populations with different needs, allowed earlier tensions to be reinterpreted rather than resolved. At the same time, the discussion expanded the locus of responsibility for the transition by foregrounding the role of instructional design at the university level, particularly in relation to how expectations, rigor, and uncertainty are structured in the first course. Importantly, transformation did not take the form of agreement on specific instructional solutions. Instead, it emerged through a change in how the problem was understood. From a boundary crossing perspective, these findings suggest that dialogical learning may involve partial and localized transformations that reshape problem definitions without necessarily leading to

immediate changes in practice. Productive boundary crossing, in this sense, depends less on consensus and more on sustained engagement with differences as resources for joint inquiry.

Future research could further explore the conditions that support shifts between different dialogical learning mechanisms, including the possible role of brokering and facilitation in sustaining productive engagement with differences.

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