INTENDED AND ENACTED REASONING IN PROSPECTIVE MATHEMATICS TEACHERS' TASK DESIGNS

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This study examines reasoning opportunities in tasks designed by prospective mathematics teachers and their conceptions of how they promote reasoning and justification. Twenty-eight prospective teachers designed curriculum-based tasks and submitted self-reports. Tasks were analysed using the Integrated Framework for Analyzing Ways of Reasoning (Sevinç et al., 2022), and self-reports were thematically analysed. Results indicate that most tasks primarily involved empirical reasoning, whereas teachers frequently described their tasks as promoting explanation and justification. Comparison of intended and enacted reasoning revealed diverse alignments, highlighting challenges in translating intentions about reasoning into task design, and have implications for teacher education.

INTRODUCTION

Developing students' reasoning and justification is a central goal of mathematics education. Mathematical reasoning allows learners to establish connections, articulate relationships, and justify conclusions, rather than simply recalling procedural steps (Mata-Pareira & da Ponte, 2017). However, designing classroom tasks that foster reasoning is a challenge for teachers and prospective teachers (Henningsen & Stein, 1997; Steen, 1999).

The quality of the tasks that the teacher brings to the classroom and the implementation of these tasks are important in developing students' reasoning skills and play an important role in conceptual learning of mathematics (Stein et al., 1996). Therefore, teachers and prospective teachers need to know which tasks and how they support the reasoning process. For this reason, teacher education programs should focus on developing tasks that have high cognitive demand levels and include reasoning and proof skills, and to analyse existing tasks and revise them when needed (Melhuish et al., 2018).

In this study, 28 pre-service teachers were asked to develop geometry tasks that promote reasoning and justification. They were also expected to explain how these tasks encouraged reasoning. The tasks were analysed according to the Integrated Framework for Analyzing Ways of Reasoning (Sevinc et al., 2022), and their statements about how they promoted reasoning were analysed with the emerging themes. The research questions of the study are formulated as:

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- What ways of reasoning are embedded in the instructional tasks formulated by preservice mathematics teachers?
- How do preservice teachers conceptualize the ways in which their tasks promote reasoning and justification?
- To what extent are the actual and intended ways of reasoning in the tasks of preservice teachers aligned?

LITERATURE REVIEW

Mathematical reasoning can be conceptualized as the systematic construction of justified inferences through deductive, inductive, and abductive modes of reasoning. It encompasses the formulation of mathematical questions, the development of problem-solving strategies, making generalizations, formulating and testing conjectures, and justification of claims (Mueller et al., 2014).

Mathematical tasks strongly shape students' cognitive engagement and learning outcomes by influencing both the content learned and the ways students think about and make sense of mathematics. Cognitively demanding tasks promote explanation, sense-making, and conceptual understanding, and improve mathematical reasoning (Stein et al., 1996).

Research has indicated that the tasks employed by teachers are primarily composed of tasks that are adopted in their original form or with minor adaptations as deemed necessary (Jones & Pepin, 2016). Steen (1999) posited that teachers face challenges in the development and implementation of tasks that foster mathematical reasoning. It is imperative to assess the ability of prospective teachers to design tasks that facilitate reasoning and to provide support in areas of struggle. To determine how designed tasks support reasoning, the "Integrated framework for analysing ways of reasoning" developed by Sevinç et al. (2022) can be utilized. This framework was primarily developed to analyse the tasks presented in mathematics textbooks and consists of 7 categories (see Table 1).

Table 1: Integrated framework for analysing ways of reasoning (Sevinç et al., 2022)

Ways of reasoning	Characteristics
1. Appeal to authority	No explanation or reasoning
2. Simple (1-step) deductive reasoning	A single deduction from one or more premises
3. Mathematising	The explanation/ justification of a word problem to a strictly mathematical form
4. Reasoning by analogy	Making a conjecture based on similarities between two cases.

5. Reasoning with empirical arguments/ specific cases a. Making claims and generalizing b. Justification of claim	Reasoning begins with specific cases and produces a generalization from these cases.
6. Developing conclusions/ justifying/ refuting through deductive reasoning a. Generic example b. Counterexample c. Systematic enumeration d. Other	Conclusions are derived from known information (premises) based on formal logic rules.
7. Other	Abductive reasoning

METHODOLOGY

Participants and Context

28 preservice mathematics teachers participated in the study. The participants were the junior students who enrolled in a Teaching Geometry course in a public university. Three participants were studying to become high school teachers, while the rest were receiving training to become middle school mathematics teachers.

This study was conducted as part of a coursework in the Teaching Geometry course. The aim of this course is to provide prospective teachers with a foundation in both content knowledge and pedagogical content knowledge based on Euclidean geometry. Throughout the semester, the prospective teachers were exposed to comprehensive course content that included proof and reasoning processes and emphasized their implementation at the middle and high school levels.

Data Sources

Two data sources were used in the study: The geometry tasks developed by prospective teachers with predetermined geometry objectives and their self-reports in response to the question “In what ways does your task promote justification and reasoning?”

Data Analysis

The analysis of tasks was realized through the “Integrated framework for analyzing ways of reasoning” by Sevinc et al. (2022). For coding, activity sheets prepared by prospective teachers and details on the implementation of tasks were utilized. The coding was also performed by a second evaluator; the inter-rater agreement was calculated as 75% (21 out of 28 codes for complete agreement). The disagreements were revisited and a consensus was achieved on all codes. A thematic analysis was

conducted to analyse the self-reports of prospective teachers. Keywords and conceptual themes were sought in the responses to the question "In what ways does your task promote justification and reasoning?" Comparisons of intended and enacted reasoning opportunities were realized through matching the categories obtained in the previous analyses and examining the alignments between two categorizations for each task.

RESULTS

The findings of the enacted tasks according to the framework by Sevinç et al. (2022) showed that prospective teachers mostly created tasks involving empirical arguments and specific cases (42.9 %) as the majority of them aimed at making claims and generalizations. Tasks requiring deductive reasoning in multiple steps constituted 39.2 % of all tasks (see Table 2).

Table 2: Ways of reasoning promoted in the tasks

Ways of Reasoning	Sub Dimensions	f	%
Appeal to authority	-	1	3.6
Simple (1-step) deductive reasoning	-	3	10.7
Mathematizing	-	-	-
Reasoning by analogy	-	-	-
Reasoning with empirical arguments/ Specific cases	Making claims and generalizing	11	39.3
	Justification of a claim	1	3.6
	Generic example	7	25
	Counterexample	-	-
Developing conclusions/ justifying/ refuting through deductive reasoning	Systematic enumeration	1	3.6
	Other	3	10.7
Other	Abductive reasoning	1	3.6

The findings for the analysis of intended tasks revealed that the majority of the prospective teachers expected students to explain and justify their results in their tasks (27.8%). This was followed by inclusion of experimentation and measurements in the tasks (22.8%). Expecting students to establish cause-effect relationships, emphasizing mathematical/geometrical rules and principles, fostering social learning, and conjecturing were other prominent ways claimed by the participants (see Table 3).

Table 3: Promotion of reasoning and justification on participants' self-reports

Theme	*f*	%
Asking for explanation and justification	22	27.8
Conjecturing	6	7.6
Empirical evidence	18	22.8
Follow the proof procedure	1	1.3
Generalizations	4	5.1
Logical reasoning	8	10.1
Looking for patterns	4	5.1
Mathematical language	2	2.5
Social interaction	7	8.9
Using mathematical/geometry facts and principles	8	10.1

The alignment of actual and intended ways of promotion of reasoning was examined through comparison of categories obtained through the “Integrated Framework for Ways of Reasoning” and themes appeared in participants’ self-report. The analyses revealed that 14 tasks had perfect alignment, 11 tasks showed partial alignment, and 3 of them had misalignments.

In one of the 14 cases of perfect alignment, the task was described as: “The task consists of two stages. In the first part, after completing a given triangle into a rectangle on GeoGebra, the relationship between the areas of the triangle and the rectangle is discussed. In the second part, students are expected to draw different types of triangles on grid paper and work on completing them into rectangles as in the previous stage. They are then asked to make a conjecture about the area relationship of two geometric shapes and justify whether their conjecture is generalizable to all triangles. Finally, students are expected to arrive at a general rule.” This task was coded as “Developing conclusions/ justifying/ refuting through deductive reasoning/other” since several steps requiring deductive reasoning were employed during the task. The prospective teacher explained how the task supported reasoning in the following way:

> The 1st question engages students in investigation and invites them to look for patterns. The 2nd question encourages them to identify a relationship between the area of a rectangle and triangle. And asks for a conjecture. The 3rd question asks them to test and justify their conjecture. This is important because they will understand why the rule for the area of a triangle works, which will save them from rote memorization of the formula without understanding. The 4th question wants them to express this regularity with a mathematical rule, based on their conjecture. Overall, this task will be an effective learning opportunity

> because it relates to existing knowledge of rectangle area, and eventually, pushes them to explore a rule to find the area of any triangle.

The themes emerged from this explanation were coded as (1) Looking for patterns, (2) Conjecturing, (3) Asking for explanation and justification, and (4) Using mathematical/ geometry facts and principles. It was observed that the coding based on the framework demonstrated a high degree of compatibility with the specified themes.

In the 11 tasks with partial alignment, it was found that prospective teachers either did not explicitly state some features supporting the reasoning required by the task or added certain elements not included in the instructions to the task's reasoning-promoting moves. A task exemplifying this situation is described as follows: "Students were given cylinders and coloured papers. They were asked to cover surfaces of the cylinder with papers and then calculate how much area was covered by the papers. They were given rulers to calculate the surface area of the cylinder and were asked to summarize in a table what the shapes were, the areas of the shapes and how they were calculated. The students were made to think about the expansion of a right circular cylinder and then presented its representation on GeoGebra". This task was coded as "Developing conclusions/justifying/ refuting through deductive reasoning (generic example)." The prospective teacher explained how the task supported reasoning in the following way:

> Students are asked to cover the cylinders in their hands with paper. They work together with their group mates to cover the cylinders and find the area of the cylinder. After finding it, they justify their results by examining and evaluating their results. They explain the strategies they followed while covering the cylinders to their friends and the teacher, considering which geometric shapes they obtained.

The self-report was coded as "Asking for explanation and justification". However, the task does not only require students to explain and justify. Students were expected to generalize from a specific example of cylinder and the cylinder under consideration is more than just a particular example; it embodies the properties that all right cylinders share. For this reason, this task, which requires deductive reasoning, can also be considered a generic example.

In the 3 tasks with misalignment, the statements in the prospective teachers' self-reports did not correspond to the reasoning-promoting opportunities afforded by the tasks. It was observed that prospective teachers either expressed reasoning-supporting moves that were not present in the task or failed to articulate the reasoning opportunities provided by the task. A task representing this situation is described as follows: "The activity of drawing a perpendicular from a point outside the line segment using a compass was considered. 4 steps of marking certain distances with a compass and then connecting the intersection point with the point outside the line segment. Students were only asked to follow the instructions given". The task was coded as "Appeal to authority." The prospective teacher stated in his self-report that:

> The task promotes justification and reasoning by requiring students to explain why a line is perpendicular and how they determined the right angle accurately. Students have to explain their thought processes and the geometric principles applied.

The explanations of the preservice teacher were assigned with 2 codes: (1) Asking for explanation and justification and (2) Using mathematical/ geometry facts and principles. This task requires only the following and execution of the steps provided in the instructions. No statements supporting reasoning were identified. Therefore, the prospective teacher's explanations are not consistent with the task's instructions.

DISCUSSION

The most prevalent code observed in the tasks, "Reasoning with empirical arguments/Specific cases," may be attributable to the content of the geometry learning objectives assigned to prospective teachers. Certain geometry objectives may be more conducive to require deductive reasoning. The majority of the tasks were aligned with middle school geometry objectives, and the prospective teachers, who were expected to design tasks accordingly, may have been hesitant to prepare reasoning- and proof-based tasks due to considerations of students' cognitive capacities. On the other hand, a considerable number of prospective teachers included deductive reasoning and generic examples in the tasks they prepared. This indicates that teacher candidates took into account the emphasis on deductive reasoning and proof-based geometry teaching in the Teaching Geometry course they were enrolled in and developed themselves in terms of pedagogical content knowledge (PCK).

Most prospective teachers stated that their tasks supported reasoning by directly expecting explanations or justifications from students, followed by using evidence from measurements and calculations—likely due to the visual nature of geometry topics and middle school objectives encouraging empirical tasks. Their responses were mostly cognitive, but two notable themes also emerged: social interaction (classroom discussions, peer convincing, group work) and using mathematical language properly. These findings highlight the need for teacher education programs to focus not only on cognitive skills but also on social and technical competencies, which are vital for developing proof skills.

To prepare reasoning-supporting tasks, prospective teachers need both subject matter knowledge and pedagogical content knowledge. This study's tasks required knowledge of geometry as well as reasoning and proof and the relevant pedagogical knowledge. Results indicate that while prospective teachers' geometry knowledge is sufficient, their ability to support reasoning needs development. The dominance of relying on empirical arguments in tasks may reflect either insufficient knowledge and skills or hesitation about implementing such tasks at the middle and high school level. Future studies should examine teacher knowledge and beliefs alongside task development, and having prospective teachers implement their tasks could better reveal their knowledge on promoting reasoning and justification skills. Also, incorporating more

task development and analysis exercises in teacher education programs can enhance the realism and depth of prospective teachers' reflections on how their tasks support reasoning, thereby increasing their knowledge and awareness of tasks' reasoning potential.

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