RELATIONSHIP BETWEEN CONTEXTUALIZATION AND COGNITIVE DEMAND IN GEOMETRY TASKS IN TEXTBOOKS FROM BRAZIL AND NORWAY

Gabriel Alves, Rúbia Amaral, Beatriz Litoldo & Danyal Farsani

UNESP- Brazil, UNESP- Brazil, UFTM- Brazil and NTNU- Norway

Contextualization is often linked to deeper cognitive engagement in mathematics, but evidence is scarce. Against this background, this study addresses the research question: How do different contextual references in geometry tasks relate to the levels of cognitive demand mobilized by these tasks in mathematics textbooks from Brazil and Norway? Using a qualitative Theorem System framework, tasks were classified by context and demand and compared. Findings reveal the dominance of Purely Mathematical contexts and low-demand tasks in both countries. Semi-reality contexts show relatively higher demand but are not predominant. Overall patterns remain consistent across the two collections. The study points out that cognitive demand is more associated with task planning intentions than with the nature of the context itself.

INTRODUCTION

This Research Report (RR) aims to analyze the relationship between contextualization and the levels of cognitive demand mobilized by geometry tasks in mathematics textbooks used in Brazil and Norway, seeking to understand how these two dimensions are articulated.

Textbooks are an important resource to be analyzed, as they establish the parameters for teaching and learning in the classroom, organizing ideas/information and defining the legitimate content to be taught (Fan *et al.*, 2013). In general, these materials present a significant number of tasks, which are selected by teachers for classroom work (Özgeldi & Esen, 2010). Tasks can influence and structure the way the teacher organizes and teaches their classes (Stein *et al.*, 2000). They can be understood as structures that allow for the analysis of the connections between teaching and learning in the classroom, aiming to draw students' attention to a specific mathematical idea, subject, and/or concept. Thus, the analysis of tasks present in mathematics textbooks allows us to glimpse part of what is offered to students in the classroom in these two countries. Among the various dimensions that constitute a task, this study focuses on two specific ones: i) contextualization, understood from the different contextual references mobilized in the tasks, and ii) the levels of cognitive demand, associated with the complexity of the cognitive processes involved in and for the resolution.

Since geometry has been the focus of study for the authors for years, this research focuses on contextualization and cognitive demand when examining how these two

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M.S. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: Research Reports*, pp. 43-50. PME.

dimensions appear in geometry tasks in mathematics textbooks. In this context, the research question of this study is: How do different contextual references in geometry tasks relate to the levels of cognitive demand mobilized by these tasks in mathematics textbooks from Brazil and Norway? This question is pertinent, since literature has questioned the possible implications of contexts associated with everyday life on tasks with high cognitive demand (Van Den Heuvel-Panhuizen, 2005; Jackson *et al.*, 2013). Results presented in this RR are part of a broader study, carried out during an international research internship, whose objective was to comparatively analyze geometry textbooks for secondary education in Brazil and Norway. The excerpt presented here focuses specifically on the analysis of the relationship between contextualization and cognitive demand of geometric tasks.

THEORETICAL FRAMEWORK

Contextualization in tasks

Contextualization in teaching can be understood as a process that involves, on the one hand, presenting a specific element to the student and, on the other, situating this element within disciplinary content, in articulation with conceptual objectives. In Mathematics Education, this concept has been discussed from different theoretical perspectives, resulting in a variety of classifications of the context references mobilized in tasks (Skovsmose, 2001; Dekker & Querelle, 2002; Lana & Carrião, 2015; Litoldo, 2021). Based on an analysis of these contributions, Alves *et al.* (2024) proposed a refinement of the classifications related to different context references, constituting a theoretical framework that underpins the task analysis conducted in this study. In this model, context references are organized into four categories: *Purely Mathematical* (PM), *Artificial* (Ar), *Reality* (Re), and *Semi-reality* (SR), the latter being subdivided into *Reasonable* (SRR) and *Not Reasonable* (SRN).

PM context reference is characterized by the absence of subjective elements, being restricted exclusively to the domain of Mathematics. In such tasks, statements consist only of signs, symbols, numbers, figures, and letters. Re, in turn, depict scenarios explicit derived from real-life situations, or contain information and actual quantifications, or even real images in their statement. SR is characterized by the presence of a synthetic reality, constructed, for example, by the author of a mathematics textbook from elements that are close to daily life, but do not emerge directly from an explicit real context (Alves *et al.*, 2024). This type of context is frequently employed when real situations would be excessively complex for mathematical treatment and are therefore adapted or simplified in order to enable the exploration of mathematical concepts (Dekker & Querelle, 2002). Litoldo (2021) emphasizes the importance of reasonableness within this type of context, distinguishing between SRR and SRN according to the degree of coherence between the task statement and the real situations it intends to represent.

Finally, Ar refer to tasks that mobilize unreal elements or belong to fictional worlds, being characterized by situations that do not correspond to real-world phenomena

(Alves *et al.*, 2024). This type of context demands that students be willing to use their imagination to produce meaning for the task and assume the proposed situations as real in their minds, experiencing them as real for themselves (Dekker & Querelle, 2002).

Cognitive demand in tasks

One way to describe students' cognitive engagement with tasks is through levels of cognitive demand, which refer to the mental processes mobilized by students during task solving, such as the type of reasoning required when solving a task (Jackson *et al*., 2013). In this sense, when a task is presented in curricular materials, such as textbooks, the cognitive demand refers to the types of cognitive processes expected to be mobilized and, most often, revealed by students to solve it. In other words, task designers develop them always assuming certain thought processes that are involved in solving such tasks, generally aligned with the mathematical and pedagogical objectives for which they were proposed (Litoldo, 2021).

Stein *et al.* (2000) classify the cognitive demand of tasks into two levels: *Lower-level demands* and *Higher-level demands*, each of which is further subdivided. Tasks involving *Lower-level demands* include two types. The first, *memorization* (Mem), only requires students to recall and reproduce certain concepts, rules, definitions, and/or formulas. The second type, *procedures without connections* (PwoutC), involves the application of routine and mechanized procedures, with little or no requirement for conceptual understanding. In these tasks, the procedure to be used is typically evident from prior instruction or closely resembles examples previously presented in the textbook.

Tasks requiring Higher-level demands, in contrast, involve making decisions about how to use mathematical knowledge and skills in specific situations in order to interpret and solve the tasks. This level is subdivided into two types: procedures with connections (PwC) and doing mathematics (Domath) The first involves procedures that, although similar to the previous level, are not presented clearly or directly by the task, requiring students to independently establish connections with unstated procedures (Jackson *et al*., 2013; Litoldo, 2021). The second, in turn, requires complex and non-algorithmic thinking, without a predictable or direct solution path, involving justification, investigation, and decision-making by students. In line with this perspective, Sierra (2025) emphasizes that tasks with Higher-level are essential for deeper learning, as they promote stronger connections between mathematical procedures and concepts, in contrast to tasks characterized by Lower-level demands.

METHODOLOGY

This study adopts a qualitative approach and makes use of the Theorem System (Amaral *et al*., 2022), a methodological framework designed for textbooks analysis, structured into three systematic stages that support data production and analysis while ensuring analytical rigor and interpretative coherence. In the *Planning*, the textbook was considered as an object of study in itself, guiding the definition of analytical procedures according to the internal structure and organization of the material. The

Exploration of the material involved the selection of mathematics textbooks, with a specific focus on Geometry, followed by an examination of their structural characteristics. At this stage, data were produced in accordance with the research objectives, considering the tasks proposed in the selected books. In the *Data Processing* stage, a stratified analytical approach was adopted, in which the strata consisted of the tasks themselves. Each task was analyzed according to its context references and levels of cognitive demand, as defined in the theoretical framework[1].

For this study, two textbook collections were analyzed: the Brazilian collection *Diálogos* (Teixeira, 2020a, 2020b) was selected due to its wide national adoption. In Brazil, a national program evaluates and distributes textbooks free of charge to schools, which allowed the identification of the most widely distributed collection; all volumes were available online in PDF format. The Norwegian collection *Matemagisk* (Wallace & Kongsnes, 2021a, 2021b, 2021c) was selected based on availability, as access to textbooks was limited; this collection was the only one found in print at the at the Norwegian University of Science and Technology (NTNU) library where the research was conducted. In terms of data production and organization, the Google Colaboratory environment integrated with Google Cloud through Python scripts, enabling the systematic automating of data entry and organization in Google Sheets. This procedure ensured efficiency in processing the large number of analyzed tasks and facilitated the generation of charts and contingency tables used in subsequent analyses.

RESULTS AND DISCUSSION

Regarding the distribution of the different context references, Table 1 presents the quantification of tasks in both textbook collections. The data reveals a marked predominance of PM contexts in both collections, with relatively few tasks contextualized in Re or SR.

This pattern suggests a limited presence of contextual references that explicitly relate mathematical content to situations beyond the mathematical domain. Although the Brazilian collection exhibits the same general tendency, it shows a more diversified distribution of context references when compared to the Norwegian collection. In the Brazilian case, tasks involving daily life other than purely mathematical ones account for approximately 40% of the total, whereas in the Norwegian collection such contexts represent a less than 18% of the tasks. This dimension may suggest that mathematics is being taught as a science restricted to itself, which may be worrying when we consider the opportunities that students are not having to reflect on their surroundings through it.

Furthermore, it is worth noting that the Norwegian collection does not include tasks classified as SRN, in contrast to the Brazilian collection. This difference suggests that, although the Norwegian collection presents a smaller proportion of tasks situated

[1] Tasks that did not fit into any of the context references or levels of cognitive demand were classified as ‘Not Applicable’ in both analytical dimensions.

outside PM, the SR are constructed in a way that preserves internal coherence and realism, avoiding SR situations that may appear artificial or implausible to students.

Table 1: Distribution of contextualization in the analyzed tasks.

		Diálogos		*Matemagisk*	
Context Reference		*n*	%	*n*	%
Purely Mathematical		566	57.2%	914	82.3%
Artificial		0	0.0%	0	0.0%
Reality		102	10.3%	54	4.9%
Semi-reality	Reasonable	257	26%	138	12.4%
	Not reasonable	7	0.7%	0	0.0%
Not Applicable		57	5.8%	5	0.4%
Total		989	100%	1 111	100%

Considering the distribution of cognitive demand levels, Table 2 presents the classification of tasks according to their levels. The results indicate a predominance of Lower-level demands tasks in both collections. In the *Diálogos* collection, tasks classified as memorization and procedures without connections account for 80.9% (800) of the total, whereas in the *Matemagisk* collection these levels represent 73% (811). This distribution suggests that, in both collections, a considerable proportion of tasks is oriented toward objectives related to the recall of definitions, properties, formulas, and the execution of routine procedures.

In contrast, tasks requiring Higher-level demands (procedures with connections and doing mathematics) correspond to 13.3% (132) of the tasks in the Brazilian collection and 26.6% (295) in the Norwegian collection, revealing a substantial difference between the textbooks. A comparative analysis shows that, although low cognitive demand tasks predominate in both collections, the Norwegian includes almost twice the proportion of high cognitive demand tasks when compared to the Brazilian collection. This pattern indicates a stronger presence of tasks that require students to engage in higher levels of reasoning, thereby offering more frequent opportunities for the development of mathematical thinking.

Table 2: Distribution of cognitive demand levels in the works.

	Diálogos		*Matemagisk*	
Level	*n*	%	*n*	%
Mem	406	41.1%	415	37.4%
PwoutC	394	39.8%	396	35.6%
PwC	114	11.5%	251	22.6%
Domath	18	1.8%	44	4%
Not Applicable	57	5.8%	5	0.4%
Total	989	100%	1 111	100%

When the two analytical characteristics examined in this study are considered jointly, a differentiated profile emerges for each collection. The *Diálogos* collection stands out for its higher proportion of tasks that attempt to articulate mathematics with contexts of Re and SR, whereas the *Matemagisk* collection is characterized by a substantially higher proportion of tasks classified at Higher-level demands. This contrast provides the basis for examining how different context references are related to levels of cognitive demand, as addressed in the research question guiding this study (Table 3).

Table 3: Distributions of cognitive demand levels according to the context.

		Purely Mathematical		Semi-reality				Reality	
				Reasonable		Not reasonable			
		n	%	*n*	%	*n*	%	*n*	%
Diálogos	Mem	286	50.5%	73	28.4%	1	14.3%	45	44.1%
	PwoutC	202	35.7%	144	56%	5	71.4%	44	43.1%
	PwC	66	11.7%	36	14%	1	14.3%	11	10.8%
	Domath	12	2.1%	4	1.6%	0	0.0%	2	2%
	Total	566	100%	257	100%	7	100%	102	100%
Matemagisk	Memorization	370	40.5%	33	23.9%	0	-	12	22.2%
	PwoutC	302	33.0%	65	47.1%	0	-	29	53.7%
	PwC	208	22.8%	32	23.2%	0	-	11	20.4%
	Doing Math	34	3.7%	8	5.8%	0	-	2	3.7%
	Total	914	100%	138	100%	0	-	54	100%

Across both collections, the predominance of Lower-level demands tasks persists in all context references, although with relevant variations among PM, SR (SRR and SRN), and Re. In both collections, PM concentrates the highest percentages of tasks classified at the memorization level, reinforcing a pattern in which tasks detached from external references tend to privilege recall-based objectives. However, particularly in the Norwegian collection, this context also shows a more balanced distribution between low and Higher-level demands levels, indicating that purely mathematical tasks are not inherently restricted to Lower-level demands. SR presents a higher concentration of tasks as the type as procedures without connections. Although such contexts are frequently associated in literature with richer learning opportunities (Skovsmose, 2001; Van den Heuvel-Panhuizen, 2005), the data show that their presence mainly corresponds to a shift from memorization toward procedural activity, rather than to a consistent increase in Higher-level demands.

These results directly address the research question by showing that the relationship between context references and cognitive demand is not linear or deterministic. While tasks situated in SR are more likely to present Higher-level demands than PM tasks, they do not predominantly foster tasks classified as procedures with connections or

doing mathematics. Similarly, Higher-level demands tasks can also be found in PM, particularly in the Norwegian collection.

Moreover, tasks classified as doing mathematics remain rare across all context categories and in both collections, reinforcing the persistence of instructional patterns centered on routine and procedural activity rather than on mathematical reasoning, exploration, and justification (Stein *et al.*, 2000). The level of cognitive demand appears to be primarily associated with the mathematical intentions inherent in the task's questioning, and not with the nature of the context in which it is embedded. This result reinforces the need to examine not only where tasks are contextualized, but fundamentally how they structure opportunities for mathematical thinking.

CONCLUSION

The analyses conducted in this study indicate that the two textbook collections investigated present distinct profiles in the ways contextualization and cognitive demand are articulated in Geometry tasks. The Norwegian collection shows a comparatively higher proportion of tasks that engage students in processes such as interpretation, justification, and strategy development, whereas the Brazilian collection concentrates a substantial number of tasks at levels associated with *memorization* and routine procedural application, which may limit opportunities for more elaborate forms of mathematical reasoning.

At the same time, the findings reveal that in both collections (and particularly in the Norwegian one) most tasks remain situated in PM contexts. The distributions observed across contexts and levels of cognitive demand indicate that the mere presence of contexts beyond the mathematical domain does not systematically correspond to higher levels of cognitive demand. In this sense, it is observed contextualization does not operate as a determining factor in the constitution of cognitive demand, but rather as one element among others in tasks planning.

A more detailed analysis, however, shows that the SR concentrates proportionally a higher incidence of cognitively demanding tasks than other contexts in both collections. This result qualifies part of the literature that characterizes SR primarily as a synthetic reality mechanism (Skovsmose, 2001). The findings suggest that SR may offer limited opportunities in terms of contextualization with the real world, but relevant opportunities in terms of cognitive demand, by mobilizing more complex mathematical reasoning.

In this sense, SR seems to function as an intermediate space: sufficiently structured to support mathematical exploration, but open enough to demand interpretation, information selection, and decision-making – characteristics that align with higher levels of cognitive demand. Overall, the study contributes to ongoing debates in Mathematics Education, demonstrating that the quality of task planning, and not the nature of the context itself, plays a central role in shaping student’s cognitive engagement.

References

Alves, G. A., Amaral, R. B., & Litoldo, B. F. (2024). *Teorizando contextos: Fundamentos para a análise de livros didáticos de matemática do Novo Ensino Médio*. In *Anais do 2º Colóquio de Livros Didáticos de Matemática*. UFMS.

Amaral, R. B., Mazzi, L. C., Vieira, L., & Perovano, A. P. (2023). *Livro didático de matemática: Compreensões e reflexões no âmbito da educação matemática*. Mercado de Letras.

Dekker, T., & Querelle, N. (2002). Chapter 6 – Context. In *Great assessment problems*. Freudenthal Institute.

Fan, L., Zhu, Y., & Miao, Z. (2013). *Textbook research in mathematics education: Development status and directions*. *ZDM*, 45(5), 633-646.

Jackson, K., Garrison, A., Wilson, J., Gibbons, L. & Shahan, E. (2013). Exploring relationships between setting up complex tasks and opportunities to learn in concluding whole-class discussions in middle-grades mathematics instruction. *Journal for Research in Mathematics Education, 44*(4), 646-682.

Lana, M. A., & Carrião, A. (2015). *A contextualização das atividades no livro didático de matemática do ensino médio*. In *Anais do VII Encontro Mineiro de Educação Matemática*.

Litoldo, B. F. (2021). *A contextualização e os níveis de demanda cognitiva de tarefas de geometria presentes em livros didáticos de matemática sob a perspectiva do opportunity-to-learn* (Tese de Doutorado). UNICAMP.

Özgeldi, M.; Esen, Y. Analysis of mathematical tasks in Turkish elementary school mathematics textbooks. *Procedia-Social and Behavioral Sciences*, v. 2(2), n. Elsevier Ltd, p. 2277-2281, 2010.

Sierra, C. D. (2025). High cognitive demand didactic experience for teaching inferential statistics. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.), *Proceedings of the 48th PME: General Contributions*.

Skovsmose, O. (2001). Landscapes of Investigation. *ZDM* 33, 123-132.

Stein, M. K., Henningsen, M. A.; Smith, M. S. & Silver, E. A. (2000). Implementing standards-based mathematics instruction: a Casebook for Professional Development. New York: Teachers College Press.

Teixeira, L. A. (2020a). *Diálogos matemática e suas tecnologias: Geometria plana*. Moderna.

Teixeira, L. A. (2020b). *Diálogos matemática e suas tecnologias: Geometria espacial*. Moderna.

Van den Heuvel-Panhuizen, M. (2005). The role of contexts in assessment problems in mathematics. *For the Learning of Mathematics, 25*(2), 2-9.

Wallace, A. K., & Kongsnes, A. L. (2020a). *Matemagisk 8*. Aschehoug Undervisning.

Wallace, A. K., & Kongsnes, A. L. (2020b). *Matemagisk 9*. Aschehoug Undervisning.

Wallace, A. K., & Kongsnes, A. L. (2020c). *Matemagisk 10*. Aschehoug Undervisning.