

FRACTION LEARNING IN A DIGITAL LEARNING ENVIRONMENT: A PILOT STUDY

Heli Nurminen¹, Hilma Halme², Arto Kortelainen¹, Jake McMullen^{1,3}, Jo Van Hoof¹

¹ Turku Research Institute for Learning Analytics, University of Turku

² Special and Inclusive Education, University of Oulu

³ Department of Teacher Education, University of Turku

This pilot study examined 6th graders' (n=29) understanding of and misconceptions with fractions over a five-week teaching period. We examine the students' knowledge before and after their use of a curriculum-based fraction learning environment conducted on a digital learning platform. Utilizing a pre-post-test research design, we measured students' basic fraction arithmetic and fraction comparison skills. Results indicate a significant improvement in arithmetic and comparison skills, but there was no decrease in fraction magnitude misconceptions. Thus, a curriculum-based digital learning environment, which relies heavily on procedural practice, may not be enough to refute misconceptions about fraction magnitude. Implications are discussed.

INTRODUCTION

Having a good understanding of the magnitude of fractions and fraction arithmetic is an important foundation for learning more complex mathematical concepts (McMullen & Van Hoof, 2020; Siegler & Pyke, 2013) and predicts later math achievement (Siegler et al., 2011). Despite this, many learners have difficulty developing a strong understanding of fractions (D'Erchie et al., 2025a; Gabriel et al. 2023). Previous studies have identified multiple reasons why fractions are difficult to learn. One of those reasons is the natural number bias (further abbreviated with NNB), where one overapplies the properties of natural numbers to rational numbers (e.g. Gabriel et al. 2023; Gómez & Dartnell, 2015). Before learning about fractions, students typically develop an understanding of numbers through experiences with natural numbers, as curricula tends to start there. However, this previous natural number knowledge can interfere with fraction learning and has been shown to lead to systematic errors and misconceptions (e.g., Gabriel et al., 2023; Gómez & Dartnell, 2015). For example, many children consider $5/11$ to be larger than $2/3$ as 5 is larger than 2 and 11 is larger than 5. Given the importance of a good fraction understanding, students' persistent difficulties with fractions, and the limited knowledge on effective classroom interventions; the present study examines students' knowledge before and after their use of a curriculum-based fraction learning environment in a digital learning platform.

Fraction magnitude misconceptions

Research has proposed three types of NNB: typical, reversed and gap thinking (e.g. Gómez & Dartnell, 2019). In the typical NNB, students systematically state that the

fraction with the largest natural number in its numerator/denominator is the one with the largest value. For example, they incorrectly reason that $\frac{5}{9}$ is larger than $\frac{3}{4}$, since 5 is larger than 3 and 9 is larger than 4. In contrast, students with reverse NNB show the opposite answering pattern, reasoning that the fraction with the smallest numerator and/or denominator is the largest one. For example, they incorrectly reason that $\frac{7}{8}$ is smaller than $\frac{2}{3}$, given that 2 and 3 are smaller than 7 and 8. Third, students with gap thinking believe that a fraction is larger if the numerical difference (the gap) between the numerator and denominator is smaller; reasoning that since the gap between numerator and denominator is smaller the “countable steps required” to get into the whole is smaller. For example, students with gap thinking consider $\frac{1}{2}$ as larger than $\frac{7}{9}$, because there is only one step needed from $\frac{1}{2}$ to go the whole ($\frac{2}{2}$), while $\frac{7}{9}$ requires two steps to reach the whole ($\frac{9}{9}$). Numerous studies have shown that these magnitude misconceptions are common throughout basic schooling and can even be found in adults (Gabriel et al., 2023; Reinhold et al., 2023).

These natural number based misconceptions are often explained by the conceptual change theory, which examines processes of learning that require the restructuring of already existing knowledge and mental models (e.g., Vamvakoussi et al., 2010). In the case of fractions, students are already familiar with natural numbers before receiving formal instruction on fractions. As a result, they have already developed a concept of numbers grounded in natural numbers, before learning about fractions. When fractions are later introduced, the rules and properties of natural numbers are not always applicable, which can lead to misconceptions unless students modify their concept of numbers.

The literature has discussed that reverse NNB and gap-thinking biases could be seen as more advanced knowledge, since in both cases students already know that rational numbers differ from natural numbers (see Rinne et al. 2017; D’Erchie et al., 2025b). Reverse NNB and gap-thinking can reflect an intermediate state of conceptual change, which suggests that conceptual change is a gradual and slow process rather than an abrupt transformation (e.g., Vamvakoussi et al., 2010; Van Hoof et al. 2021).

Fraction interventions

Ample studies conclude that approaches including teaching mathematical language, and the use of multiple external representations are especially effective when aiming to improve students’ general fraction understanding (e.g., Krowka et al., 2024). However, only a handful of intervention studies have targeted students’ natural number based misconceptions with varying success. While Van Hoof et al. (2021) did not find additional benefits of the use of refutation text on top of explanatory text, the use of correct and incorrect worked examples with corrective feedback (Barbieri & Devlin, 2023) and number line estimations (D’Erchie et al., 2025b) in digital learning environments were successful in refuting misconceptions about fractions.

PRESENT STUDY

Given the importance of a good fraction understanding and students' difficulties with learning them, the present study investigates how sixth grade students' understanding of and misconceptions with fractions develop before and after a five-week fraction teaching period.

Specifically, we examine students' basic fraction arithmetic skills as well as their misconceptions regarding fraction magnitude before and after they follow a five-week curriculum-based fraction learning environment in a digital learning platform. The following research questions are addressed

RQ1: Can we replicate previous findings by showing natural number based misconceptions in Finnish 6th graders, an under-investigated group in the global literature on the NNB?

RQ2: What are the improvements in students' basic fraction arithmetic skills, fraction magnitude understanding, and fraction magnitude misconceptions between the pre- and post-test?

METHODS

Participants were 29 Finnish 6th graders, who followed a curriculum-based fraction learning environment in ViLLE once a week for five weeks. Ethical guidelines from the Finnish National Board on Research Ethics were followed.

ViLLE is a widely used digital learning platform in Finland. The exercises on ViLLE are curriculum-based, co-created with teachers, and shown to positively influence learning (Laakso et al. 2018). Approximately 70% of Finnish schools use ViLLE and students typically work on mathematical tasks in ViLLE for one lesson weekly, as in the present study. Regular use of ViLLE lessons in math classes has shown positive effects on students' math fluency and performance (Kurvinen et al. 2020). In this light, ViLLE offers a real-world platform for gathering data from actual learning environments, providing high ecological validity for the research results. The ViLLE lessons are carried out with laptops and tablets from school during regular school hours under the supervision of the teacher. ViLLE provides automatic feedback (right or wrong) as well as gamified and visual elements (including external representations).

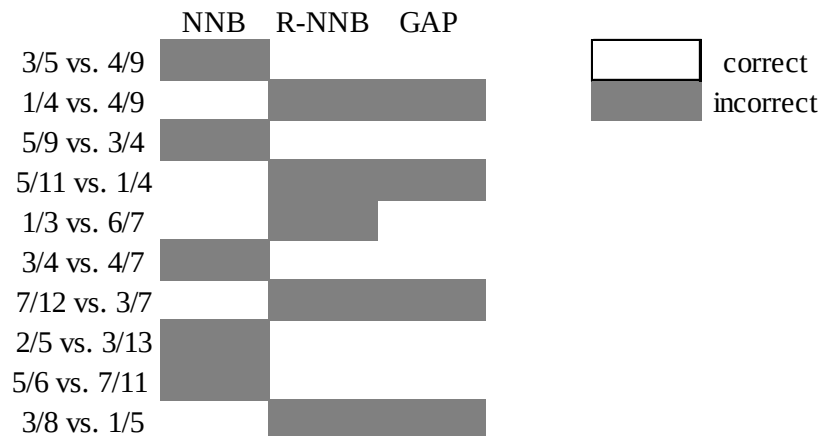
In this study, the curriculum-based fraction learning environment included visual representations of fractions with area models. Additionally, short video tutorials were used in three lessons (e.g., conversion to mixed fraction, arithmetic operations) that students could rewatch while practising. Each lesson took 45 minutes and on average students spent 30 minutes time on the tasks. On top of this, students solved fraction tasks in ViLLE at home for 15 minutes each week, as homework.

The pre- and post-test measures included a fraction comparison task and a basic fraction arithmetic task. In the comparison task, students must determine which of the two irreducible fractions is larger. The used fraction pairs are either congruent or

incongruent with the typical NNB misconception. Thus, a student with a typical NNB profile answers correctly for congruent fraction pairs and incorrectly in incongruent ones (with opposite pattern for the reverse NNB profile). For the typical NNB a pair of fractions is congruent if the larger fraction has both a larger numerator and denominator (e.g., $3/8$ vs. $1/5$). On the contrary, if the larger fraction has smaller numerator and denominator values, it's called incongruent with typical NNB (e.g., $5/6$ vs. $7/11$). Students were considered to have a fraction magnitude misconception when they answered 9 out of 10 fraction comparisons in line with a specific misconception (for coding scheme, see Figure 1, which shows when the misconception leads to a correct [white] or incorrect [black] answer). On top of these misconception profiles, students were considered to have a good understanding of fraction magnitude ("Accurate" profile) when they scored at least 9 out of 10 on the fraction comparison test. As shown in Figure 1, there is considerable overlap between the reverse NNB and gap thinking answer patterns. Consequently, to be classified under a specific profile, the distinguishing item had to be answered in line with that profile's criteria.

In the basic fraction arithmetic task, students solved five fraction problems, including three additions and two subtractions with and without a common denominator (e.g., $1/5 + 3/5$, $4/6 - 1/2$). For the analysis we used the sum score of correctly solved items.

Figure 1. Fraction comparison coding scheme. Expected response patterns by misconceptions



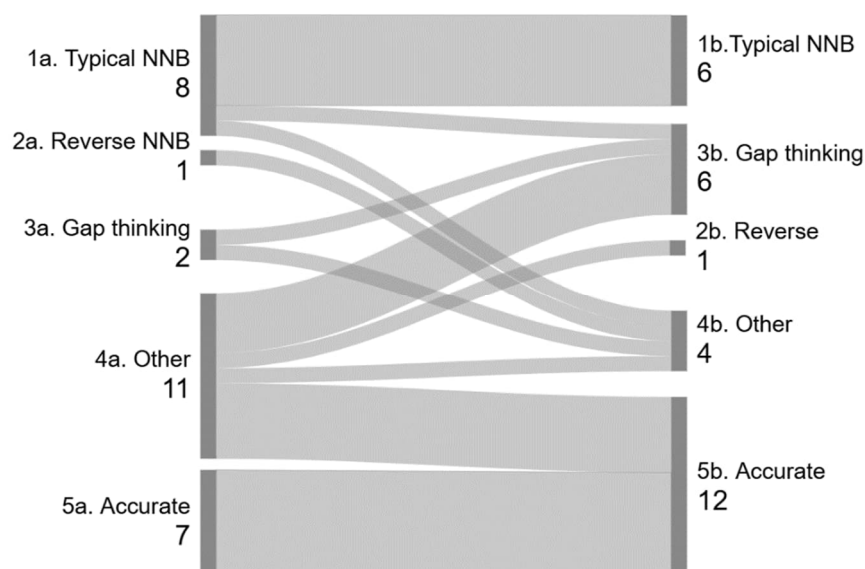
RESULTS

Regarding RQ1 (Can we replicate previous findings showing natural number based misconceptions in Finnish 6th graders, an under investigated group in the global literature on the NNB?), 11 out of the 29 participants showed a natural number based misconception pattern in pre-test, with eight students showing typical NNB, one reverse NNB, and two gap-thinking. A visualisation of the students' natural number based misconceptions at the pre-test is presented on the left side of the Figure 2.

To answer RQ2 (What are the improvements in students' basic fraction arithmetic skills and fraction magnitude misconceptions between the pre- and post-test?), students' sum scores on the basic arithmetic task were compared between pre- and post-test. The mean accuracy increased from 2.5 at pre-test to 4.7 at post-test (maximum 5). Because the assumption of normality was violated (Shapiro-Wilk test, $p < .05$), a Wilcoxon signed rank test was conducted, which indicated a significant increase from pre-test ($Md = 3.00$) to post-test ($Md = 5.00$), $W = 2.00$, $p < .001$. Secondly, students' sum scores on the fraction comparison task were compared between pre- and post-test to examine improvements in students' fraction magnitude understanding. The mean accuracy increased from 7 at pre-test to 8 at post-test (maximum 10). A Wilcoxon signed-rank test was conducted, since the assumption of normality was violated (Shapiro-Wilk test, $p < .05$). The results demonstrate a significant increase from pre-test ($Md = 6$) to post-test ($Md = 7$), $W = 25.5$, $p = .007$.

Concerning changes between misconception profiles, results showed that at the post-test, two more students (13 out of 29) exhibited NNB misconceptions. Seven students stayed in the same profile, six of whom showed typical NNB and one gap thinking. From the other four students having a misconception at the pre-test, no-one developed a correct understanding during the five-week teaching period. Moreover, five new students developed an NNB misconception, four gap-thinkers and one with reverse NNB. For visualisations of the development of students' fraction magnitude understanding, see Figure 2.

Figure 2. Distribution of participants into misconception profiles for pre- and post-test and their transitions between profiles.



DISCUSSION

A key finding from this pilot study is that, although students showed significant gains in overall fraction arithmetic and magnitude performance, the curriculum-based fraction learning environment, may not be sufficient to address or prevent misconceptions about fraction magnitude. A possible explanation for these findings is that, in many countries, including Finland, the teaching materials are more focused on procedural than conceptual fraction knowledge. Therefore, also the curriculum-based fraction learning environment mainly emphasized procedural practice, which might explain the overall found learning gains, but the absence of overcoming magnitude based fraction misconceptions.

In this light, an important educational implication of the study is that more focus is needed on supporting students' conceptual understanding of fractions. One way to achieve this is to move beyond the usual superficial part-whole schema (e.g., Gabriel et al., 2023) and incorporate more in-depth conceptual approaches, such as number line estimations (see for example D'Erchie et al., 2025b) and visual representations into the fraction classroom (e.g. Van Hoof et al., 2021).

Interestingly, the number of students with natural number based misconceptions increased from pre- to post-test. Moreover, none of the students with a misconception at pre-test developed an accurate understanding of fraction magnitude at the post-test, which is in line with earlier findings supporting the persistence of these misconceptions (e.g., González-Forte et al. 2022) and the fact that the process of conceptual change is slow and gradual (e.g. Vamvakoussi et al., 2010; Van Hoof et al., 2021). Especially concerning was that five new students exhibited a misconception at the post-test, reminding us that learning is not a linear process (see Rinne et al. 2017).

It should be noted that the pilot nature of this study and its accompanying small sample size, limits the interpretations of the findings. Therefore, future research should include larger samples to generalise the findings. Nonetheless, an important implication of this study is that overlooking qualitative differences in students' (fraction) knowledge, such as whether or not they hold misconceptions, can result in misinterpretation of the results. For example, in the present study, the overall increase in accuracy on the fraction comparison task could have masked the continued presence of fraction magnitude misconceptions.

Put together, the present study confirmed previous findings that a large group of students show consistent misconceptions of fractions, and that these were not sufficiently addressed by a curriculum-based learning environment. This confirms the need for intervention studies investigating how didactical interventions can help students to overcome these misconceptions (see also Gabriel et al., 2023).

Additional information

The study is funded by the Research Council of Finland (EDUCA Flagship #358924, #358947, and SCAFFLEX #363159) and the Ministry of Education and Culture (Doctoral school pilot #VN/3137/2024-OKM-4).

ChatGPT was used during the writing process to improve the readability and language of the research report. The authors take full responsibility for the content of the publication.

References

- Barbieri, C., & Devlin, K. (2023). Targeting fraction misconceptions and reducing high-confidence errors in an online tutor. *Journal of Computer Assisted Learning*. <https://doi.org/10.1111/jcal.12877>
- D’Erchie, M., Forsmann, C., Schneider, M., & Obersteiner, A. (2025a). Predictive relationships between conceptual and procedural knowledge across the domains of fractions and algebra. In *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 179–186). PME.
- D’Erchie, M., Rosenkranz, J., Schwarzmeier, S., Dresler, T., & Obersteiner, A. (2025b). Magnitude-based reasoning can reduce the natural number bias when comparing fractions: An intervention study. *Learning and Instruction*, 98, Article 102109. <https://doi.org/10.1016/j.learninstruc.2025.102109>
- Gabriel, F., Van Hoof, J., Gómez, D. M., & Van Dooren, W. (2023). Obstacles in the development of the understanding of fractions. In *Mathematical cognition and understanding: Perspectives on mathematical minds in the elementary and middle school years* (pp. 209–225). Springer. https://doi.org/10.1007/978-3-031-29195-1_11
- González-Forte, J. M., Fernández, C., Van Hoof, J., & Van Dooren, W. (2022). Incorrect ways of thinking about the size of fractions. *International Journal of Science and Mathematics Education*. <https://doi.org/10.1007/s10763-022-10338-7>
- Gómez, D. M., & Dartnell, P. (2015). Is there a natural number bias when comparing fractions without common components? A meta-analysis. In *Proceedings of the 39th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 1–8). PME.
- Gómez, D. M., & Dartnell, P. (2019). Middle schoolers’ biases and strategies in a fraction comparison task. *International Journal of Science and Mathematics Education*, 17, 1233–1250. <https://doi.org/10.1007/s10763-018-9913-z>
- Krowka, S. K., Jayanthi, M., Schumacher, R. F., Gersten, R., Haymond, K. S., & Newman-Gonchar, R. A. (2024). Identifying key instructional practices in rational number interventions: A systematic review and meta-analysis. *Learning Disability Quarterly*. <https://doi.org/10.1177/07319487241288823>
- Kurvinen, E., Kaila, E., Laakso, M.-J., & Salakoski, T. (2020). Long-term effects on technology-enhanced learning: The use of weekly digital lessons in mathematics. *Informatics in Education*, 19(1), 51–68. <https://doi.org/10.15388/infedu.2020.04>

- Laakso, M.-J., Kaila, E., & Rajala, T. (2018). ViLLE – Collaborative education tool: Designing and utilizing an exercise-based learning environment. *Education and Information Technologies*, 23, 1655–1675. <https://doi.org/10.1007/s10639-017-9659-1>
- McMullen, J., & Van Hoof, J. (2020). The role of rational number density knowledge in mathematical development. *Learning and Instruction*, 66, Article 101228. <https://doi.org/10.1016/j.learninstruc.2019.101228>
- Reinhold, F., Leuders, T., & Loibl, K. (2023). Disentangling magnitude processing, natural number biases, and benchmarking in fraction comparison tasks: A person-centered Bayesian classification approach. *Contemporary Educational Psychology*, 75, Article 102224. <https://doi.org/10.1016/j.cedpsych.2023.102224>
- Rinne, L. F., Ye, A., & Jordan, N. C. (2017). Development of fraction comparison strategies: A latent transition analysis. *Developmental Psychology*, 53(4), 713–730. <https://doi.org/10.1037/dev0000275>
- Siegler, R. S., & Pyke, A. A. (2013). Developmental and individual differences in understanding of fractions. *Developmental psychology*, 49(10), 1994. <https://doi.org/10.1037/a0031200>
- Siegler, R. S., Thompson, C. A., & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Cognitive Psychology*, 62, 273–296. <https://doi.org/10.1016/j.cogpsych.2011.03.001>
- Vamvakoussi, X., Christou, C., & Van Dooren, W. (2010). Greek and Flemish students' understanding of the density of rational numbers: More similar than different. In *Proceedings of the 34th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 353–360). PME.
- Van Hoof, J., Engelen, A.-S., & Van Dooren, W. (2021). How robust are learners' misconceptions of fraction magnitude? An intervention study comparing the use of refutation and expository text. *Educational Psychology*, 41(5), 524–543. <https://doi.org/10.1080/01443410.2021.1908521>