BRIDGING REPRESENTATIONS IN FRACTION OPERATIONS: EXPLORING THE DECISION-MAKING OF FUTURE TEACHERS IN GERMANY AND SPAIN

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Noticing is a teacher competence that implies attending, interpreting, and making decisions in teaching-learning situations, and that can be developed in initial training programs by analysing vignettes. Since vignettes often are contextually framed, research is needed to explore noticing of pre-service teachers (PTs) from different cultural and institutional contexts. Participants were 73 Spanish and 56 German PTs who had to plan a follow-up lesson after analysing a cartoon-based vignette depicting a teaching–learning situation about fraction operations. Results show that most PTs from both groups were able to provide a problem according to the objective of the follow-up lesson, but they had difficulties in justifying their decisions.

INTRODUCTION AND THEORETICAL BACKGROUND

Noticing is considered a fundamental professional competency for teachers, essential for responsive and effective mathematics instruction (e.g., some reviews or special issues: Dindyal et al., 2021; Weyers et al., 2024; Zhang et al., 2025). Although it has been conceptualised from different perspectives, it implies the processes of attending, interpreting, and deciding to (responding to) important aspects of mathematics classroom situations. In this study, we focus on the process of deciding, since previous studies have indicated that this aspect of noticing is the most challenging (e.g., Barnhart & van Es, 2015).

However, as highlighted by Dindyal et al. (2021), although noticing can be developed in training programs, "the consideration of context is critical because teaching and learning is located in a complex cultural and political ecosystem." (p. 11). For instance, Yang et al. (2019) conducted a comparative study with in-service teachers from China and Germany using videos. Their results revealed that German and Chinese teachers differed in their identification, interpretation, and decision-making. Therefore, "noticing is not merely an individual cognitive skill but is fundamentally shaped by context" (Zhang et al., 2025, p. 925).

Vignettes, understood as scenarios depicting situations in the mathematics classroom, have been shown to be effective tools for enhancing and assessing teachers' noticing (Buchbinder & Kuntze, 2018; Herbst & Kosko, 2014). One of the potentialities of vignettes is the reduction of the complexity of real classrooms, as this focus may help direct pre-service or in-service teachers' attention to significant aspects of the

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mathematics teaching and learning situation. Another potential for initial teacher education programmes is that vignettes allow for connecting theory with practice, and particularly, that they can strengthen pre-service teachers in their classroom-related decision-making and to base decisions on an analysis of classroom situations.

However, vignettes are often designed by teacher educators and researchers, usually making implicit cultural context assumptions (Zhang et al., 2025) and deciding which events in mathematics classrooms are important or which subsequent "good" teaching actions are effective for mathematics instruction. Moreover, across different cultures and countries, these assumptions may differ. Dreher et al. (2021) showed that German and Taiwanese researchers evaluated students' thinking differently in the same vignette, revealing that experts' norms for responding to students' mathematical thinking may vary. Therefore, cultural context appears to interdepend with teachers' perceptions of classroom situations, raising additional questions about how we can understand teacher noticing cross-culturally. These studies hence have emphasised the need for collaboration between mathematics teacher educators from different cultures and countries to explore the competency of noticing and its sub-aspects, such as classroom-related decision-making.

Although differences between teachers from different continents can be expected (see Yang et al., 2019), it is also necessary to explore possible differences between less distant samples (e.g., Spain and Germany), in particular differences and commonalities in decision-making of PTs from different contexts. Beyond the context of different countries, different institutional cultures or different perspectives adopted in teacher training may lead to differences in PTs noticing (Ivars et al., 2025).

Our Study: Objective and research questions

Considering the need for collaboration between mathematics teacher educators from different countries and the need for further intercultural noticing research, the objective of our study is to examine how pre-service teachers from two universities (Alicante-Spain and Ludwigsburg-Germany) that are culturally different in terms of country, culture, or university settings plan the follow-up lesson after analysing a cartoon-based vignette depicting a teaching–learning situation about fraction operations. This cartoon vignette was designed using graphical elements from the DIVER CREATE digital tool, developed in the CoReflect@maths Erasmus+ project (www.coreflect.eu), and in collaboration with mathematics teacher educators from both universities.

In line with our focus on PTs' *decision-making*, we focus on the problem/activity proposed by PTs for the follow-up lesson and PTs' justifications of their decisions. Then, the objective has been particularised in the following research questions:

- What decisions (problem and justification) did PTs make for the follow-up lesson after analysing a classroom situation?
- To which extent can differences in the decisions made for the follow-up lesson between the Spanish and German university PT groups be detected?

DESIGN AND METHODS

Participants and context

Participants were 129 PTs; 73 Spanish PTs from the University of Alicante (coded as "S-PT") and 56 German PTs from Ludwigsburg University of Education (coded as "G-PT"). The Spanish PTs were attending a course on the *Teaching and Learning of Mathematics in Primary Education*, which is part of their third year of the degree to become a primary school teacher. The main objective of the course is to develop pre-service teachers' skills as mathematics teachers: noticing students' mathematical thinking, analysing and designing mathematical activities, and planning a lesson. The German PTs were participants in a one-semester seminar on representations. In this seminar, PTs worked on analysing cases of classroom situations, focusing on how mathematical objects can be represented.

PTs from both universities solved the same vignette as part of the course they were attending. The vignette depicts a teaching–learning situation involving subtraction with fractions.

Instrument

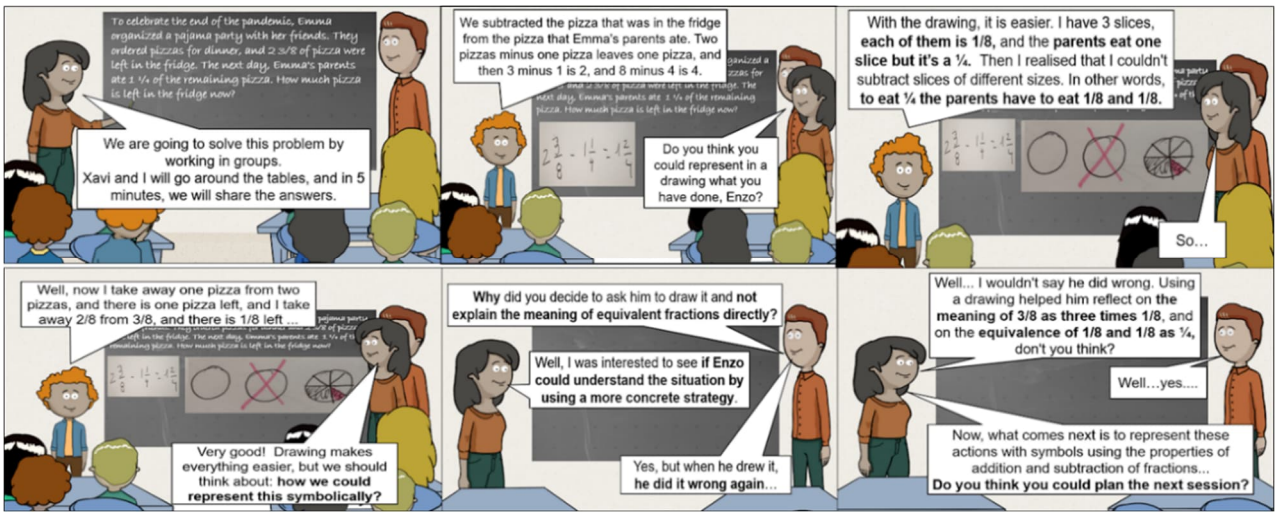
The vignette depicts an interaction between a primary school teacher and a student. The problem formulated by the teacher was: "To celebrate the end of the pandemic, Emma organised a pyjama party with her friends. They ordered pizzas for dinner, and $2\frac{3}{8}$ of the pizza was left in the fridge. The next day, Emma's parents ate $1\frac{1}{4}$ of the pizza. How much pizza is left in the fridge now?" In the classroom discussion, Enzo provides a symbolically incorrect answer based on "subtracting numerators and denominators separately". After Enzo's response, the teacher asks him to represent the problem graphically, and Enzo has difficulty identifying the fraction that represents a portion of the pizza (indicating ¼). After focusing on the graphical representation and with the teacher's guidance, Enzo realises that each part of the pizza represents $\frac{1}{8}$ rather than $\frac{1}{4}$, and he can provide a correct answer from the graphical representation. The vignette continues, and the teacher asks the practising teacher (who is assisting her in the classroom) to plan the follow-up lesson (Figure 1).

PTs had to imagine to be in the role of the practising teacher and plan the follow-up lesson considering the following learning objective: "Connect the representations and actions in the drawings with arithmetic expressions (supporting the progression to more sophisticated strategies)". PTs had to answer the following prompt: "Propose the following problem and justify the choice of numbers". Spanish and German PTs' open-format answers to this prompt are the data of this study.

We provided PTs with a theoretical document containing information related to the learning of fractions and ideas for supporting the progression of students' mathematical thinking from less to more sophisticated strategies based on different representations (Gibson et al., 2018). With the learning objective provided, PTs were expected to

formulate a problem/activity to continue working the addition/subtraction of fractions (context of the vignette) and provide numbers that help students to link the graphical and symbolic representation (e.g., fractions with the same denominators; fractions with different denominators but denominators that are multiples of each other that easily allow a graphical representation…).

Figure 1: Excerpt of the vignette used as an instrument to collect data.



Analysis

To ensure consistency in codification criteria across languages, the data were translated into English before the analysis. The PTs' answers were analysed deductively considering the following codes focused on: (i) whether PTs decisions (problems/activities provided) were aligned with the analysis of the situation (context, fraction addition and subtraction), (ii) whether PTs decisions were related to the learning objective (linking graphical and symbolic representations), (iii) whether PTs justifications of the numbers used considered the difficulty on the content, and (iv) whether PT justifications consider how the numbers used facilitate linking the different representations. The codes were refined by an inductive analysis of the data. Cases of initial disagreement were discussed until a consensus had been reached.

- Code 0. PTs provided a problem/activity not related to the learning objective. That is, the problem (number chosen) does not facilitate linking different representations. Furthermore, the code also covered PTs' answers that, although the problem/activity facilitates linking the graphical and symbolic representations (learning objective), were not focused on the context of the vignette, namely fraction addition or subtraction.
- Code 1. PTs provided a problem/activity related to the learning objective with difficulties. PTs showed mathematical knowledge difficulties formulating the problem/activity (e.g., the use of different wholes to add or subtract; the problem does not imply the use of fractions (e.g., natural numbers, etc.).

- Code 2. PTs provided a problem/activity related to the learning objective without a justification of the numbers used, or the justification provided was general without any relation to the learning objective (e.g., the fractions have different denominators).
- Code 3. PTs provided a problem/activity related to the learning objective and a justification of the numbers (e.g., because the fractions are easier to represent; the fractions are easier because they have the same denominator; the fractions used are familiar…). However, PTs did not explicitly comment on how the numbers used will facilitate linking the different representations or how they will link them.
- Code 4. PTs provided a problem/activity related to the learning objective and a justification of the numbers used. The PTs explicitly commented on how the numbers used will facilitate linking different representations and/or how they will link them.

RESULTS

PTs decisions and justifications

Our results show that 104 of 129 PTS provided a problem/activity related to the learning objective for the next lesson (codes 2, 3, and 4). However, only a part of them (56 PTs) could justify the numbers used in the problem. A few (7 PTs) were able to provide not only a justification of the numbers but also an explanation about how the numbers used will facilitate linking the graphical and symbolic representations or/and how they will link the different representations. The following examples of PTs' answers are provided (regardless of country) as instances of each code.

S-PT12 formulated a problem according to the learning objective: "To celebrate Juan's birthday with all his friends, his parents bought four cakes. Marta ate $1\frac{1}{4}$ cakes, and Juan ate $2\frac{4}{8}$. How much cake did they eat in total?" Nevertheless, S-PT12 did not justify the numbers (he/she only wrote the problem) and, therefore, this answer was coded by 2.

Regarding code 3, an example is the following answer of G-PT27. This PT proposed to solve the following subtraction $2\frac{5}{6}$ - $1\frac{1}{3}$ and justified that he has chosen thirds and sixths since "they are familiar and easier to represent graphically". Also, the answer of S-PT48 was coded by 3: This PT formulated the follow-up problem: "At Juan's house, everyone ate $2\frac{7}{8}$ pizzas with ham and cheese and $1\frac{4}{8}$ pizzas with four cheeses. How much pizza did they eat in total?" This PT justified the numbers as follows: "I will use fractions with the same denominator to help children in their initiation."

An example of an answer coded as 4 is G-PT7, which provided a sequence of four activities in which the fractions used ranged from the easiest to the most difficult. The first activity was an activity of relating the graphical representation of a fraction to its

symbolic representation. The second activity implied adding fractions with the same denominator. The third activity involved the addition of fractions with different denominators that were multiples ($\frac{3}{4}+\frac{4}{8}$), and the fourth activity asked students to add fractions with non-common denominators that were not multiples ($\frac{2}{3}+\frac{3}{4}$). This PT designed the activities to link both representations (graphical and symbolic), justifying it as follows: "Activity 2 consists of linking the graphical representation with the arithmetic expression […] the parts represented graphically are of the same size since the fractions have the same denominator. This allows students to solve the activity by counting the parts of the pizza".

Another example coded by 4 is S-PT61's answer. The problem formulated was "Last night Ana ordered pizza for dinner. She and her sister ate $1\frac{3}{8}$, and her parents ate $2\frac{4}{8}$. How much pizza did they eat in total?". S-PT61 justified the numbers used and explicitly commented how the numbers will allow linking different representations "I have chosen fractions with the same denominator so that, at this early stage, it is easier for children to connect the graphical representation with the symbolic one. In this way, I believe it will be easier for them to gradually extrapolate these concepts to fractions with different denominators and more complex problems."

Nevertheless, a group of PTs in both countries faced difficulties by not providing a problem/activity related to the learning objective or to the vignette's context (code 0; 12 PTs), or by proposing a problem/activity related to the learning objective but with difficulties (code 1, 13 PTs). These last PTs showed difficulties with mathematical knowledge in formulating the problem. For instance, G-PT14 proposed an activity of transforming mixed numbers into improper fractions. This activity neither works the connection between the graphical and symbolic representation nor the context of the vignette: fraction addition or subtraction. It was coded by 0.

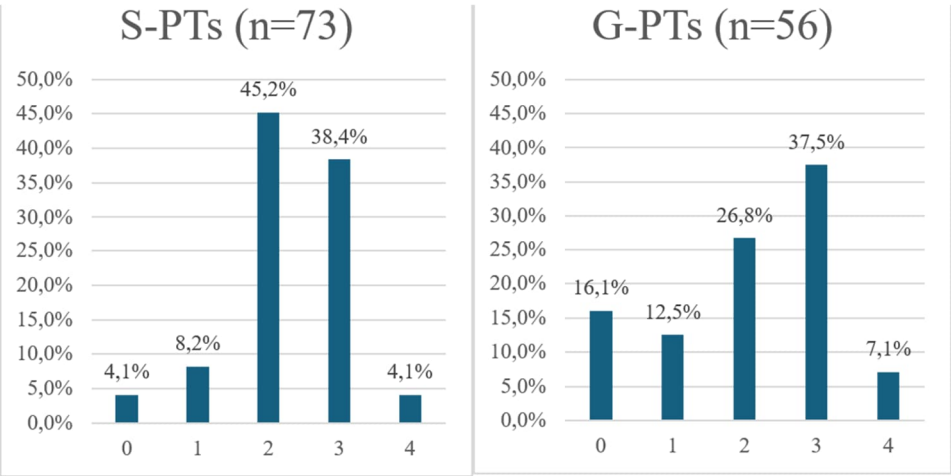
The answers coded by 1 displayed PT difficulties in formulating the problem/activity. For instance, S-PT32 proposed a problem related to the learning objective, but the formulation of the question shows difficulties. As the problem asks about "how many pieces", it can be solved using natural numbers without fractions. "We have two cakes. Maria has eaten $\frac{3}{6}$, and Juan has eaten $\frac{8}{6}$. How many pieces in total have they both eaten?" Also, S-PT26, who formulated a problem related to the learning objective, was coded by 1 since this PT showed difficulties regarding the meaning of improper fractions, when proposing "$\frac{6}{4}$ of a cake": "Irene went to a birthday party last week. All of her friends ate $\frac{6}{4}$ of a cake, but one of them had to leave early and took $\frac{4}{8}$ of the cake. How much cake did they all eat?"

Similarities and differences between PTs' decisions from the two countries

Figure 2 shows the frequencies of codes for each PT group. The results obtained are similar for both groups of PTs: 71.4% of the G-PTs and 87.7% of the S-PTs were able

to propose a problem related to the learning objective (codes 2, 3 and 4) while the rest of them (28.6% of the G-PTs and 12.3 % of the S-PTs) provided a problem or activity not related to the learning objective, or if related, showing mathematical difficulties.

Figure 2: Frequencies of codes in both PTs groups.



The main difference in frequency between the groups is observed for codes 0 and 2 (45.2% S-PTs vs. 26.8% G-PTs and 4.1% S-PTs vs. 16.1% G-PTs). However, if we examine deeply PTs responses, S-PTs proposed a problem without justifying the numbers in the context of the vignette (operation with fractions) but that allows to work the learning objective (connecting the graphical and symbolic representations) and G-PTs proposed a problem/activity without considering the context of the vignette (fraction operations), but this activity allows to work the learning objective (e.g., an activity of relating the graphical representation with the symbolic representation). Our results highlight how difficult it was for both S-PTS and G-PTs to explicitly provide arguments for how the numbers used will allow linking the graphical and symbolic representations or for how they will link the different representations (code 4).

DISCUSSION AND CONCLUSIONS

Results have shown that most of the PTs in both groups were able to provide decisions that allow linking the representations since they provided problems/activities according to the learning objective showing subtle differences. This fact might be related to the vignette-based learning experience of the PTs in both groups. The findings further suggest that co-designing vignettes with researchers/educators from different cultural backgrounds can ensure relevance as well as fairness (Zhang et al., 2025), and could allow for minimising differences between PTs from different cultural and institutional contexts.

Nevertheless, findings have also shown the difficulties PTs faced to justify the decisions they made, particularly, about the numbers used and how these numbers help link the different representations. These difficulties highlight the significance of noticing, and particularly of the aspect of decision-making, as a knowledge-based reasoning process and, therefore, the importance of using theoretical lenses (Ivars et

al., 2019) to help PTs acquire and use the mathematical knowledge needed to notice relevant aspect of a teaching-learning situation and to justify the decisions needed for planning follow-up teaching.

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