HOW A PRIMARY SCHOOL TEACHER ENHANCES DIFFERENT CORE PRACTICES DURING A PROFESSIONAL DEVELOPMENT COURSE

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To date, the research has focused on how mathematics teachers acquire isolated core professional practices. Hitherto unexplored, however, is how these practices develop jointly. This case study focused on how a primary school teacher developed different core practices during a professional development course on the teaching and learning of magnitudes. The study participant was selected based on the specific context of a single-teacher school. The teacher noticed students' mathematical thinking and anticipated students' answers integrating the lenses provided in the course (Hypothetical Learning Trajectories and curricular guidelines). The lesson planning, however, remained closely aligned with institutional requirements (curricular guidelines) and was therefore work context-sensitive.

INTRODUCTION AND THEORETICAL FRAMEWORK

Practice-based education for mathematics teachers aims to integrate knowledge needed to teach mathematics with the development of core professional practices (Grossman, 2018). Some of these core practices are: (i) analysing students' answers and designing activities based on their understanding; (ii) analysing and selecting mathematical activities; (iii) managing mathematical discussions within the classroom; (iv) planning a lesson; or (v), analysing one's own teaching practice.

Recent research has built an understanding of how pre-service or in-service teachers develop these core practices during teacher education programmes. Studies have been performed, for example, on the following subjects: the development of student mathematical thinking noticing (e.g., Weyers et al., 2024); the analysis and selection of mathematical activities (e.g., Amador et al., 2017); lesson design (Smith et al., 2018); or reflection on their own practice and that of others (e.g., Font et al., 2024; Moreno et al., 2025).

While an extensive body of research has focused on each core practice individually, a critical knowledge gap remains on how these core practices jointly develop during professional development courses. In the present work, we examined how different core practices evolved within a professional course on magnitudes and their measurement. This subject area was particularly relevant because these specific topics have been found to present teaching difficulties (e.g., Callejo et al., 2022).

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This study is part of a larger project aimed at characterising how in-service primary school teachers acquire core practices in Costa Rica. Specifically, we identified development characteristics of three core practices: noticing students' mathematical thinking; planning a lesson; and analysing one's own implementation (practice). In this research report, some of these characteristics are presented based on a case study.

Noticing students mathematical thinking and analysing one's own practice

The core practice of noticing students' mathematical thinking was conceptualised in conformity with Jacobs et al. (2010): (i) identifying important details in students' strategies; (ii) interpreting students' mathematical thinking considering the details identified; and (iii), deciding about subsequent instruction steps based on students' mathematical thinking. Previous research has contributed specific tools allowing to promote the acquisition of this core practice within teacher education programmes. For instance, the use of theoretical lenses, such as Hypothetical Learning Trajectories (HLT), helps to focus the teachers' attention on the nuances of teaching and learning situations (Fernández & Choy, 2020). Additionally, prompts and vignettes (Buchbinder & Kuntze, 2018) fulfil a scaffolding function, helping teachers to deepen their analysis of classroom situations.

Regarding the core practice of analysing one's own practice, we centred on students' mathematical thinking. We thus considered the same skills: identifying, interpreting and deciding. However, since teachers are present in the classroom, teachers are considered as "narrators of their own practice". Narratives help teachers to notice the mathematical thinking of their students. In the narrative, teachers describe a situation where students are learning a specific mathematics concept, interpret students' mathematical thinking, and decide about the subsequent instruction step to take to enhance their students' mathematical thinking (Ivars & Fernández, 2016).

Planning a lesson

When planning a lesson, teachers must design instructional activities that align with learning objectives and mathematical content. This process also requires anticipating student responses and assessing student understanding. In this study, we adapted the lesson planning protocol proposed by Smith et al. (2008) because this latter protocol emphasises the role of students' thinking.

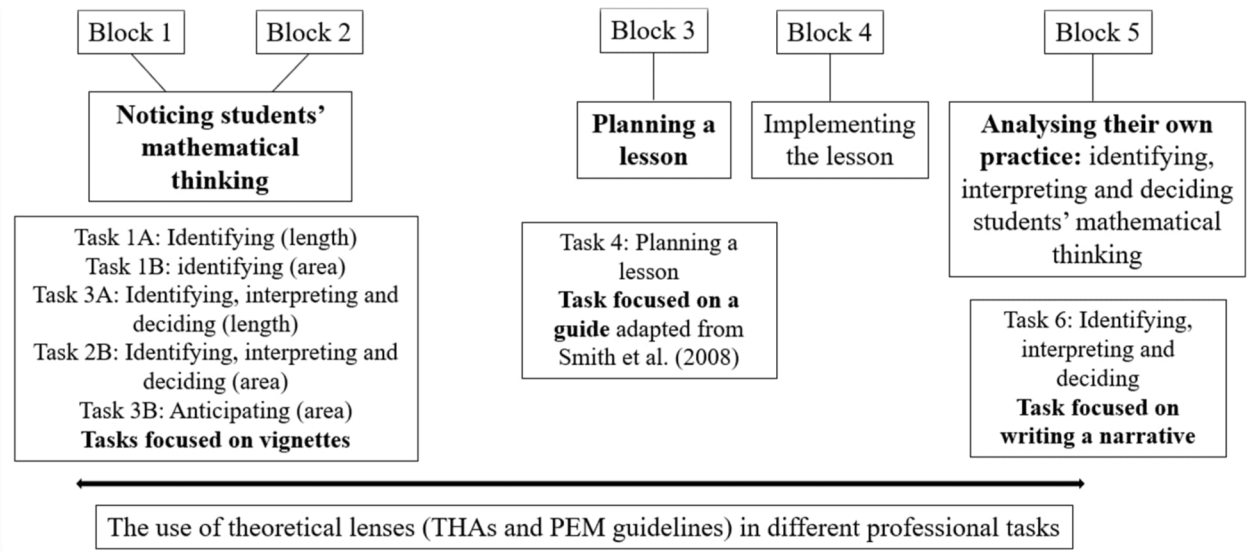
RESEARCH QUESTION

The study objective was operationalised through the following research question: How does an in-service teacher enhance these three core practices (analysing student mathematical thinking, planning a lesson, and analysing one's own practice) during a professional development course that integrates diverse theoretical lenses? More specifically, we focused on the subject area of length and area magnitudes.

METHOD

The participants consisted of seven in-service primary school teachers who were exercising within public education institutions in Costa Rica. These teachers attended a professional development course aimed at developing core practices for the mathematics teaching in the subject area of *Measurement*. Course participation was voluntary and free of charge. The 50-hour professional development course was structured into five blocks and integrated theoretical lenses and professional tasks. The theoretical lenses included a HLT focused on length magnitude and its measurement (Callejo et al., 2022) as well as a HLT on area magnitude and its measurement (Sarama & Clements, 2009) and the guidelines from the Mathematics Study Programmes of the Ministry of Public Education in Costa Rica (PEM). Each HLT included: (i) the learning goal; (ii) a hypothetical student learning process (students' levels of understanding); and (iii), a set of learning activities designed to help students progress through different levels of thinking (Simon, 1995). The professional tasks were designed to facilitate the enhancement of the specific core practices: noticing students' mathematical thinking (attending, interpreting, and deciding); planning a lesson; and analysing the lesson implementation focusing on the students' mathematical thinking (Figure 1).

Figure 1. General structure of the professional development course.



The methodological design was a qualitative case study. We focused on how a single teacher (DP) performed the professional tasks in order to characterise how this teacher integrated the HLTs and the PEM guidelines into the different core practices. DP was selected owing to the specific work context: DP worked at a single-teacher school, that is, DP was teaching all subjects across six primary education levels (from 1st to 6th grade) and was the school's head teacher.

The data was analysed in two phases. First, the teachers' task answers were examined by block in order to identify the teacher's characteristics regarding each core practice skill (identifying, interpreting and deciding student's mathematical thinking,

anticipating, planning, and identifying, interpreting and deciding their own students' mathematical thinking). These characteristics emerged through an inductive thematic analysis performed by three researchers. Any disagreements were discussed until a consensus was reached, ensuring the validity of the identified characteristics. Second, the teachers' general characteristics in each skill were collected throughout the course and compared in order to determine cases and subcases. In this research report, we present one of the cases.

RESULTS

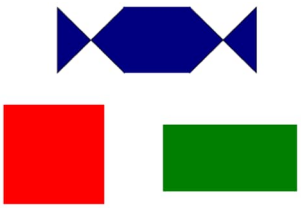
The results are presented in three subsections: first, how DP (the teacher) identified, interpreted, as well as decided about students' mathematical thinking (using vignettes) and anticipated student answers; second, how DP planned de lesson; and third, how DP identified, interpreted and decided about the students' mathematical thinking once the lesson had been implemented.

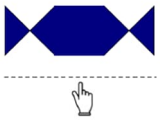
How DP identified, interpreted, decided and anticipated

The characteristics identified were: (i) DP identified important mathematical details in students' answers in both magnitudes; (ii) DP interpreted students' understanding based on the HLT information in both magnitudes; (iii) DP made instructional decisions aligned with the HLT, selecting learning objectives and corresponding mathematical activities in both magnitudes; and (iv), DP anticipated student answers with different levels of understanding based on the HLT. Evidence supporting these characteristics is presented below.

In the area magnitude vignette (Task 2B), DP identified important mathematical details in the students' answers. Based on these details, DP interpreted that neither José nor Sofia displayed any characteristic associated with the HLT levels of understanding (Table 1).

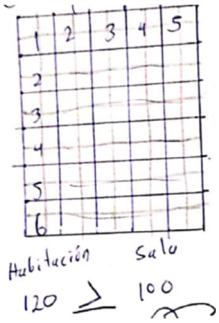
Table 1. Part of the vignette related to the area magnitude and the DP's answer.

Activity	Vignette	DP answer
Sort the figures according to their surface area. 	Teacher: Which shape has the largest surface area? Sofía: <u>The blue one, I think</u>. Teacher: José, do you agree with your classmate? José: Yes, if <u>we look at the length of the shape</u> (the image below shows what José is referring to), this is the largest.	This DP highlighted the sentences (adding the emphasis shown in the vignette) and wrote "students are not proficient in any element" Furthermore, this DP interpreted that "Sofia and José did not show any characteristic of the understanding levels since

		they did not recognise the surface attribute”

DP proposed the subsequent objective of “recognising the surface attribute” (mathematical element of the level 1 in the HLT) and noted that Sofia and José had not demonstrated an understanding of this characteristic yet. Moreover, DP provided an activity aligned with the objective: “The teacher shows the students several objects, and they must indicate their width, length and surface area.” In the anticipating vignette (Task 3B), DP anticipated for Anita (a student at HLT level 5) the answer in Figure 2: the use of rows and columns arrangement to count the number of measurement units in item *a;* the use of the inverse relationship to obtain the answer in *b*; and the use of the same measurement unit to compare in c.

Figure 2. Anita answer anticipation in the items *a*, *b* and *c*.

Sofia and her dad are going to lay the floor tiles in a bedroom. The surface area to be covered is a 5 m-long by 6 m-wide rectangle. a) If the tile sides are 1 square m, how many tiles will they need to cover the bedroom floor? b) If the tile sides are ½ square m, how many tiles will they need to cover the bedroom floor? c) If 100 tiles with sides measuring half a metre were used in the living room, which is larger, Sofía’s bedroom or the living room?	 a. 30-meter tiles b. 120 half-meter tiles c. Sofia’s room is larger	She can arrange rows and columns (image). Furthermore, because she understands the inverse relationship between the number and the unit measurement, she is able to deduce that half a meter can be represented by dividing each square in half (both in the columns and rows).

How DP planned a lesson

DP planned a lesson (Task 4) to develop length magnitude in 1st grade students. The plan followed the guidelines established in the PEM, yet it failed to incorporate the specific HLT progression. Specifically: (i) DP proposed a general learning objective that mirrored a general skill in the PEM; and (ii), DP designed an activity related to that general objective without aligning it with the mathematical elements and HLT levels.

Although DP noted that students in this grade should focus on magnitude perception and conservation (mathematical elements of the levels 1 and 2 in the HLT), DP proposed a general objective included in the PEM “understanding the measurement concept and enabling children to calculate, estimate, compare, and apply these concepts in different contexts.” Consequently, DP did not define a specific learning objective in accordance with the mathematical elements provided in the HTL. The

designed activity was aligned with the general formulated objective rather than with the identified HLT elements. The teacher suggested an activity where students throw a coin toward a line on the floor from a fixed starting point. Students were tasked with measuring the distance between their coin and the line, first using an anthropomorphic measure (their hands), and then a measurement unit provided by the teacher (a small paper strip). Finally, the teacher asked comparative questions: "Which coin landed furthest (closest) to the line? Which coins landed closest to each other?"

How DP identified, interpreted and decided about their own students' mathematical thinking

The main characteristics identified regarding DP's analysis of the students' thinking in Task 6 (own practice) were: (i) DP identified relevant mathematical elements in students' answers; (ii) interpreted students' understanding based on the HLT information; (iii), formulated learning objectives derived from that interpretation; and (iv), proposed specific activities to support and advance students' understanding.

DP identified the mathematical elements and understanding level of each student's answer and organised them in a table. The example of Justin and Joshua is presented in Table 2.

Table 2. Elements and level of understanding identified by DP for Justin and Joshua.

Mathematical elements	**Justin**	**Joshua**
Unit of measurement (select a single unit of measurement)	X	X
Iteration (iterate the unit without gaps or overlap)	X	X
Accumulation (count the number of iterations to give a measurement)	X	X
Level of understanding	4	4

DP observed that Justin and Joshua used identical mathematical elements as measurement units. Specifically, both students maintained an extended hand throughout all iterations – thereby selecting a single measurement unit –, iterated the unit appropriately, and counted the total number of iterations. DP provided an excerpt of the class dialogue as evidence (Figure 3).

Figure 3. Class dialogue excerpt (evidence provided by DP).

Teacher: What is the distance between the coin and the line if you use your hand? **Justin:** 5 hands (places his hand, repeats the action successively without overlapping or skipping, see image) **Teacher:** Very good. You can go and write down the data in the record table.	

As previously noted, DP interpreted the students' level of understanding. For example,

DP interpreted that Justin and Joshua displayed level 4 characteristics because they successfully applied the measurement unit (selection, iteration, and accumulation). DP indicated: "Justin used the unit of measurement, performed iterations, and applied accumulation; therefore, I placed him at level 4. Joshua demonstrated the same elements as Justin (iteration and accumulation)".

The DP proposed a follow-up activity that required students to select their own measurement unit from a set of options. DP argued that "the HLT level 4 needs to be reinforced" because, in the previous task, the students "did not choose the units of measurement themselves" since they were given a piece of paper as a measurement unit. This new activity was designed to address the uniqueness of the unit: "Students can measure objects such as the blackboard, the door width, or another classroom object with three different pieces of paper, but they must recognise and select a single unit of measurement."

DISCUSSION AND CONCLUSIONS

DP noticed the students' mathematical thinking (identifying, interpreting, and deciding) and anticipated student answers using the theoretical lenses provided in the professional development course (HLT and PEM guidelines). The latter was visible throughout the course and in the subsequent lesson analysis. However, a significant disconnect was observed during the planning phase: the teacher failed to use the HLT information to plan the lesson, focusing exclusively on the PEM guidelines. The reason may have been the dominant influence of ministerial requirements on lesson planning. These findings thus suggest that it remains a challenge for teachers to integrate different lenses in the planning – such as research-based trajectories and national curricula. Such results support previous findings according to which integrating HLTs and curricular guidelines can support the enhancement of core practices (Fernández & Choy, 2020), but this integration is non-uniform and complex.

Furthermore, considering DP's specific context (a single-teacher school), we found that planning a lesson in specific classrooms requires more than the integration of theoretical lenses (HLTs) and curricular standards (PEM). It also demands a deep consideration of the teacher's unique working context (Scheiner, 2023). In a multigrade classroom, the challenge is to adapt these theoretical and institutional frameworks to the simultaneous management of multiple educational levels and diverse student needs.

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