THE ROLE OF DRAWING TYPE FOR DRAWING CONSTRUCTION AND USE IN MATHEMATICAL MODELLING

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Different types of drawings influence their effectiveness in solving modelling problems, however, the underlying processes are still not fully known. This study investigates how a drawing's level of abstraction relates to students' drawing construction, drawing use, and modelling performance in geometry. Sixty-six ninth- and tenth-graders solved six geometry modelling problems with instructions to make a drawing while their gaze movements were recorded. Path analysis showed that students whose gaze movements indicated more effective drawing construction created more abstract drawings. More abstract drawings were indirectly related to better modelling performance via more effective drawing use. Thus, instructional support should focus on fostering both the construction of abstract drawings and their effective use in modelling.

INTRODUCTION

Many students have difficulties solving modelling problems (Galbraith & Stillman, 2006; Niss et al., 2007), which involve applying mathematics to real-world situations (Niss & Blum, 2020). Learner-generated drawing may support modelling, particularly in geometry (Bassi & Brunetto, 2025). However, its effectiveness depends, among other factors, on the drawing type, including its level of abstraction (Rellensmann et al., 2017). Yet, drawing type does not operate in isolation: a drawing's level of abstraction may emerge from how the drawing is constructed, and, in turn, shape how it is used. To date, little is known about how drawing type relates to the processes of drawing construction and drawing use and how they jointly contribute to modelling performance. In this study, we investigate the relationships between drawing type, drawing construction, drawing use, and modelling performance.

THEORETICAL BACKGROUND

Mathematical Modelling

An exemplary modelling problem, the *Fire brigade* problem, is displayed in Figure 1. In this task, students are required to model a real-world rescue situation by translating it into a mathematical model, using this model to determine quantities such as heights and distances, and relating the results back to the real-world context. Prior studies have demonstrated that students may have difficulties at various steps when solving modelling problems (Galbraith & Stillman, 2006; Niss et al., 2007), such as building an adequate representation of the problem structure or identifying the mathematical

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relationships needed to set up equations. Learner-generated drawing may support geometry modelling (Bassi & Brunetto, 2025), but its effective implementation remains challenging for learners.

Figure 1: The modelling problem *Fire Brigade* (Holzäpfel & Leiss, 2014)

Fire-Brigade
The Münster fire department operates 16 stations in the city center, ensuring that they never have to travel more than 6 km to reach a burning building. A central component of Münster's fire-fighting units is a turntable ladder truck that is 11 meters long, 2.55 meters wide, and 3.3 meters high. With its 30-meter-long ladder, the fire department can rescue people from the upper floors of burning buildings. The rescue is carried out using a basket attached to the end of the ladder. During a rescue operation, the turntable ladder truck must maintain a safety distance of 7 meters from the burning building and a 10-meter clearance at the rear of the vehicle to allow rescued persons to exit the basket without obstruction. From what height can the Münster fire department rescue people using the turntable ladder truck?

Learner-generated drawing

Learner-generated drawing refers to both the product, a visual representation of the problem created by the learner, and to the processes involved in constructing and using the drawing to facilitate understanding, exploration, and problem solving (van Meter & Firetto, 2013; Zimmermann & Cunningham, 1991). Empirical evidence on the effectiveness of learner-generated drawing for problem solving remains inconsistent. For instance, when students were asked to create drawings to solve geometry problems, de Bock et al. (2003) reported negative effects on students' problem-solving performance. As another example, Rellensmann et al. (2022) found no effect of drawing instructions when asking students to create situational drawings, but a positive effect for instructions to make mathematical drawings. Variations in students' drawings, particularly in drawing type, may partly account for mixed findings.

Drawing Type

Drawing type refers to the drawing's level of abstraction. In situational drawings, the drawing represents the objects visually according to their real-world appearance (e.g., a house with windows as exemplified in the left panel of Figure 2). In mathematical drawings, the drawing abstractly depicts the mathematically relevant features of the objects (e.g., figures and lines as exemplified in the right panel of Figure 2) (Rellensmann et al., 2017). Situational, mathematical, and mixed drawing types can accurately represent the problem (Rellensmann et al., 2017). Prior research indicates that higher levels of abstraction contribute to better performance (e.g., van Garderen et al., 2013). Hegarty and Kozhevnikov (1999) and Boonen et al. (2014) found that less abstract drawings impaired real-world problem-solving performance, whereas more abstract drawings enhanced it. One explanation is that situational objects misdirect attention to the object's irrelevant real-world features and away from the problems' mathematical structure, which hinders problem solving (e.g., Boonen et al., 2014). However, drawing type may be the result of how a drawing is constructed and shape how the drawing is used during problem solving. Thus, to understand the role of

drawing type in modelling, it requires considering drawing construction and drawing use alongside the drawing product.

Drawing Construction and Use

The Cognitive Model of Drawing Construction (van Meter & Firetto, 2013) describes effective drawing construction as a process involving identifying important information, the spatial and relational organisation of objects, their connection with prior knowledge, visual encoding, and externalisation through drawing. This process is iterative, as insights gained from the drawing provide feedback that enhances students' problem understanding, which in turn shapes subsequent refinements of the drawing. Effective drawing use involves utilising the drawing as a cognitive tool for problem solving such as extracting new information, recognising figures, shapes, or relational aspects, making inferences, and monitoring (e.g. Cox, 1999). Previous research suggests that the way a drawing is constructed shapes its type, which in turn affects how the drawing is used in problem solving. For example, Stylianou (2011) reported that students who relied on less abstract drawings tended to focus on superficial information and were distracted from the underlying mathematical structure. Students who effectively constructed and used more abstract drawings showed expert-like iterative behaviour, repeatedly abstracting, gaining new insights, and incorporating these into the drawing. Similarly, van Garderen et al. (2013) showed that students who constructed more abstract drawings were more likely to use them effectively, such as for identifying geometric figures, and achieved better problem-solving outcomes. However, it remains unclear how drawing construction, drawing type, and drawing use are related and how they contribute to modelling performance.

Research questions

In this study, we investigated the following research question: *How does drawing type relate to drawing construction, drawing use, and modelling performance?* Based on the theoretical background, we specified a path model (Figure 4) and tested the following hypotheses: (H1) More effective drawing construction predicts a higher level of abstraction in drawings. (H2) A higher level of abstraction in drawings predicts more effective drawing use. (H3) A higher level of abstraction in drawings predicts better modelling performance. (H4) The relationship between the drawing abstraction and modelling performance is indirectly mediated by drawing use.

METHODOLOGY

Participants and Procedure

Sixty-six ninth- and tenth-grade students participated in this study (30 female, 36 male). Following the completion of a demographic and motivation questionnaire, students solved six geometry modelling problems with instructions to make a drawing, using paper, pencil, a set square, and a calculator, while their eye movements were tracked. After excluding recordings due to technical issues, 342 drawings and solutions as well as recordings of drawing construction and use remained for the data analysis.

Some of the data used in this study has previously been examined by Baumann et al. (under review) and Schoenherr et al. (under review). While this earlier analyses focused on the relationships among drawing construction and use, drawing accuracy, modelling performance, and strategy-based motivation, the present analysis specifically investigates the relationships with drawing type.

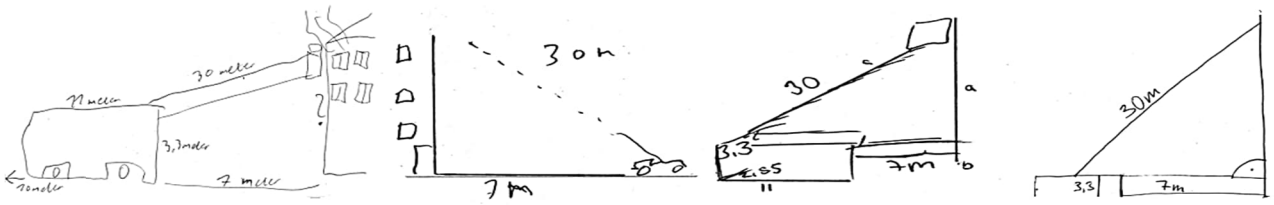
Materials

The materials consisted of an information sheet on drawings for modelling problems, along with six geometry modelling problems, each presented on an individual sheet of paper. One of these problems was the *Fire-Brigade* problem (Figure 1). Each worksheet required students to make a drawing for the modelling problem, solve the problem, and provide a final answer.

Scales

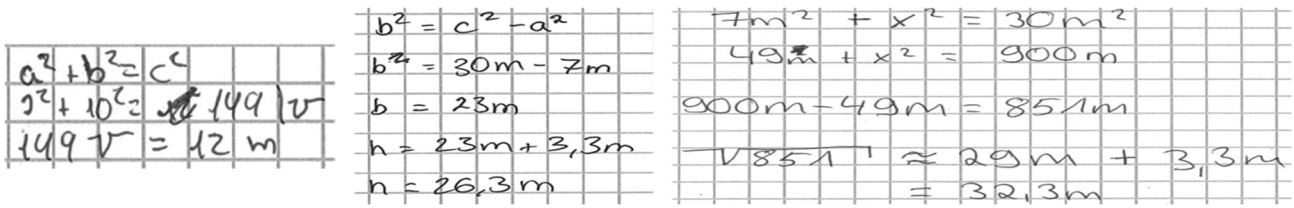
Drawing type was rated from 0 (all objects depicted as real-world objects) to 3 (all objects depicted as abstract objects). Interrater reliability was high, with Cohen's κ = .91, based on independent double coding of 35% of the data. Exemplary drawings and their coding are presented in Figure 2.

Figure 2: Drawing type coded with 0, 1, 2 and 3 from left to right



Modelling performance was assessed with scores ranging from 0 (inappropriate mathematical model) to 2 (appropriate mathematical model and correct result), where a score of 1 indicated calculation errors despite a correct mathematical model (Cohen's κ = .92, based on independent double-coding of 35% of data). Examples are presented in Figure 3.

Figure 3: Modelling performance coded with 0, 1 and 2 from left to right



Drawing construction and use

Eye-tracking data were collected using Tobii Glasses 3 (sampling rate: 50 Hz; gaze estimation accuracy: 0.6° under laboratory conditions) with the Tobii Attention filter. Average accuracy was determined using a nine-point calibration grid, yielding a spatial accuracy of 1.16°, equivalent to approximately 1.08cm on the worksheet. The eye-tracking glasses were recalibrated using a one-point procedure prior to each problem.

To assess drawing construction and use, we used three eye movement indicators, respectively. The construction and validation of these eye-tracking indicators and scales have been reported by Baumann et al. (under review). They were selected based on previous research, which suggests that fixation patterns and gaze transitions reflect core cognitive processes of drawing construction and use, such as selecting relevant information, integrating verbal and visual representations, and translating visual relations into mathematical expressions (e.g., Hellenbrand et al., 2019; Mason et al., 2013; Jian, 2018; Baumann et al., under review). The worksheet comprises four areas of interest: Text, including solution-relevant text, drawing, and solution area. *Drawing construction* was assessed using three eye-tracking indicators starting with the first reading and ending when the last drawn object: (a) proportion of gaze transitions between important text passages and the drawing area across all gaze transitions to and from the drawing area; (b) proportion of fixations on important text passages across all fixations in text; and (c) proportion of fixation time on important text passages and the drawing area across total fixation time (e.g., Hellenbrand et al., 2019; Mason et al., 2013). Across indicators, higher proportions indicate more effective drawing construction, reflected in stronger selection and organisation of relevant information (b, c) and more efficient verbal–visual integration (a, c). *Drawing use* was assessed using three eye-tracking indicators indexing the use of the drawing for problem solving: (a) proportion of gaze transitions between the drawing and the solution area across all transitions to and from the drawing area; (b) proportion of fixations on the drawing and solution areas across all fixations; and (c) proportion of fixation time on the drawing and solution areas across total fixation time (e.g., Jian, 2018; Mason et al., 2013). Across indicators, higher proportions indicate more effective drawing use, reflected in recognising figures and relations, making inferences from the drawing (b, c) and translating geometric relations into algebraic expressions (a).

RESULTS

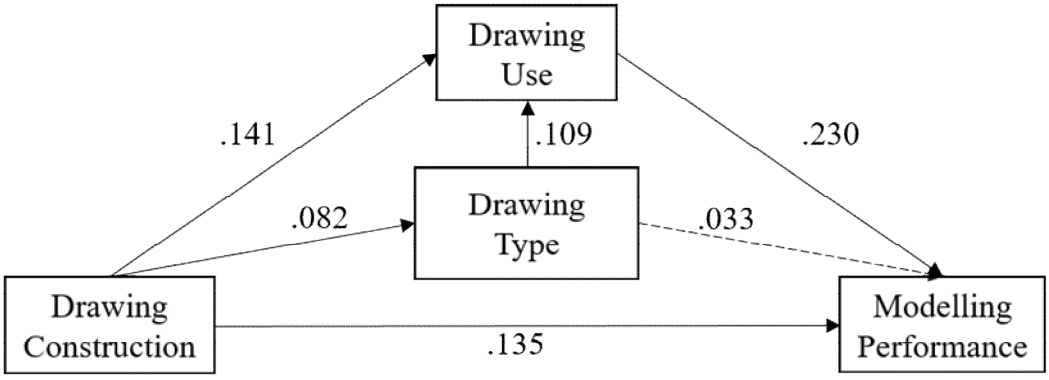
Table 1 shows the descriptive statistics and correlations for all measures.

Table 1: Descriptive statistics and Pearson correlations. *$p < .05$; **$p < .01$; *** $p < .001$ (two-tailed). Drawing construction and drawing use are standardized.

	M	SD	TYPE	CON	USE	PERF
Drawing Type (TYPE)	2.398	.944	1	.072	.121	.071
Drawing Construction (CON)	.000	.967		1	.150*	.171**
Drawing Use (USE)	.000	.952			1	.254***
Modelling Performance (PERF)	1.015	.952				1

To investigate our research questions, we tested the hypothesized path model (Figure 4). The hierarchical structure of the data, with multiple problem-solving processes being nested within students, was addressed by employing robust standard error estimation. Given our directed hypotheses, we used one-tailed tests.

Figure 4: Estimated standardized path coefficients for direct effects. Dashed lines indicate $p > .05$, solid lines indicate $p < .05$; p-values are one-tailed



Model fit could not be assessed, as the model was saturated ($df = 0$). Figure 4 illustrates the direct effects. In line with hypothesis 1, drawing construction had a positive direct effect on drawing type ($\beta = .082$, $p = .011$). That is, students who constructed their drawings more effectively created more abstract drawings, i.e., with fewer real-world objects. Consistent with hypothesis 2, drawing type had a positive direct effect on drawing use ($\beta = .109$, $p = .031$), indicating that students used abstract drawings more effectively than drawings including more real-world objects. Drawing type did not have a direct effect on modelling performance ($\beta = .033$, $p = .259$). Contrary to hypothesis 3, drawing type did not have a total effect on modelling performance ($\beta = .058$, $p = .140$). However, supporting hypothesis 4, drawing type had an indirect effect on modelling performance via drawing use ($\beta = .025$, $p = .042$). That is, students who created more abstract drawings used their drawings more effectively to solve the problems, which in turn was associated with higher modelling performance.

DISCUSSION

Our aim was to investigate the role of drawing type in constructing and using drawings to solve modelling problems. Path analysis showed that students who constructed their drawings more effectively produced more abstract drawings. One explanation is that effective drawing construction involves selecting, organising and integrating solution-relevant information from the problem text into a coherent drawing, as reflected by frequent gaze transitions between text and drawing, which in turn promotes the omission of less relevant real-world details. Furthermore, students used more abstract drawings more effectively to support the modelling process. This finding is in line with a case study of Stylianou (2011), who reported that abstract drawings supported recognition of geometric properties. Abstraction from irrelevant real-world details may allow students to focus attention on mathematical features and facilitate perceptual

insights. For example, a right-angled triangle may be more easily recognized when extraneous details like the windows and wheels of a fire truck in their drawing for the *Fire-Brigade* problem are omitted. Although drawing type did not show a total effect on modelling performance, it had an indirect effect via drawing use. This aligns with prior research indicating that mathematical drawings are one step toward better modelling performance, but that making an accurate drawing is also essential (Rellensmann et al., 2017). By linking students' drawing products with their drawing processes, this study provides new insights into the conditions that support effective drawing. Accordingly, instructional approaches should explicitly support both the construction and effective use of abstract drawings in mathematical modelling.

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