ELICITING THE DISCERNMENT OF VARIABLES IN YOUNG CHILDREN THROUGH BETTA-THE-BEE GAME

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Grounding on Variation Theory, this study examines how four-year-old children identify and manage critical aspects of visual stimuli during gameplay in a kindergarten setting, and how teacher scaffolding supports this process. Data were drawn from the implementation of the Betta-the-Bee card game, a mathematically rich game designed to engage children with the discernment of numeric and spatial variables depicted on the game cards. Findings from a case-study analysis indicate that structured, variation-rich games can effectively foster dynamic discernment processes, with teacher guidance playing a crucial role in supporting these emerging skills.

INTRODUCTION

Recognizing the critical aspects of mathematical objects is central to mathematical learning, as what learners understand depends on which features they discern and attend to when working on tasks or applying definitions. Research in mathematics education highlights the role of noticing—learners' selective attention to relevant features of situations and representations—in shaping mathematical understanding (Radford, 2000). Studies of task design and learners' activity highlight how the features students notice and generalize from tasks shape the meanings they construct, and how deliberate attention to critical aspects can support deeper understanding (Watson & Mason, 2006). Semiotic approaches further emphasize that understanding mathematical objects involves discerning invariant properties across representations (Duval, 2006).

From the perspective of Variation Theory (VT), learning is seen as becoming aware of the critical aspects that define an object of learning (Marton & Booth, 1997). For example, to understand what an altitude of a triangle is, it is not enough to memorise a verbal definition; rather, learners need to discern at least two invariant critical aspects: (i) the segment must be perpendicular to a side (or its extension), and (ii) it must pass through the opposite vertex. When these aspects are made visible through carefully chosen examples and non-examples (e.g., triangles in non-standard positions and misleading candidate segments), students are supported in distinguishing what is mathematically essential from superficial features of the drawing (Huang & Li, 2017, p. 17).

Grounding on this perspective, we argue that it is important to start fostering such processes of discernment from early childhood. If mathematical learning requires

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learning to "see" what matters—i.e., to identify the critical aspects that constitute an object and govern its use—then kindergarten should offer learning environments that are appropriate for young learners, in which these aspects may emerge through meaningful actions and accessible language. To this purpose, we designed mathematically rich (Russo, 2018) card games (they may be downloaded for free from the website https://www.dfedidamath.unito.it/home-page/giochi-di-carte). In this report, we focus on the card game Betta-the-Bee and examine the discernment of critical aspects with reference both to the design of the game and to its enactment in kindergarten setting.

THEORETICAL FRAMEWORK

Variation Theory conceptualises learning as a qualitative change in how an object of learning is experienced: learners come to understand "what counts" by discerning critical aspects of the object (Marton & Pang, 2013; Marton, 2015). In this view, learning is not primarily accumulation, but differentiation: when learners do not learn what was intended, it is because they have not discerned the necessary aspects. A distinctive tenet of VT is that seeing differences precedes seeing sameness: meanings are more likely to be acquired by experiencing differences against a background of invariance than by starting from sameness and adding differences (Marton & Pang, 2013). In other words, learners often grasp a concept more powerfully when they can contrast it with what it is not, rather than only comparing multiple similar instances.

Crucially, discernment presupposes the experience of variation: to notice an aspect, the learner must have encountered variation in that aspect (Kullberg et al., 2017). Teaching can thus be understood as the organisation of a space of variation—a set of dimensions and values through which critical aspects become available to notice and coordinate. Learning depends not only on which dimensions of variation are opened (that is, the content), but also on how they are opened, including their sequencing and simultaneity. Runesson (2006) refers to patterns of opened variation as a potential space of learning, emphasizing attention to which aspects are varied and which are kept invariant across the set of examples, tasks, representations, or interactions offered to learners.

Pang and colleagues describe four classic patterns, often used for instructional design and for post-hoc analysis, that specify how discernment can be supported (Pang & Ki, 2016; Pang et al., 2017): *contrast*, *generalization*, *separation*, and *fusion*. Contrast involves making a critical aspect discernible by juxtaposing what an object is and what it is not, or by comparing different values of a variable while keeping other aspects invariant. Separation refers to isolating a single dimension of variation by varying one aspect at a time, allowing learners to attend explicitly to that aspect. Generalization entails experiencing an invariant meaning or principle across a range of varied instances, so that learning is not bound to a specific example. Fusion involves the simultaneous awareness and coordination of multiple critical aspects, enabling learners to integrate them into a unified understanding (Huang & Li, 2017; Runesson, 2006). In Marton's account, separation functions as a higher-order pattern: a critical aspect

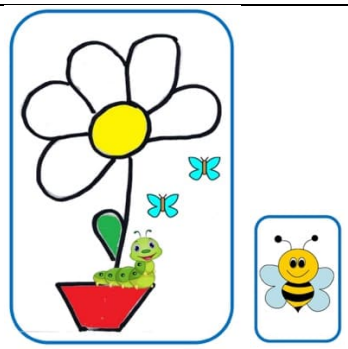
becomes discernible when variation is arranged so that the learner can distinguish what "makes a difference" from what does not; in this sense, contrast (seeing "what it is" against "what it is not") and generalization (seeing the same defining aspect across different instances) can be understood as complementary moves that together accomplish separation (Marton, 2015; Pang et al., 2017).

While VT is usually employed to focus on conceptual objects of learning (such as the altitude of triangle, discussed above) by structuring discernment, we use it here to focus on discernment itself and to analyse how it can be developed from early childhood through a purposely designed game, *Betta-the-Bee*.

THE BETTA-THE-BEE CARD GAME

Betta-the-Bee is a "Guess Who?"–inspired card game for young children (ages 4–7) in which players identify a selected card by asking yes–no questions. The deck includes 24 flower cards plus a smaller Betta card. Each flower card depicts the same overall scene (a flower in a pot, butterflies, a caterpillar, and a leaf), while a limited set of features varies across the deck. These varying features constitute the game key variables, each displayed through two or three modalities (Table 1). In terms of the Variation Theory, each variable represents a critical aspect of the flower card that players can experience through its dimension of variation, by contrasting and generalizing across the different modalities in which the variable is manifested. For example, the critical aspect "number of butterflies" is presented through the dimension of variation consisting of 0, 2, or 4 butterflies, with each modality depicted on different cards in the deck.

Table 1: One of the flower cards (left) and the Betta-the-Bee card (right)



Variables and modalities variation within the flower cards

Number of butterflies (0, 2, or 4)

Number of petals (3,4 or 5)

Position of the caterpillar (on the pot, on the leaf, or on the petal)

Position of the leaf (on the right or on the left of the stem)

Pot colour (red, blue or pink)

Gameplay plays a crucial role in prompting players' discernment of the cards' critical aspects, as the goal of the game is to discover under which flower card Betta is hidden by asking questions about those aspects. At the beginning of a round, one player takes the role of the hiding player and secretly hides the Betta card under one of the face-up flower cards, while the others act as seeking players. The seekers' goal is to identify the hiding place by asking questions about the cards' critical aspects. Crucially, each question must refer to a single critical aspect, which must be separated from the others and phrased so that only a "yes" or "no" answer is possible. After the hiding player

responds, the group deterministically eliminates cards: a “yes” implies that Betta is under a card that possesses the queried dimension of variation, so all cards lacking that dimension are discarded; a “no” implies that Betta is under a card without the queried dimension of variation, so all cards possessing it are discarded. The round ends when only one card remains; this card depicts the fusion of all the critical aspects addressed during gameplay. Across these phases—question formulation, answering, and discarding—the game engages children with core logical ideas such as the law of excluded middle (yes/no constraint), negation and material implication (rules for elimination), and emerging case-based reasoning, as players choose questions that efficiently reduce the space of possibilities (Casi et al., in press).

The present study draws on VT to address to investigate how discernment of critical aspects unfolds during gameplay in kindergarten settings. Specifically, it addresses the following research question:

What types of challenges do four-year-old children face in discerning and managing the critical aspects of visual stimuli during Betta-the-Bee gameplay, and how can teacher scaffolding support them?

METHODS

The *Betta-the-Bee* card game was designed by the ‘Matematica 011’ research group at the University of Turin, a mixed team including university researchers, kindergarten and primary teachers, and pre-service teachers. Its development followed principles inspired by Educational Design Research (EDR; McKenney & Reeves, 2019) through iterative cycles alternating research-group design meetings, experimental teaching sessions with children, and subsequent meetings in which the team discussed the experimental sessions. Across these cycles, classroom trials were audio- and video-recorded; audiovisual data and field notes were then used both to refine the game and related resources and to deepen understanding of the learning opportunities afforded to children by the activity.

In this paper, we adopt VT both as a design language (previous section) and as an analytic lens. In doing so, we use VT constructs to describe and interpret how variation may be generated by children as they explore, compare, and refine questions during play, with the guidance of the teacher. This approach aligns with the broader VT perspective that both teachers and students can open up dimensions of variation through individual and collective work (Runesson, 2006; Kullberg et al., 2017).

Data are drawn from an implementation session of the game in a kindergarten classroom, conducted by the classroom teachers with one of the authors participating as an observer. The selected session serves as the students’ first encounter with the game, allowing us to capture children’s initial discernment of critical aspects and to examine the teacher’s role in eliciting and supporting this process.

The study takes the form of a case study focusing on a subgroup of four children, for whom all relevant ethical procedures were fulfilled.

In our analysis, we use the patterns of variation as a descriptive and interpretive vocabulary for how mathematical meaning is made available through the game and the teacher–children interaction, rather than as a prescriptive script.

ANALYSIS

The data come from a session in which four four-year-old children encounter the game for the first time. We will call them *Fede*, *Marghe*, *Michi*, and *Zero*. The teacher (T.) guides the session, and the third author is present as a participant-observer (C.). After displaying the cards, the teacher focuses the children attention on the critical aspects of the deck through questions such as "What is showed on the cards?", "Can you see any similarities between some of the cards?", and "Are there any differences?". The rules of the game are then explained, and the children participate in a first round of play in which the teacher hides Betta and the children ask questions. The first question is asked by Michi, as reported below:

1	Michi:	Is it on the flower with three petals, and has it the blue pot? (*He is looking at a card with these critical aspects*)
2	T.:	No, you may ask only a question: with three petals? (*Michi remains silent*)
3	T.:	Or do you mean with the blue pot? Make a decision.
4	Michi:	Has it three petals?

In this first excerpt, a difficulty in the process of separating critical aspects becomes evident. While attempting to formulate a question, Michi simultaneously refers to two distinct critical aspects: the number of petals (dimension of variation: three petals) and the colour of the pot (dimension of variation: blue pot). The teacher intervenes by scaffolding Michi's reasoning, explicitly prompting the separation of these aspects and encouraging a focus on a single critical aspect when formulating the question.

After the teacher answers "yes" and asks what should be done next, Fede and Marghe begin counting the number of petals on different cards, demonstrating that they can separate the variable "number of petals" and check its modalities. Meanwhile, Michi suggests turning over the cards with three petals—precisely the opposite of the correct move. As noted in a previous study (Casi et al., in press) difficulties in selecting the cards to remove after a "yes" answer reflect challenges in interpreting logical aspects of negation. From the perspective of VT, this can be referred to difficulty in generalization: in this case, to discard cards players have to discern the complementary subset of the one that has the property named in the question.

The teacher scaffolds the children's reasoning, after which the game continues with another question regarding the critical aspect of the pot colour. Finally, Fede is invited to ask the next question:

5	Fede:	Betta the bee is under the card with the caterpillar? *(silent moment)*
6	C.:	The caterpillar is on all of them…
7	T.:	*(to Fede*) Instead, ask where the caterpillar could be.

8 Fede: Here *(pointing to a card with the caterpillar on the pot)*

9 T.: So, [the caterpillar] on...?

10 Fede: The pot

Since the caterpillar is present on all remaining cards, Fede asks an inefficient question—one that does not advance gameplay. In other words, at this point in the game, the critical aspect no longer lies in the presence or absence of the caterpillar (as it might have earlier), but in its position. The researcher's response implicitly signals that presence cannot serve as a contrasting aspect (line 6), and the teacher then scaffolds Fede's reasoning, guiding the reformulation of the question to focus on a variable that still allows meaningful variation (lines 7–9).

In subsequent plays, the children take turns assuming the role of hiding player. We focus on the final phase of one of these plays, when only three cards remain on the table (Figure 1), and the children again encounter difficulties in separating a critical aspect.

Figure 1: The cards left on the table

11 T.: Zero what do you notice? (*three seconds of silence*)

12 T.: So… the butterflies are everywhere, right? On all the cards… the petals are…?

13 Michi: All the same

14 T.: They all have four [petals]… So what could be different?

15 Zero: The caterpillar

16 Teacher: The caterpillar! What can we ask? Is Betta the little bee…

17 Zero: The caterpillar on the pot

Since Zero shows difficulties in formulating a question, the teacher begins by prompting a generalization through the observation of aspects that are no longer critical for the game, as all remaining cards share the same dimensions of variation (line 12): all cards display four butterflies and four petals. She then explicitly invites the children to look for differences (line 14). Zero attempts to isolate a critical aspect by referring to the caterpillar (line 15), but does not yet specify the relevant dimension of variation. With the teacher's support, the question is then formulated on one of the dimensions of variation of this aspect, namely the position of the caterpillar on the pot (line 17).

This allows one card to be discarded, leaving only two cards on the table (the first and the third cards in Figure 1).

CONCLUSIONS

The observational data indicate that four-year-old children faced several challenges during gameplay. One major difficulty is separating and focusing on individual critical aspects of the cards, a key process highlighted by Variation Theory. For example, Michi struggles to isolate a single variable, referring simultaneously to petals and pot colour (lines 1–4), showing difficulty in distinguishing which aspects to attend to. Fede encounters a different challenge by asking about a feature—the caterpillar—whose presence is not informative (lines 5–10), showing trouble in recognizing which variables still vary meaningfully. In the final phase of a round, Zero initially identifies a potential critical aspect but focuses on features that are uniform across the remaining cards (lines 11–14), highlighting challenges in detecting meaningful differences.

Across these episodes, the teacher's interventions revel crucial in helping the children overcome these challenges by guiding attention toward the relevant dimensions of variation, as seen when Zero successfully formulates a question about the caterpillar's position (lines 15–17). We need to remind ourselves that the analysed data come from the first encounter of the children with the game. Observation from subsequent sessions show that gradually children become able to formulate effectively the questions and logically eliminate the cards.

Through their engagement in the gameplay, children not only develop variable discernment, but also learn to apply it in purpose-directed actions—whether in formulating questions, answering them, or selecting cards to eliminate based on the responses. In other words, discernment is practiced as a fundamental process for playing and enjoying the game. This discernment must be continuously updated according to the game situation; we may therefore speak of a "dynamic discernment" fostered by the game dynamics. As the episodes have shown, young children encounter difficulties and challenges in this dynamic process and require teacher guidance to navigate it successfully. Nevertheless, we argue that this endeavour is worthwhile, as both mathematical processes and mathematics learning rely on the development of abilities related to dynamic discernment, including pattern recognition and generalization, logical reasoning with conditional statements, classification and sorting, and problem-solving with multiple variables. As it may too early to address these mathematics domain at the kindergarten level, our research shows that this is not the case with structured rich game such as Betta-the-bee.

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