UNIVERSITY-BASED MATHEMATICS TEACHER EDUCATORS' USE OF AN IDEATIONAL RESOURCE IN PROVIDING LESSON OBSERVATION FEEDBACK TO PRE-SERVICE TEACHERS

Nicola Bretscher[1], Jill Adler[2]

[1]University College London, [2]University of Witwatersrand

We investigate how university-based mathematics teacher educators (MTEs) use an ideational resource when providing feedback to pre-service teachers after observing them teach a lesson in secondary school. The specific ideational resource was a framework for teaching mathematics introduced into university-taught sessions of our pre-service teacher education programme to support lesson-planning. We selected two MTE-pre-service-teacher pairs as telling cases to illuminate when and how MTEs use elements of the framework in providing lesson observation feedback. For each pair, we analysed two lesson observations, drawing on notions of situated abstraction and transparency. We discuss our initial findings and implications.

INTRODUCTION

Research on mathematics teacher educators (MTEs) is of growing importance within mathematics education research (e.g. Barbosa, 2025). We investigate how university-based MTEs use an ideational resource in their written feedback provided to pre-service teachers after observing them teach a lesson in school. We define an *ideational resource* as a conceptual frame (Pepin & Gueudet, 2020) composed of ideas about mathematics and mathematics teaching intended to support teachers in their professional development. Ideational resources abound in teacher education: both mathematics-specific ideational resources, such as the Knowledge Quartet (Rowland et al., 2005) or the Mathematics Teaching Framework (MTF, Adler, 2021), and those reflecting general teacher competences, such as the Initial Teacher Training and Early Career Framework (Department for Education, 2024) in England. A central issue, then, is whether and how such ideational resources should be used, alone or in combination, in mathematics teacher education. In this paper, we aim to describe university-based MTEs use of a particular ideational resource, the MTF (Adler, 2001), for providing lesson observation feedback and identify differences in use over two lesson observations, as a starting point for addressing this issue. We previously applied the notions of *transparency* (Lave & Wenger, 1991; Wenger, 1998) and *situated abstraction* (Noss & Hoyles, 1996) as a theoretical framework for describing MTEs' use of the MTF in their lesson observation feedback (Bretscher et al., 2025). Our particular interest in this current work is in addressing the question: *How does the transparency framework enable description and identification of differences in the guidance on mathematics pedagogy provided by university-based MTEs?* In the next

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M.S. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: Research Reports*, pp. 219-226. PME.

section, we describe the transparency framework before describing our methods, results and discussing our initial findings and next steps.

THEORETICAL BACKGROUND

We draw on the notion of *transparency* (Lave & Wenger, 1991; Wenger, 1998) to describe the relationship (Adler, 2021) between a user and an ideational resource in practice. Resources, and perhaps ideational resources especially, are imbued with communal knowledge and practices. Thus, resource-use is described in terms of how *transparent* such embedded knowledge and practices are for the user. Transparency is underpinned by the dual notions of *visibility*, whether the ideas embedded in a resource are available for the user to "see", and *invisibility*, whether the user can "see through" these ideas to interpret and mobilise them in their own practice (Lave & Wenger, 1991, pp. 101-102). For the purposes of our study, we are interested in whether university-based MTEs recognise or "see" the practices highlighted by the MTF categories, naming them in their written feedback, and whether they "see through" these ideas, interpreting them in relation to the lesson observed.

In other words, where university MTEs make explicit use of MTF terminology in their written feedback, then the MTF is visible to them. To define the notion of invisibility, we draw on the idea of *situated abstraction* (Noss & Hoyles, 1996) which describes mathematical knowledge in teaching as at once embedded within specific situations, occurring in the context of mathematics teaching, and yet also 'abstract' or generalised across lessons or teaching episodes for example. An ideational resource can be said to be invisible, if the university-based MTE situates the 'abstract' or *general* ideas about mathematics teaching embedded in that resource, interpreting them in relation to the *specific* practices of mathematics teaching observed in the lesson. The transparency framework is summarised in Table 1.

Table 1: the transparency framework.

Transparency		Invisibility
		'sees through' the resource to a greater or lesser extent
Visibility	'sees' the resource	Explicit use of resource; Resource use is situated more or less in teaching
	does not 'see' the resource	Tacit or no use of resource; Resource use is situated more or less in teaching

In terms of identifying differences in the guidance on mathematics pedagogy over time, we might expect to see MTEs move towards more explicit use of the MTF that is situated to a greater extent in teaching as using the MTF becomes more embedded in their practice. However, the findings from Bretscher et al. (2025) indicate that the lesson context affects the transparency of an ideational resource and hence the degree to which an ideational resource supports MTEs to provide guidance on pedagogy.

METHODS

We briefly set out the context for the study, explaining how and why the MTF was introduced as an ideational resource into the pre-service teacher education programme at University College London, before describing our participants, data collection and analysis.

The purpose of introducing an ideational resource into our teacher education programme was to support pre-service teachers in focusing on core mathematics teaching practices, and their coherence when planning lessons. The MTF was chosen because the framework was developed for this purpose and for use with secondary school teachers in a context where mathematics teaching needed significant development. Hence the framework seemed appropriate in the context of our programme, where pre-service teachers arrive with varied mathematical backgrounds and with little or no experience of teaching mathematics in secondary schools. The MTF highlights that a lesson must have a learning goal, i.e. a specification of what learners must know and be able to do by the end of the lesson. This goal is mediated through three categories of core practices specific to mathematics teaching: *Exemplification* being the teacher's choice of mathematical examples, representations and tasks; *Explanatory communication* being what the teacher says and writes, including how their explanation is justified; and *Learner participation* being how the learners are invited to participate in doing and talking mathematics. Finally, the MTF emphasises that each of the three categories should be coherent in focusing learners' attention on the learning goal.

We selected two MTE-pre-service-teacher pairs as telling cases (Miles et al., 2020) of using the MTF in lesson observation feedback: the first pair was MTE1 and pre-service teacher Oliver; the second pair was MTE2 and pre-service teacher Mohammed. The cases differed in two important ways: firstly, MTE1 was part of the programme leadership that chose to introduce the MTF as an ideational resource, whereas MTE2 was part of the wider programme team. University-based MTEs are required to link their feedback to non-subject specific teacher competences. As such, there was no requirement that MTEs would use the MTF when providing feedback on lesson observations in school. However, an implicit goal of the programme leadership was that the MTF would inform MTEs' practices with pre-service teachers throughout the programme. Hence, we expected MTE1 might make more explicit use of the MTF in lesson observation feedback than MTE2. Secondly, MTE1 observed Oliver, a pre-service teacher who was perceived to have good mathematical knowledge, but his understanding of pedagogy was underdeveloped. By contrast, MTE2 observed Mohammed, a pre-service teacher who was perceived to have good understanding of general pedagogy, but his understanding of mathematics-specific pedagogy was underdeveloped. We thus expected the opportunities for focusing on core mathematics teaching practices might vary across the pre-service teachers' lessons.

For each pair, we collected the written feedback over two lessons, one lesson from each of the pre-service teachers' two practicum placements. The first observation usually takes place in November during the first practicum placement, and the second observation takes place during the second placement in March or April. MTEs' written feedback on lesson observations has four main components: (1) a brief note of context details about the lesson including the year group and lesson content, (2) real-time notes about what happened in the lesson, (3) the strengths of the lesson with regard to non-subject specific teacher competences as perceived by the MTE, and (4) the targets for development, again linked to non-subject specific teacher competences, as discussed and agreed by the MTE with the pre-service teacher. As such, one potentially interesting issue to explore is how MTEs' use of the two ideational resources interacts. In other words, if MTEs did use MTF categories in their feedback, we were interested in how these interacted with the non-subject specific teacher competences they were required to use.

As described in Bretscher et al. (2025), in our first step of data analysis, we broke down the text into lesson observation episodes and then into feedback instances within episodes. In terms of visibility, feedback instances were coded as having an explicit focus on the MTF if terminology from the MTF was used; an implicit focus if the feedback instance related to an MTF category but the framework terminology was not used; and no focus on the MTF if the feedback instance did not relate to the framework categories. In terms of invisibility, feedback instances were coded as mathematics-specific if they were fully-situated in mathematics pedagogy, where mathematical examples, vocabulary or learner actions related to the feedback instance were specified within the lesson observation episode. Feedback instances were coded as mathematics-general if they were partially-situated in mathematics pedagogy, where the instance related to mathematics pedagogy in general but not specified in terms of particular mathematical examples and so on. Finally, feedback instances were coded as general pedagogy where they were situated in general teaching practices, not specific to mathematics teaching. Strengths and targets were analysed in the same way and recorded separately. Coding was checked against Bretscher et al. (2025) for reliability over time. Our coding is exemplified in Bretscher et al. (2025).

RESULTS

We first present the overall results of analysing the feedback instances from MTE1's observation of Oliver's two lessons across his first and second placement. We then similarly present the results of analysing MTE2's observation of Mohammed's lessons. In each case, we relate these results to the lesson context, drawing on the strengths and targets for development for the pre-service teacher as identified by the MTE.

Table 2 and 3 show the summary of analysing the real-time notes in MTE1's written observation feedback of Oliver's lessons in the first and second school placement respectively. In both lessons, there were feedback instances in which MTE1 focussed on an MTF category, situated in mathematics-specific pedagogy. However, there is a

difference in that MTE1's observation feedback in the second placement has a more explicit focus on the MTF than the first placement. In the first placement, only 3 out of 10 feedback instances with any kind of focus on the MTF made that focus explicit. By contrast, in the second placement, 6 out of 10 feedback instances with any kind of focus on the MTF made that focus explicit. In both lessons, the majority of feedback instances related to general pedagogy rather than having a subject-specific focus.

Table 2: Summary of analysis of MTE1's observation feedback on Oliver's lesson in the first practicum placement.

	Transparency	Invisibility: 'sees through' MTF		
		Maths-specific	Maths-general	General pedagogy
Visibility: 'sees' the MTF	Explicit focus on MTF	3	.	.
	Implicit focus on MTF	4	2	1
	No focus on MTF	.	.	16

Table 3: Summary of analysis of MTE1's observation feedback on Oliver's lesson in the second practicum placement.

	Transparency	Invisibility: 'sees through' MTF		
		Maths-specific	Maths-general	General pedagogy
Visibility: 'sees' the MTF	Explicit focus on MTF	4	2	.
	Implicit focus on MTF	2	1	1
	No focus on MTF	.	.	15

The context of the two lessons is important in understanding these results. Oliver's first lesson was significantly disrupted by behavioural issues, obscuring the mathematical content of the lesson, hence much of the feedback necessarily focussed on his handling of these issues. As such, all the targets for development from this first lesson related to classroom management, with no focus on the MTF, e.g. one such target was "use praise to reinforce expected behaviours". In his second lesson, Oliver's classroom management had improved, allowing MTE1 to focus instead on the mathematical aspects of the lesson. In the second lesson, Oliver was teaching pupils how to solve linear equations with the unknown on both sides. As a strength, MTE1 noted that Oliver's "modelling of solving linear equations was consistent and enabled pupils to succeed" (implicit *Explanatory Communication*, maths-general). However, the subsequent tasks he posed pupils increased in difficulty too quickly, e.g. including unknowns with negative coefficients, leaving pupils confused. Thus, as a target, MTE1 that in future Oliver should "choose mathematical examples with deliberate variation" and provided a set of equations to illustrate this target (explicit *Exemplification*, maths-specific). Behaviour management was also noted by MTE1 as a strength in the second

lesson: the high number of feedback instances relating to general pedagogy serving to capture the progress Oliver had made in this respect.

Tables 4 and 5 show the summary of analysing the real-time notes in MTE2's written observation feedback of Mohammed's lessons in the first and second school placement respectively. Again, in both lessons, there were feedback instances in which MTE2 focussed on an MTF category, situated in mathematics-specific pedagogy. However, there is no clear difference in feedback between the first and second placement, in terms of explicit focus on the MTF, as with MTE1. Instead, the pattern of feedback instances is similar with a slight decrease in the proportion of feedback instances with an explicit rather than implicit MTF-focus. For example, in the first placement, 6/16 (38%) of feedback instances with any kind of focus on the MTF made that focus explicit. In the second placement, this dropped to 3/13 (23%) of feedback instances with any kind of focus on the MTF made that focus explicit. The number of feedback instances relating to general pedagogy remained constant across the two lessons.

Table 4: Summary of analysis of MTE2's observation feedback on Mohammed's lesson in the first practicum placement.

	Transparency	Invisibility: 'sees through' MTF		
		Maths-specific	Maths-general	General pedagogy
Visibility: 'sees' the MTF	Explicit focus on MTF	2	4	.
	Implicit focus on MTF	6	4	.
	No focus on MTF	.	3	8

Table 5: Summary of analysis of MTE2's observation feedback on Mohammed's lesson in the second practicum placement.

	Transparency	Invisibility: 'sees through' MTF		
		Maths-specific	Maths-general	General pedagogy
Visibility: 'sees' the MTF	Explicit focus on MTF	2	1	.
	Implicit focus on MTF	4	3	3
	No focus on MTF	.	.	8

Again, the context of the two lessons is important in understanding these results for MTE2's observations of Mohammed. In the first lesson, the strengths MTE2 noted were predominantly about general pedagogy, focusing on "positive relationships" with pupils and other school colleagues. However, all three targets contained an explicit or implicit MTF-focus: two related explicitly to *Explanatory Communication*, focusing on improving the mathematical content of Mohammed's board work and his use of mathematical language; the third related implicitly to *Exemplification*, focusing on introducing contextualised (rather than context-free) examples when teaching about data. These strengths and targets align with MTE2's stated focus for the observation

on the general teacher competences entitled *positive relationships*, *knowledge and curriculum* and *pedagogy and planning* (note MTE1 did not state a focus). In the second lesson, MTE2's stated focus changed to the general teacher competences entitled *pedagogy and planning*, *assessment* and *inclusive teaching* as is typical practice in the second placement. For both MTEs, the *knowledge and curriculum* and *pedagogy and planning* competences were most aligned with an explicit focus on the MTF whereas *assessment* and *inclusive teaching* were more likely to align with an implicit or no focus on the MTF and *positive relationships* only with no MTF-focus. Hence the slight drop in proportion of feedback instances with an explicit MTF-focus was likely related to the change in MTE2's observation focus. Three out of five strengths in Mohammed's second lesson included an MTF focus: two of these were MTF-explicit and linked to *pedagogy and planning;* the third was implicit and linked to *assessment*. All three targets linked to either *assessment* and *inclusive teaching*: only one of them contained an MTF focus and this was implicit.

DISCUSSION AND CONCLUSION

Our first finding is that the transparency framework was productive in describing and identifying differences in how university-based MTEs use an ideational resource to provide guidance on mathematics pedagogy to their pre-service teachers. MTE1 made more explicit use of the MTF in their feedback on Oliver's second lesson observation compared to the feedback on the first lesson and compared to MTE2's feedback on Mohammed's lessons. The explicit MTF focus in MTE1's second lesson observation feedback highlighted Oliver's choice of examples and consistent modelling of solving linear equations, linked to general teacher competences of *knowledge and curriculum* and *pedagogy and planning*. Similarly, in Mohammed's first lesson, where MTE2's stated focus was on *knowledge and curriculum* and *pedagogy and planning*, a greater proportion of MTE2's feedback instances with a focus on the MTF were explicit.

Our second finding is that understanding the reasons underlying differences in how MTEs use an ideational resource are complex and dependent on several factors. These factors include the lesson context, perceptions of pre-service teacher's progress and interactions between ideational resources. For example, disruptive behaviour in Oliver's first lesson meant that feedback focused on general pedagogy with no MTF-focus was foregrounded, hence guidance with an explicit MTF-focus was backgrounded. The lesson content of Oliver's second lesson (solving linear equations) made it easier for MTE1 to record his choice of examples in written feedback and hence make *Exemplification* an explicit focus compared to the topic of loci in Oliver's first lesson or to stem-and-leaf diagrams and frequency tables in Mohammed's first lesson.

These findings contribute to understanding how ideational resources are used in mathematics teacher education. We have shown that the transparency framework reveals differences between MTEs' use of a specific ideational resource, the MTF. These differences are important in providing opportunities for reflection on resource-use to encourage more deliberate use of ideational resources with the long-term goals

of making core practices explicit and developing a shared language of mathematics teaching. For example, differences in MTF-use related to the interaction between general teaching competences and mathematics pedagogy create an opportunity to reflect on which practices are foregrounded and what is backgrounded by each ideational resource. Such reflections can then support deliberate decision-making about when and how it is useful to use these ideational resources, alone or in combination, in mathematics teacher education.

Acknowledgements

We gratefully acknowledge the support of pre-service teachers and MTEs on the PGCE Secondary Mathematics programme at the Institute of Education, University College London in carrying out this research.

References

Adler, J. (2021). Levering change: the contributory role of a mathematics teaching framework. *ZDM Mathematics Education*, *53*(6), 1207–1220. https://doi.org/10.1007/s11858-021-01273-y

Barbosa, J.C. (2025). Bridge-building as a dimension of mathematics teacher educators' expertise. In C. Cornejo, P. Felmer, D. M. Gómez, P. Dartnell, P. Araya, A. Peri, & V. Randolph (Eds.). *Proceedings of the 48th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 83–90). PME.

Bretscher, N., Adler, J., Saunders, P., & Ghosh, S. (2025). Exploring university tutors' use of a mathematics teaching framework when providing lesson observation feedback in initial teacher education. In T. Fujita (Ed.), *Proceedings of the British Society for Research into Learning Mathematics*, 45(3).

Lave, J., & Wenger, E. (1991). *Situated learning: legitimate peripheral participation*. Cambridge University Press. https://doi.org/10.1017/CBO9780511815355

Miles, M.B., Huberman, A. M., & Saldaña, J. (2020). Qualitative data analysis: a methods sourcebook. Fourth edition. SAGE.

Noss, R., & Hoyles, C. (1996). Windows on mathematical meanings: Learning cultures and computers. Kluwer Academic Publishers.

Pepin, B., & Gueudet, G. (2020). Curriculum resources and textbooks in mathematics education. In S. Lerman (Ed.), *Encyclopedia of Mathematics Education* (pp. 172–176). Cham: Springer International Publishing. https://doi.org/10.1007/978-3-030-15789-0_40

Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education*, *8*(3), 255–281. doi:10.1007/s10857-005-0853-5

Wenger, E. (1998). *Communities of practice: learning, meaning, and identity*. Cambridge University Press. https://doi.org/10.1017/CBO9780511803932