INSTRUMENTAL GENESIS IN LINEAR ALGEBRA: A CASE STUDY OF A STUDENT WITH VISUAL IMPAIRMENT

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This research report examines instrument-mediated activity in linear algebra learning in a non-visual context. Using the instrumental approach, we analyse a task-based interview with a student with visual impairment solving matrix-related tasks. The analysis focuses on processes of instrumentation and instrumentalisation in accessing mathematical structure through sequential representations. Results show how digital artefacts are transformed into cognitive instruments supporting control and coherence of mathematical meaning. The study suggests that non-visual contexts pose explicit challenges for linear algebra that are often implicit in visually mediated learning.

INTRODUCTION

Students with visual impairments face numerous challenges during schooling, which can negatively affect their academic trajectories. In particular, when they enrol in university, they may encounter a series of barriers: students with visual impairments achieve significantly lower academic results than their sighted peers, and this gap is most pronounced in mathematics (Mason & McCall, 2013). Historically, STEM subjects have been inaccessible to students with visual impairments. However, there is no evidence that visual impairments lead to reduced quantitative reasoning abilities, suggesting that poor mathematical aptitude is not the cause of these outcomes.

THE ROLE OF THE SIGHT IN MATHEMATICAL LEARNING

Blind people rely primarily on touch to learn about reality, resulting in limitations in the availability and accessibility of certain types of stimuli compared to visual experience (Radford, 2013). Furthermore, the lack of visual mediation limits access to social practices that are fundamental to learning, such as imitation and social reference, which are often already compromised by the reduced opportunities for interaction typical of some disability contexts (Hodges & Keller, 1999).

According to De Freitas and Sinclair (2014), the loss of one sense does not simply result in a deficiency, but can alter the way in which the other senses are involved, potentially generating new perceptual opportunities. From this perspective, even mathematics itself changes depending on different sensory organisations (De Freitas & Sinclair, 2014). In the context of access to mathematics for blind people, keyboard input has long been considered dominant, but it is not the only option. In this regard, tactile gestures have been identified as a promising way to enable blind students to do mathematics using mobile devices (Abrahamson et al., 2019).

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In terms of output, Braille is generally the most effective reading mode for those blind people mastering it; however, little mathematical content is available in Braille, and the plurality of existing codes poses compatibility problems (Archambault et al., 2007). Added to this are institutional barriers, as most mathematical material is still available exclusively in paper format. Even when content is accessible in digital format, human readability remains problematic: visual representation does not always explicitly convey mathematical structure, requiring additional strategies for comprehension.

Reading and navigating mathematical expressions without visual support requires a significant amount of time and cognitive effort, especially when trying to maintain an overview during calculations that involve frequent returns to intermediate steps (Maffia et al., 2025). Standard mathematical representations are predominantly two-dimensional and visual-spatial, relying on complex graphic conventions (indices, fractions, roots) that convey structural information difficult to render in linear formats. Braille representations and speech synthesis cannot exploit the two-dimensional layout and necessarily operate sequentially, requiring alternative strategies to make the structure of mathematical expressions accessible. Organising calculations in such a way as to preserve their overall consistency is a significant challenge for all students, but it is particularly complex for those who only have access to sequential information (Archambault et al., 2007). To address these critical issues in educational design for students with diverse sensory abilities, it is necessary to question established assumptions about mathematical learning and the role of sensory perception.

LINEAR ALGEBRA EDUCATION

Linear algebra, together with calculus, is one of the main mathematical disciplines taught in science faculties. In recent decades, the teaching of linear algebra has become an active field of research in several countries (Pedersen, 2007). Linear algebra is generally the first course that students encounter as a complete mathematical theory, i.e. systematically constructed from the ground up, with all its rigour in making all assumptions explicit, justifying statements with reference to definitions and facts already proven. Therefore, at a certain level, students' difficulties with linear algebra simply stem from their inexperience with proofs and theories based on proofs. The other important aspect of a mathematical theory is its generality. Knowing linear algebra at this level requires the student to start thinking about the objects and operations of algebra not in terms of relationships between particular matrices, vectors, and operators, but in terms of whole structures of these elements.

Research in tertiary mathematics education initially focused on calculus, but over the last 20 years, more and more studies have been conducted on the teaching of linear algebra (e.g., Dorier, 2003). The research available on mathematics teaching, while important, has not yet fully accounted for the challenges of learning linear algebra.

Real and lasting improvement requires a collective effort and a flexible, personalised approach, especially when dealing with students with visual impairments. When

teaching students with visual impairments, it is necessary to rethink previously established practices and to propose teaching methods that are effective for these learners. In fact, what may be intuitive for a sighted student may not be immediately so for a student with visual impairments who, in addition to the difficulties mentioned above, must also cope with their disability (Abrahamson et al., 2019). Some objects of linear algebra, such as matrices, are designed for sighted students and, since Braille readers access them sequentially through codes such as LaTeX (Maffia et al., 2025), without adequate support from teachers, could be real obstacles for a blind student.

THEORETICAL FRAMEWORK

This work falls within the theoretical framework of the instrumental approach, which aims to understand how individuals use and transform artefacts into instruments capable of mediating cognitive and practical activities (Rabardel & Samurçay, 2001). As Rabardel states, an artefact is not an instrument in itself, but becomes one through the activity of the subject (Vérillon & Rabardel, 1995).

The relationship between the subject and object of the action is generally mediated by instruments: any action on a vector or a matrix by a blind subject is mediated by codes read through a screenreader. The IAS (Instrument–Activity–Situation) model by Vérillon and Rabardel (1995) describes this mediation through the dynamic interaction between three poles: the subject (S), the instrument (I) and the object of the activity (O). The instrument mainly performs a mediating function with respect to the object, although within a system of reciprocal relationships.

Rabardel distinguishes between different forms of mediation: epistemic, oriented towards understanding the object; pragmatic, oriented towards its transformation; and reflective or heuristic, which concerns the subject's relationship with itself mediated by the instrument (Rabardel & Samurçay, 2001). Each instrument can potentially support multiple forms of mediation, giving rise to phenomena of multimediacy. Within this framework, artefacts play a central role in cognition. The interaction between thought and artefacts is bidirectional: artefacts transform the way we think and solve problems, while human activity modifies and reinvents the use of artefacts. Instrument-mediated activities are therefore characterised by a close interdependence between knowledge and action. To analyse them, Rabardel introduces the concept of *usage schemes*, inspired by Piaget's theory of schemas (1977), which includes both domain conceptual knowledge and knowledge related to the use of the instrument.

The transition from artefact to instrument occurs through two complementary processes: *instrumentation*, which concerns the development of usage schemes by the subject, and *instrumentalisation*, which concerns the adaptation and transformation of the artefact according to the user's activities (Rabardel & Samurçay, 2001). These processes jointly contribute to instrumental genesis, a dynamic and continuous process of appropriation, adaptation, and innovation of the instrument (Rabardel & Bourmaud,

2003). Thus, the study of an instrument becomes the study of its genesis of meaning for a given user and a given purpose (White, 2008).

In light of this theoretical framework, this research examines how students with visual impairments choose, use, and adapt instruments and instrument systems in learning linear algebra, with particular attention to artefacts that ensure accessibility to the mathematical content under consideration. The focus is on the emergence of instrumentation and instrumentalisation processes in non-visual contexts. This leads to the following research question: how do students with visual impairments choose, exploit, and adapt the instruments (or instrument systems) available (and necessary) to them when learning linear algebra?

METHODS

The research adopts a qualitative methodology based on task-based interviews, considered an effective instrument for investigating students' cognitive processes during the resolution of mathematical tasks (Cohen et al., 2002), and consistent with the research question that focuses on understanding students' actions and reasoning. Interviews allow data to be collected through multiple communication channels (verbal and non-verbal) and enable in-depth exploration of students' reasoning. Task-based interviews focus on observing students' actions, strategies, and interactions with tasks and instruments, rather than on the correctness of final results. The interviewer interacts with the subject by asking clarifying questions and following a semi-structured protocol adapted to emerging mathematical reasoning (Goldin, 2012). Observations include language, gestures, writing, actions on external materials, and non-verbal expressions and are supported by audio-video recordings enabling in-depth analysis. The interviews are designed according to the methodological principles proposed by Goldin (2012), with particular attention to controlling variables (mathematical content, linguistic complexity, task structure, materials, and time) and distinguishing between observations and inferences, to ensure rigour, replicability, and generalisability of results. The tasks are chosen to be meaningful, appropriate to the level of participants, and capable of revealing mathematical structures, conceptual understanding, and communication dynamics.

The analysis of the interview uses a descriptive ethnographic approach, motivated by the limited literature on the topic. It aims to collect rich data on interviewees' personal experiences, the problem-solving strategies they adopt, and the material and digital instruments they use. The interviews were recorded (audio, video and screen), fully transcribed, and annotated with elements of non-verbal communication. This enriched transcript forms the reconstructed narrative presented in the following section.

The protocol involves solving five linear algebra tasks on matrices and two final questions aimed at explaining the use of the instruments. No time or instrument constraints were imposed, and the materials were provided in accessible formats (txt, LaTeX, and PDF). The tasks were selected to investigate both procedural and

conceptual aspects, with particular attention to the specific difficulties encountered by students with visual impairments.

Three adults with visual impairments, each with different scientific backgrounds and educational paths, volunteered to participate and were selected to observe instrumental genesis that had developed through their extensive mathematical experience. For reasons of space, this paper focuses on the analysis of a single exercise solved by one of the participants, Ale Micarti.

Ale Micarti has a degree in Computer Science and a congenital visual impairment. He learned LaTeX during his bachelor's degree, knows Braille, and uses an eight-dot Braille-bar. During the interview, Ale Micarti used a text editor to solve the exercises. He explains that if he had to take a complete course on matrices, not just a few tasks, he would probably customize it with a series of more convenient commands to minimize the use of keyboard arrows when navigating the files. To read the interview text and perform the tasks, Ale Micarti used a Braille display combined with a computer keyboard. Ale Micarti moves easily between documents and reads quickly. He performs calculations mainly in his head, but explains that, in other circumstances, he would have used more compact notations to fit and read everything on a single line.

ANALYSIS

The task analysed in this contribution requires calculating the determinant of three matrices and verifying the existence of the inverse (the inverse matrix exists when the determinant is not zero), which may need to be determined. This task provides a meaningful context for analysing the processes of instrumental genesis according to Rabardel's theoretical framework. Ale Micarti's activity clearly shows how generic (non-assistive) technological instruments have been progressively transformed into cognitive instruments that are functional for mathematical activity.

Ale Micarti begins by reading the instructions with the cursor and Braille display. He copies the matrix into a work file, thus facilitating access during subsequent calculations and optimising the use of the available instruments. This behaviour highlights a process of instrumentation, understood as the construction of usage schemes that allow the subject to orient themselves in the task. According to Vérillon and Rabardel (1995), the instrument is configured as a mixed entity composed of the artefact and the associated usage schemes, and these schemes emerge progressively in the interaction with the mathematical task.

Ale Micarti recognises the size of the matrix, but since he does not immediately remember the algorithm for calculating the determinant, he initially calculates it incorrectly. The interviewer's intervention allows Ale Micarti to recognise the error and correct it. In this step, it was fundamental that Ale Micarti gradually wrote down the intermediate results he had done in his head in a line of the text editor. This transformation of the text is a clear example of instrumentalisation, as the subject acts directly on the artefact, adapting it to their own cognitive needs. As Rabardel points

out, instrumentalisation concerns the transformation of the artefact by the subject according to their own purposes (Vérillon & Rabardel, 1995). In this case, the text editor is reconfigured as an accessible mathematical space, optimised for sequential reading on a Braille display, and primarily supports epistemic mediation, as it enables Ale Micarti to understand and control the mathematical object through externalised symbolic representations.

When calculating the determinant, as previously mentioned, Ale Micarti performs most of the operations mentally, verbally expressing only the intermediate results, which are then entered into the text editor. This selective use of the instrument is indicative of a high level of internalisation of mathematical procedures. Nevertheless, the instrument does not lose its role: it supports pragmatic mediation, allowing reasoning steps to be externalised and a stable trace of the solution process to be maintained, while simultaneously interacting with the interviewee's mental activity – a clear instance of multimediacy. Externalisation makes complex cognitive processes visible and controllable (Vérillon & Rabardel, 1995).

Another important aspect is the sequential exploration of the matrix rows using the Braille display. The lack of a global two-dimensional representation is compensated for by systematic navigation schemes, which allow Ale Micarti to mentally reconstruct the matrix structure. These schemes form the core of the instrument, as their use mediates the relationship between the subject and the object of the activity (Rabardel & Bourmaud, 2003). In this sense, we observe the result of a process of cognitive instrumentalisation, in which the subject has adapted their own reasoning strategies to the structural constraints of the artefact.

DISCUSSION

The analysis of Ale Micarti's activity triggered by the determinant task shows how the processes of instrumentation and instrumentalisation are deeply intertwined, forming the basis of instrumental competence developed by an expert subject. The text editor and Braille display are not mere technical aids for accessibility, but are fully integrated into the subject's mathematical activity, instantiating Rabardel's principle that instruments are constructed in action. This aligns with previous research, showing how visually impaired students develop sophisticated instrumental patterns to manage mathematical structures (Maffia et al., 2025).

The choice to perform calculations mainly mentally, limiting the use of the tool to recording and checking results, shows a mature instrumental genesis: the subject uses the artefact in flexible and conscious ways. This behaviour is consistent with the idea that instrumentalisation does not necessarily lead to more intensive use of the tool, but rather to its more targeted use, depending on the cognitive needs of the moment.

In the absence of a global visual representation, Ale Micarti's activity demonstrates how subjects choose accessible formats (LaTeX/txt), exploit them through selective externalisation to stabilise reasoning, and adapt the sequential interface into a

functional cognitive tool. This process exemplifies multimediacy, as the artefact simultaneously supports epistemic control and pragmatic action. Ale Micarti's activity therefore shows how, through effective instrumental genesis, it is possible to deal with complex linear algebra tasks by transforming generic technological artefacts into stable and functional cognitive tools.

CONCLUSION

The presented analysis shows how mathematical activity, even in a context of strong perceptual constraints, can be supported by effective and mature instrumental genesis. The case study highlights that the success in the task does not depend exclusively on the availability of technological instruments, but on the subject's ability to transform these artefacts into cognitive instruments that are functional to the activity.

From a theoretical point of view, this study corroborates the validity of the framework proposed by Rabardel even in this very peculiar case, highlighting the dynamic and situated nature of instrumental genesis. The text editor and Braille display are not configured as instruments that are fixed once and for all, but as entities constructed in action through the emergence of usage schemes, navigation strategies and selective modes of externalisation. In particular, the case analysed well exemplifies how instrumentation and instrumentalisation are not sequential processes, but they are deeply intertwined and constantly renegotiated in the course of the activity.

From an educational perspective, the results suggest the importance of recognising and valuing the instrumental strategies developed by students with visual impairments, avoiding the consideration of assistive instruments as mere technical aids. However, as literature on the developmental processes of these strategies remains scarce, this study leaves open critical questions for future research. Mathematical competence emerges here as the result of a complex interaction between theoretical knowledge, instruments and usage schemes, which should also be supported at the level of educational design and assessment. Although limited to a single case study, this work opens up relevant research perspectives on the role of instrumental genesis in the study of linear algebra in accessibility contexts.

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