SKETCH-TO-DANCE FOR AESTHETICALLY DRIVEN COVARIATIONAL REASONING

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In this paper we report a design-based empirical study in which Grade 10 students use a digital environment, "Sketch-to-Dance", to choreograph an avatar to "match" a song by sketching graphs. Grounded in a new-materialist perspective, we treat students' covariational reasoning as an aesthetic–material practice distributed across an assemblage of bodies, sound, sketches, and animated motion. Using aesthetically driven covariational reasoning as our lens, we analyse how aesthetic intensities of fit/misfit organise meanings for how different quantities (e.g., vertical position, horizontal position, and orientation) change over time. Across three short episodes, we show how iterative attunement between graph shape and felt musical–dance fit actualises covariational meanings on period, direction, amplitude, and rate of change.

INTRODUCTION

Digital technologies continue to reshape how mathematical relationships are expressed, sensed, and used, particularly in environments that invite learners to *explore* and *make* mathematics through expressive digital media (da Silva, 2020; Gadanidis et al., 2016), calling for lenses that can account for how meaning emerges through embodied, tool-mediated, and affectively charged activity. Alongside research that has productively described functional and covariational reasoning in terms of how learners coordinate varying quantities using digital media (e.g., Margolis, 2025; Bagossi et al., 2023; Thompson & Carlson, 2017), we attend to how this coordination is materially and affectively accomplished in an aesthetically rich, creative activity. Drawing on inclusive materialism (de Freitas & Sinclair, 2014) and on work that conceptualises learning as a moving assemblage with distributed agency (Ferrara & Ferrari, 2017), we treat covariational reasoning as an aesthetic–material practice distributed across assemblages of bodies, sound, sketches, mathematical functions, and animated motion (De Freitas & Sinclair, 2014). Empirically, we study Grade 10 students working with "Sketch-to-Dance", a digital activity designed on the Amplify classroom platform, where users choreograph an avatar by sketching graphs in a Cartesian plane that control different motion dimensions (vertical position, horizontal position, orientation/rotation, and scale) over time. The environment foregrounds rapid inscription–feedback cycles: students draw, observe the enacted motion against a song, and redraw, as sensations of fit/misfit become consequential for what is changed. To analyse these processes, we use Aesthetically Driven Covariational Reasoning (ADCR) to trace how fit/misfit orients attention to graph features, and how these

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become resources for making sense of change over time. We address the research questions: (RQ1) How do assemblages of bodies, sound, sketch-graphs, and animation enact ADCR as students choreograph motion to music in Sketch-to-Dance?; and (RQ2) What covariational meanings are actualised through these aesthetic–material events?

AESTHETICALLY DRIVEN COVARIATIONAL REASONING

Covariational reasoning is commonly defined as the ability to coordinate two varying quantities and to envision how one changes in relation to the other (Thompson & Carlson, 2017). Much research in this tradition has framed covariation primarily as an individual's mental action of coordinating internal images of instantaneous-continuous changes. In this study, our theoretical stance is new-materialist: drawing on prior new-materialist studies (e.g., Ferrara & Sinclair, 2016; Sinclair et al., 2013) we treat reasoning as a material *process of becoming* that is enacted through the entanglement of bodies, gestures, tools, inscriptions, and affective forces. From this view, a "learner" is not a bounded individual but a body-as-assemblage of heterogeneous elements that co-produce action and meaning. Reasoning unfolds through inventive diagrammatic gestures (e.g., sketching, tracing, pointing) that "cut" into a field of possibilities and bring particular relations of change into visibility as diagrammatic anchors that can orient further action (de Freitas & Sinclair, 2013; Ferrara & Sinclair, 2016). To clarify what we mean by a "field of possibilities" we draw on the concept of *virtuality* (Châtelet, 2000). The virtual is the immanent creative potential of a situation: what an assemblage can do, prior to (and alongside) any single settled outcome. In this view, the virtual and the actual are two dimensions that mutually presuppose each other: the actual is the particular relation that becomes perceptible and actionable here-and-now, while the virtual is the reservoir of still-unrealised relations that remain present as tendencies, invitations, and alternative ways the situation could unfold. Diagrams, gestures, and digital networks are not just records of thinking but can actualise the virtual by bringing one relation of change into temporary visibility, while other relations remain virtually available for later uptake.

Within this perspective, we use ADCR to name how virtual covariational relations are actualised through aesthetic–material dynamics in creative activity (Karavakou & Kynigos, 2025). ADCR refers to the emergent coordination of varying quantities as students work toward a felt sense of fit between music and motion; an intensity that arises within the assemblage, rather than being imposed as external correctness criteria. Such aesthetic forces are the very driver of creative activity: they shape the felt sensations through which the assemblage forms new ways of coordinating change.

METHODS

Design-based research methodology

This study is situated within a wider design-based research (DBR) project that investigates music–movement tasks for supporting students' meanings for covariation and (in particular) periodic relationships. This study constitutes a subsequent DBR

cycle following the ones reported in Karavakou et al. (2023) and Karavakou & Kynigos (2025). Whereas the previous cycles centred on students' work with pre-defined function families, the current cycle shifts the design emphasis to freehand sketching as diagrammatic gesture: students inscribe a time-graph and immediately encounter its enacted consequences as avatar motion, in synchrony with a musical excerpt.

Participants and setting

The activity was implemented with ten 10th grade students who participated after their afternoon classes at a tutoring centre in Athens, Greece. The session lasted 120 minutes (including a 10-minute break). Students were divided into five small groups of two, each working with either a tablet or a laptop. Small-group work was used to foreground joint noticing, negotiation of fit/misfit, and collaborative revision of graphs through talk, gesture, and repeated testing. Participation was voluntary and students and guardians provided written informed consent for participation and for the use of anonymised data for research purposes.

Digital environment and activity sequence

Students worked in the designed (by the second author) "Sketch-to-Dance" (https://classroom.amplify.com/activity/686fc96f59bd55fdbf0a1955) Amplify Classroom environment, which links a time–graph (time in seconds on the horizontal axis) to a selected motion dimension on the vertical axis (vertical position, horizontal position, rotation, or scale). For each sketch, students could play the song excerpt and observe the avatar's resulting motion along with the song, enabling rapid cycles of sketching, observing, and resketching. The activity progressed through a sequence of choreographic tasks. With Song 1 (Pink Panther), students created and iteratively refined four separate animations by sketching graphs for (a) vertical position, (b) horizontal position, (c) rotation, and (d) scale. They then changed to Song 2 (Yakety Sax) and replayed their existing animation(s) with the new soundtrack, revising graphs until a satisfactory fit was achieved. The same "re-fit" process was repeated with Song 3 (The Second Waltz) and Song 4 (Clint Eastwood). Then groups chose one out of the four songs and created combined choreographies by coordinating graphs for two avatars. Across phases, students' satisfaction functioned as a locally produced stopping condition.

Data collection and analysis

Data were collected to capture the evolving 'Sketch-to-Dance' assemblage, including students' saved sketch traces, screen recordings of graphing and playback, and audio/video of small-group talk and gesture during revisions. We conducted event-oriented analyses by selecting episodes in which a sensed musical–movement fit/misfit became consequential, redirecting students' actions. For each episode, we produced multimodal accounts attentive to students' word use, gestures, and emerging routines in relation to the inscriptions and animated motion, tracing how covariational relations

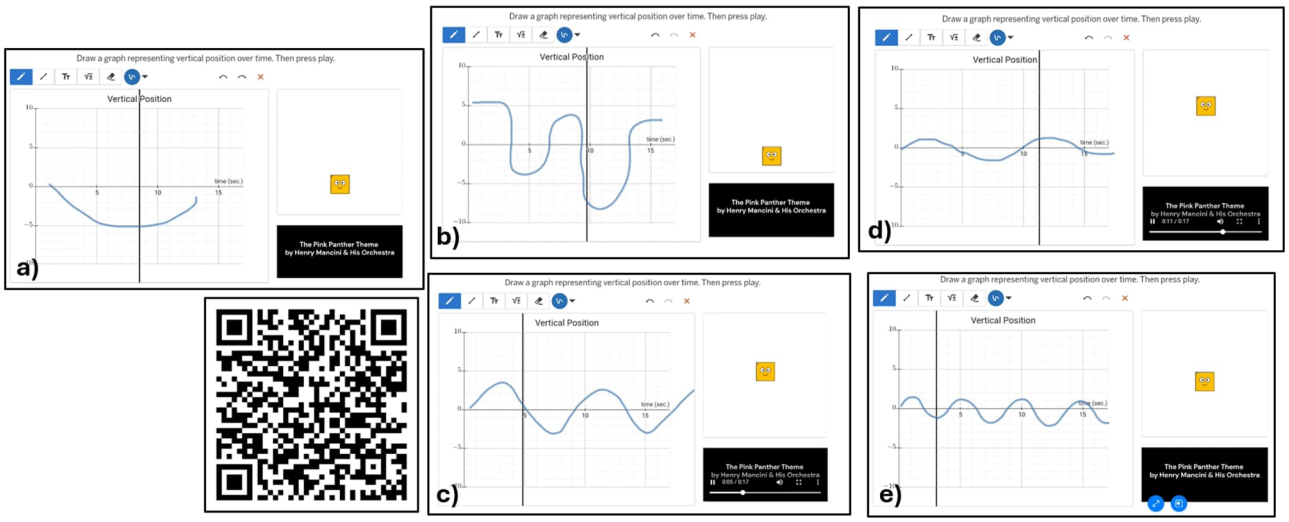
were actualised through the material entanglement of sound, animation, bodies, and graphs.

RESULTS

In the analysis, we traced three recurring ways the Sketch-to-Dance assemblage produced covariational relations as students (S1-S10) iteratively drew and revised graphs in response to what the music–animation coupling made perceptible. We describe these episodes through ADCR: what "works" emerges as students improvise with covariational features, such as periodicity, monotonicity, range (amplitude), rate of change, guided by sensations of fit and misfit among music (beat/melody), gesture, graph, and animated motion. We organise the findings into three subsections: (1) periodicising the sketch to catch the beat; (2) curving the graph to match musical smoothness; and (3) melodic tracing through monotonicity, range, and rate of change.

Periodicising the sketch to "catch the beat"

Figure 1. Group 2 periodicising the sketch to align avatar motion with the beat. Video of the animations can be seen through [this link]() or by scanning the QR code.



At the beginning of the activity (song 1; vertical position), Group 2 (S3 and S4) went through the draw–play–redraw loop to explore what might count as "matching" the music. Their attention quickly settled on the song's steady repetitive chime, which organised the task as an aesthetic problem of temporal fit: how can the avatar's vertical motion *feel* in time with the pulse? After their early attempts (Figure 1a–b), S3 expressed an idea:

> S3: I have an idea! Maybe we should make it follow the background chime. Do you hear it? (S3 replays the song.) Look at my hand. It should do this. (S3 moves her hand up and down following the chime pulse.)

S3's hand-as-metronome gesture anchored the beat as a bodily rhythm and helped open a field of possible periodic sketches. S3 then drew a more explicitly periodic trace (Figure 1c). Playback produced a sense of misfit, which S4 framed as a timing problem:

S4: It loses too much time to go so much up and down. It doesn't catch the beat.

S3: Should I make it shorter?

S4: Yes, shorten them.

Here, time became palpable as something the motion "loses" and the group's ADCR temporarily folded together amplitude, travel distance, and tempo. Their next improvisation (Figure 1d) tested whether reducing vertical travel (amplitude) would make the motion quicker, but the attempt was rejected:

S3: No, no, no, this doesn't work. It's even slower. (She sounds angry.)

S4: We should make more up and downs… And make it a bit higher.

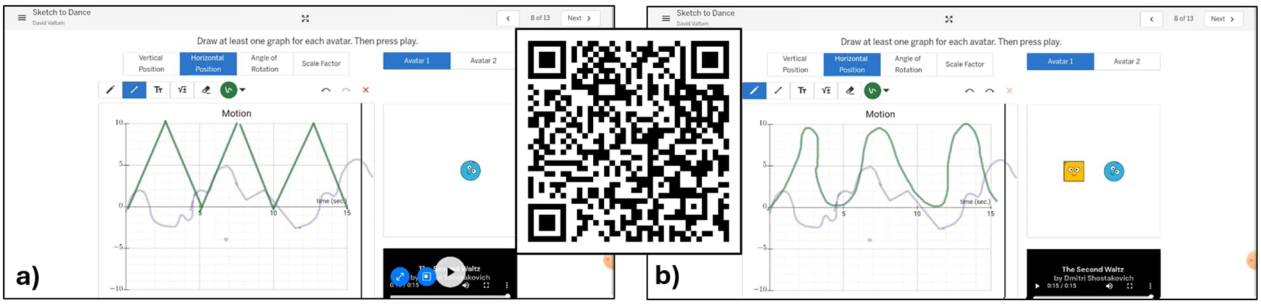
S3's disappointment marked the moment where the aesthetic force of the assemblage (the felt misfit between chime, motion, and graph) interrupted the group's attempt to arrive at a 'working' configuration. S4's next proposal shifted the locus of control from amplitude alone toward periodicity: increasing the number of oscillations within the same time span. The revision in Figure 1e realised this shift by compressing the oscillations into more frequent cycles, producing a "quicker" bobbing that better aligned with the chime tempo.

In this episode, ADCR was surfaced by the assemblage as relations between period, frequency and amplitude became actionable. The group's emerging understanding of "quicker" was progressively displaced from "shorter" toward the period of the graph. Through improvisational revisions driven by sensations of fit/misfit, the assemblage actualised one possibility from the virtual field: a more densely periodic sketch as the configuration that "works" for catching the beat.

Curving the graph to match musical smoothness

The next episode comes from Group 4 (S7, S8) during the final phase of the activity. Having already choreographed the first avatar (blue circle), they began designing the second avatar's dance (yellow square), aiming for it to "follow the steady bass" by moving left and right around the circle. In this subsequent episode, *smoothness* entered the activity as an aesthetic force. After testing the dancing animation generated by the graph in Figure 2a, S7 expressed the misfit linked to the sensation produced when the sketched trace was enacted as motion based on the sharp turning points:

Figure 2. Group 4 curving the sketch to align avatar motion with smoother melody. Video of the animations can be seen through this link or by scanning the QR code.



S7: Now you know what I don't like? The moments it changes direction. Those changes.

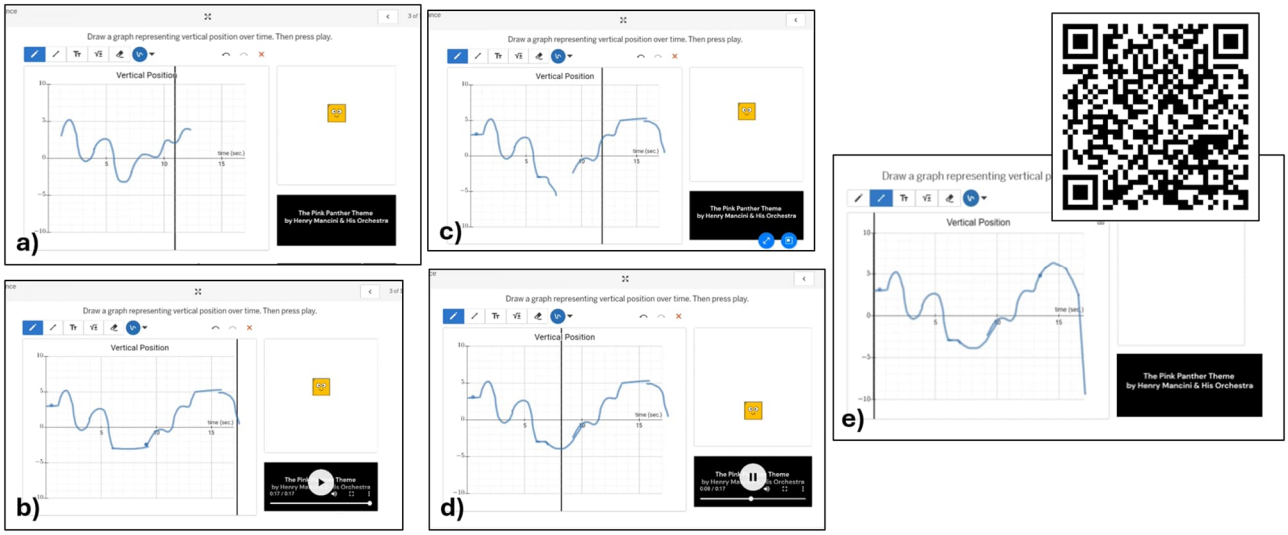
S8: They're too sudden?

S7: Yes! They need to be curvier. Smoother, to match the melody right. Now it goes like this (abrupt gesture). We want it like this (smooth gesture).

The paired gestures materially contrast two movement qualities (abrupt reversal versus continuous turning). The redraw in Figure 2b rounds the corners, redistributing directional change over time. What is produced mathematically here is a practical distinction between abrupt versus gradual change: students tune curvature and local rate of change around turning points so the motion feels smoother "to match the melody". ADCR resides in how the felt misfit ("too sudden") drives an improvisational transformation of the graph until a "working" smoothness takes hold.

Melodic tracing through monotonicity, range, and rate of change

Figure 3. Group 4 tracing the melody through monotonicity, range and curvature. Video of the animations can be seen through this link or by scanning the QR code.



Across the dataset, melodic tracing repeatedly emerged through draw–play–redraw cycles in which playback reopened possibilities and students refined what counted as "following" the melody. In these moments of ADCR, students commonly mapped melodic direction to monotonic increase/decrease, calibrated "how high/low" as range, and adjusted steepness as rate of change to align with note durations and phrase endings. In this episode, Group 4's "melodic tracing" is composed through an assemblage of song excerpt (song 1), hand-gesture, graph-as-inscription, and animation. S7 began by voicing the melodic motif and translating it into covariational terms, while drawing the first sketch (Figure 3a):

S7: It goes tourourourou, tourou. So upwards, downwards, a bit higher, a bit more lower.

The voiced contour functions as a resource for tracing directionality (monotonic segments) and relative magnitude (graph amplitude). When Figure 3a was played, the assemblage produced a felt mismatch, noticed by S7:

> S7: This note takes longer. (Re-draws the middle horizontal part shown in Figure 3b and plays the song again.) No, now the horizontal line makes it pause. It remains at the same height. We should give a smooth downward bouncing here. Like this (hand gesture), wavy. Together with the melody.

S7 replaced the "hold" with a continuous descent (Figure 3c), redistributing change over time (a modulation of rate of change through curvature). Next, S8 constrained the extent of the descent:

> S8: No, no. It doesn't go that low. Don't you hear it? It needs smaller slope.
>
> S7: Ok, ok, I got it. (Draws the graph in Figure 3d and replays the animation). At the end it needs to become even lower. We need to trace this fall by the bass. (Draws the graph in Figure 3e and replays the animation).

The disagreement was settled through listening and redrawing: the melody was invoked as an authority that constrained how far downward the graph can travel. This is a clear instance of the assemblage *producing* range as a negotiated bound on "how low" the movement should go for a specific time-segment to remain faithful to what is heard. Across sketches shown in Figure 3, "following the melody" was produced through improvisational revisions guided by sensations of fit/misfit: monotonic segments trace upward/downward motion, range is negotiated through "how low," and rate of change is tuned to avoid pausing and to shape the ending fall.

DISCUSSION

This study contributes by highlighting aesthetic fit as an organising force of mathematical sense-making. In Sketch-to-Dance, ADCR unfolded through sound–graph–motion aesthetic events in which avatar playback reconfigured what became perceptible and actionable. Fit/misfit was not simply a correctness check against a pre-given target, but an affective–material force that redirected attention and invited further action. Students' gestures-with-tools (e.g., the hand-as-metronome; abrupt/smooth contrast gestures; voicing melodic motifs) functioned as diagrammatic anchors, coupling embodied rhythm and contour with the graph-as-inscription and the avatar's enacted movement. In line with new-materialist accounts of inventive diagramming, these gestures did not merely communicate ideas but helped bring into play latent possibilities for relating time to movement variables.

Within these events, specific covariational meanings were actualised. First, period was produced when students moved from amplitude-based hypotheses toward adjusting the spacing and number of cycles so motion became "quicker" and aligned with the beat. Second, directionality/contour came to matter as students mapped melodic movement to monotonic up/down segments, using voice and gesture to trace rises and falls. Third, rate of change (and relatedly curvature) became meaningful when students worked on

how change happens: avoiding pauses by replacing holds with continuous descent, tuning "how low" via range constraints, and softening sharp direction changes so motion felt smoother with the melody. Taken together, these findings characterise ADCR as the ongoing production of covariation through iterative attunements among sound, inscription, gesture, and motion, culminating in locally workable aesthetic fits.

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