MATHEMATICS TEACHERS' NOTICING OF CRITICAL EVENTS IN CONDENSED AND EXPANDED WRITTEN VERSIONS

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This study explores how the level of detail in written critical events shapes mathematics teachers' noticing. Using the MOST framework and gap-filling theory, the study compares teachers' interpretations of the same events across condensed and expanded versions. Seven experienced secondary mathematics teachers analyzed three events through written reports and semi-structured interviews. The analysis focused on teachers' expressions of certainty and the range of interpretations generated in response to each version. Condensed versions elicited more tentative statements and greater interpretive diversity than expanded versions, suggesting that reduced detail promotes gap filling, interpretative flexibility, and openness in teachers' noticing.

INTRODUCTION

A central goal of mathematics teacher education is to develop pedagogical skills that promote students' mathematical thinking. One such skill is teachers' professional noticing, commonly conceptualized as comprising three interrelated components: attending to significant aspects of instruction, interpreting them, and considering possible instructional responses (Jacobs et al., 2010). While all three components are integral to noticing, the present study focuses specifically on the interpretive component—how teachers interpret students' mathematical thinking when engaging with representations of classroom events.

Critical events are defined as "moments in which students' mathematical thinking becomes apparent and thus can provide teachers opportunities to delve more deeply into the mathematics discussed in the lesson" (Rotem & Ayalon, 2022, p. 2). In teacher education, such events are represented in various ways, including written descriptions that are used as a basis for professional discussion and reflection (e.g., Sanchez-Matamoros et al., 2019). Written representations of events reduce the complexity of classroom interactions and may allow focused attention on selected aspects of practice. At the same time, by limiting the information available for interpretation, they may steer the range and nature of interpretations teachers may generate.

Still, little is known about how different written versions of the same event, varying, for example, in the level of detail they provide, may invite different interpretations. Examining how teachers interpret condensed and expanded versions of a single critical event may shed light on how representations of practice shape interpretive processes within professional noticing, and on how teacher educators might intentionally design materials that open or constrain teachers' sense-making.

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THEORETICAL FRAMEWORK

The study is grounded in the Mathematically Significant Pedagogical Opportunities to Build on Student Thinking (MOST) framework (Stockero et al., 2020), which conceptualizes critical events as moments in which students' mathematical thinking becomes visible, involves significant mathematical content, and offers opportunities for pedagogical engagement. This framework highlights the central role of such moments in teachers' noticing and instructional decision making.

While prior research has emphasized the importance of critical events in teacher education, less attention has been given to how characteristics of representations of these events shape teachers' noticing. The present study focuses on written descriptions of critical events and draws on theoretical perspectives on text reading to conceptualize teachers' interpretive activity. From a gap-filling perspective, written texts are understood as structured systems that include informational gaps, which readers actively complete to make sense of the situation (Perry & Sternberg, 1986). When information is underspecified, readers rely on inference and prior knowledge, and may produce multiple plausible interpretations of the same event.

Drawing on this perspective and on research on teachers' noticing, we employ three analytic dimensions to examine teachers' interpretations of written critical events: (1) anchoring interpretations in the specific mathematical context of the event, (2) the degree of certainty expressed in teachers' interpretations, and (3) the variety of interpretations generated. Together, these dimensions provide an analytic framework for examining how different written versions of the same event may shape teachers' interpretive processes.

Anchoring interpretations in the specific mathematical context refers to teachers' explicit connection between their interpretations and the mathematical ideas involved in the event. In the present study, this dimension functions as an analytic criterion to ensure that interpretations focus on students' mathematical thinking rather than on general pedagogical considerations. Such anchoring has been emphasized in prior research on noticing as essential for connecting observations of classroom events to instructional reasoning (Jacobs et al., 2010; Rotem & Ayalon, 2022).

The degree of certainty refers to how teachers express confidence or tentativeness in their interpretations. Previous research has emphasized linguistic modality—tentative language (e.g., "maybe," "it seems")—as an important resource in teachers' reasoning (Chen & Herbst, 2012). Studies examining teachers' interpretations of classroom events documented through video or comics have shown that hedging, questions, and tentative statements can reflect openness to alternative explanations and sustained engagement with uncertainty. Building on this work, the present study examines how teachers express certainty and tentativeness when interpreting written descriptions of critical events.

The range of interpretations refers to the breadth of perspectives and inferences articulated by teachers in response to an event. From a gap-filling perspective, texts with more gaps may invite a wider range of interpretations, whereas more detailed descriptions may constrain interpretation by narrowing the set of plausible inferences (Marco et al., 2021; Perry & Sternberg, 1986). Examining the range of interpretations allows us to explore how variations in the level of detail in written event descriptions shape teachers' interpretive activity.

RESEARCH AIMS AND RESEARCH QUESTION

The aim of this study is to explore how variations in the level of detail in written descriptions of critical events shape teachers' interpretations of students' mathematical thinking. Specifically, the study examines how experienced mathematics teachers interpret a critical event when it is presented in condensed and expanded written versions. The research question guiding the study is: How do experienced mathematics teachers interpret a critical event when analyzing its condensed and expanded written versions?

METHOD

Research Design. This qualitative study draws on written analyses of critical events and semi-structured interviews. Seven experienced secondary teachers analyzed three written critical events, each presented in two versions: a condensed version and an expanded version. The two versions referred to the same instructional moment but differed in the level of detail provided.

Population. The study involved seven experienced secondary mathematics teachers, each with more than five years of teaching experience. Teachers were purposively selected to enable an examination of noticing processes grounded in accumulated teaching experience.

Tools. The critical events used in this study were drawn from a previously developed corpus of classroom events hosted on the Aklim website – Critical Events in the Teaching and Learning of Mathematics (aklim.haifa.ac.il). This corpus was developed as part of a research project focusing on teacher education through the analysis of critical events. Within that project, experienced teachers and mathematics education researchers collaboratively collected and examined events from teachers' own mathematics lessons. All events were jointly identified as critical, in line with the definition of critical events as "moments in which students' mathematical thinking becomes apparent and thus can provide teachers opportunities to delve more deeply into the mathematics discussed in the lesson" (Rotem & Ayalon, 2022, p. 2).

For the purposes of the present study, three critical events were selected to represent different types of student contributions in secondary mathematics classrooms: (1) an exploratory suggestion. (2) a student error, and (3) an original student solution.

To examine the research question, two written versions were constructed for each of the three critical events. The distinction between condensed and expanded versions

was informed by the analytic framework for characterizing critical mathematical events proposed by Stockero et al. (2020). The condensed version included the minimal components required to characterize a critical event according to this framework: the classroom context, the mathematical task, a student utterance, and a teacher utterance. The expanded version retained these core components and additionally included a description of the broader classroom interaction, providing a richer contextualization of students' mathematical thinking. The distinction between the two versions reflects differences in the level of detail provided, rather than differences in the critical nature of the events themselves.

To illustrate the structure of the materials, we present below the first critical event in both its condensed and expanded versions. The event focuses on identifying the type of extremum of a quadratic function. Teachers were presented with the following task:

Figure 1: The first critical event in its condensed and expanded written versions

Condensed version.	*Expanded version.*
Teacher: Why is it that for a quadratic function $y = ax^2 + bx + c$, when $a > 0$ the extremum point is a minimum and when $a < 0$ it is a maximum? Efi: Maybe we can take the derivative. Then we get $y' = 2ax + b$, so we have an expression that depends only on a, but I am not sure how this connects to maximum and minimum…	Teacher: What do you think we should do in this situation? Shai: We can differentiate the function, set the derivative to zero, and determine the type using a sign table. Mor: We can also use a table of values of the function. Hagit: Since the function is quadratic, we can determine the type of extremum by the sign of the coefficient of x^2. Teacher: Why is it that for a quadratic function $y = ax^2 + bx + c$, when $a > 0$the extremum point is a minimum and when $a < 0$ it is a maximum? Adi: We can find the derivative in a general form and substitute the value of x into the function. Shai: That may not help. After setting $y' = 2ax + b$ equal to zero, we get $x = -\frac{b}{2a}$, but I got stuck because there are two unknown signs. Efi: Maybe we can take the second derivative. Then we get $y'' = 2a$, but I am not sure how this connects to maximum and minimum…

The interviews further examined how teachers interpreted these events and the factors that shaped their reasoning.

Procedure. The study was carried out across three separate time points. At each time point, teachers individually analyzed a critical classroom event. They first examined the condensed version of the event and submitted a written interpretation. After completing this analysis, they were provided with the expanded version of the same event, which they also analyzed. This procedure was repeated for three different events,

resulting in a total of 42 reports, as each teacher analyzed each event in both its condensed and expanded form.

Data Analysis. The analysis proceeded in four stages: (1) interpretations were screened to include only those anchored in the specific mathematical context of the event; of the 42 interpretations collected, 40 met this criterion and were included in subsequent analyses; (2) context-anchored interpretations were analyzed for the degree of certainty expressed, focusing on the use of tentative or hedging language (Chen & Herbst, 2012); (3) the range of interpretations generated in response to each event version was examined, focusing on the variety of perspectives and inferences articulated. Drawing on gap-filling theory (Marco et al., 2022), variation in interpretive range was understood as reflecting differences in the interpretive space invited by the written descriptions; and (4) the interpretations of the condensed and expanded versions of the same event were compared to examine similarities and differences in teachers' interpretations under different levels of textual detail. Interview data were used to complement the written analyses and deepen understanding of the patterns identified.

FINDINGS

Level of Certainty. Systematic differences were identified in the degree of certainty expressed in teachers' interpretations of the condensed and expanded versions of the events. Across the three events, teachers produced substantially more tentative statements when interpreting the condensed versions (45 tentative statements) than when interpreting the expanded versions (8 tentative statements). A similar pattern was observed at the level of participants. In the condensed versions, four teachers used tentative expressions in the first and second events and five teachers in the third event. In contrast, in the expanded versions, tentative language was used by only one teacher in the first and third events and by two teachers in the second event.

Four main types of tentative statements were identified: (1) investigative and clarifying questions (e.g., "Is it…?" , "Why…?"); (2) internal dialogue (e.g., "If so, then it is clear", "But then why?"); (3) hypotheses and assumptions (e.g., "One possible explanation could be…", "I assume that…"); and (4) cautious expressions of opinion (e.g., "I think…", "It seems to me…").

This pattern is illustrated in the interpretation of one participating teacher (Chen) of the condensed version of the first event, which focused on a student's attempt to determine the type of extremum of a quadratic function:

> In Grade 9, students learn that the type of the vertex of a parabola depends on the sign of the coefficient of x squared, in this case the parameter a. Nevertheless, the student did not use this knowledge and instead chose to compute the second derivative .It is not clear to me why she decided to take the second derivative because I do not know what prior knowledge she has. Does she rely on ideas she had in previous classes? Or because she wanted to perform an operation that would eliminate x and leave an expression depending only on a, and therefore she says she derives twice? If so, then it is clear that they know the role of the second derivative in identifying the type of extremum, but then why did the

> student not know how to continue? Did they not know at all what a second derivative is? If so, then how did the student think of it at all?

In this interpretation, Chen used eight tentative statements of different types, including investigative questions (e.g., "Does she rely on ideas she had in previous classes?"), internal dialogue (e.g., "If so, then it is clear that they know the role of the second derivative, but then why did the student not know how to continue?"), and cautious expressions of opinion (e.g., "It is not clear to me").

By contrast, Chen's interpretation of the expanded version of the same event, which included additional information about classroom interactions, was concise and assertive and did not include tentative statements. Her interpretation focused on identifying specific pieces of prior knowledge that students were drawing upon, without questioning alternative explanations:

> The students are trying to connect to prior knowledge they have: 1) about the importance of the sign of the quadratic coefficient of the parabola, learned in Grade 9, and 2) about the investigation table as a tool to determine the type of extremum, learned in Grade 10.

Range of Interpretations. The condensed versions consistently elicited a wider range of interpretations than the expanded versions. In the first event, three distinct interpretations were identified in the condensed version, compared with a single interpretation in the expanded version. In the second event, five interpretations emerged in the condensed version and two in the expanded version. In the third event, five interpretations were identified in the condensed version, compared with four in the expanded version. These differences are illustrated by teachers' interpretations of the first event:

When interpreting the condensed version, teachers proposed multiple explanations related to students' prior knowledge of the second derivative. Four teachers interpreted that the student had already learned the concept of the second derivative. In contrast, three teachers suggested that the student had learned the second derivative but did not know how to apply this knowledge to a parameterized problem, while one teacher attributed the difficulty to an inability to apply the second derivative to identifying minimum and maximum points.. In the expanded version, however, teachers' interpretations converged on a single one, according to which the student had not yet learned the second derivative. These interpretive differences emerged as teachers attempted to complete missing information embedded in the condensed version of the event, consistent with a gap-filling perspective on text interpretation.

To provide an overview of the differences between the condensed and expanded versions across the three events, Table 1 summarizes the number of tentative statements, the number of teachers who used tentative language, and the number of distinct interpretations identified for each event and version.

Table 1. Tentative statements, the number of teachers using tentative language, and range of interpretations across condensed and expanded versions of the three events

Event number	Version	Count of tentative statements	Count of teachers using tentative language	Count of distinct interpretations
E1	condensed	23	4	3
	expanded	3	1	1
E2	condensed	8	4	5
	expanded	2	2	2
E3	condensed	14	5	5
	expanded	3	1	4

DISCUSSION

The present study investigated how variations in the level of detail in written representations of critical events relate to experienced mathematics teachers' interpretations of students' mathematical thinking. Comparison of condensed and expanded versions showed that condensed representations elicited substantially more tentative language and a broader range of interpretations than expanded versions, where interpretations were more constrained (Jacobs et al., 2010; Sánchez-Matamoros et al., 2019).

In condensed versions, teachers were required to infer students' prior knowledge, intentions, or conceptual understanding based on limited information. This aligns with the gap-filling perspective, according to which texts contain informational gaps that readers must complete using inference and prior knowledge (Perry & Sternberg, 1986). Furthermore, the findings resonate with Marco et al. (2021), who demonstrated that when information in geometry tasks was omitted, students engaged in active gap filling, generating multiple plausible solutions. Similarly, in the condensed critical events, teachers engaged in gap-filling processes to interpret students' thinking, illustrating how inferential reasoning can play a role in noticing.

The higher prevalence of tentative language – questions, hypotheses, internal dialogue, and hedging – in interpretations of condensed versions indicates sustained engagement with incomplete information and consideration of multiple possibilities (Chen & Herbst, 2012; Herbst & Kosko, 2013). In contrast, expanded versions led to fewer tentative expressions and a narrower range of interpretations, showing that richer contextual information can limit the interpretive space available to teachers.

From a pedagogical perspective, these findings suggest that that the design of representations of practice is consequential for teacher learning. Specifically, teacher educators may use condensed representations to open interpretive space and invite

hypothesis-building and exploration of alternatives, or expanded representations to focus teachers' attention on particular aspects of students' mathematical thinking. In this sense, representations are not neutral carriers of information but epistemic tools that shape how teachers make sense of classroom activity. Future research could examine how preservice and novice teachers engage in similar gap-filling processes, and how work with different types of representations relates to the development of teachers' response skills within the broader noticing cycle.

In conclusion, reducing the level of detail in written representations of critical events does not constrain noticing; rather it promotes tentative reasoning and a broader range of interpretations. These results represent a first step toward connecting gap-filling with noticing, with the intention of further exploring and elaborating this connection in future research.

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