“LIKE A LITTLE GLIMMER OF LIGHT”: UNDERSTANDING AND VALIDATING STUDENTS’ CREATIVE PROCESSES

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As societies continue to evolve, creativity—particularly mathematical creativity—has become a crucial 21st-century skill. While prior research has primarily emphasized creative outcomes, the cognitive processes underlying mathematical creativity remain insufficiently understood. The aim of this study is to characterize the creative process of school students. In our study, 15 upper secondary school students worked on a creativity problem while wearing eye-tracking glasses and participated in a stimulated recall interview. Using a combined deductive–inductive analytic approach based in Hadamard’s stage model of creativity, we were able to characterize and develop a model of the students’ creative process. The model substantiates previous models, while also revealing additional phases as well as reinterpretations.

INTRODUCTION

Preparing contemporary learners for complex global challenges requires a focus on flexible agency and creative problem-solving (OECD, 2024), a necessity that is increasingly evident in mathematics education. Hence, it is essential to prepare today’s students accordingly and to foster their mathematical creativity. For this, an evidence-based understanding of how mathematical creativity manifests in learners is crucial. One field of research in mathematics education is accordingly dedicated to mathematical creativity, whose focus is primarily on the understanding of creative *products* and their connection to other constructs such as intelligence, or abilities. Creative *processes*, on the other hand, are less often studied (Joklitschke et al., 2022). This research gap was addressed by Schindler and Lilienthal (2020, 2022), who developed a tentative model of the creative process, drawing parallels to Hadamard’s (1945) model of creativity with the stages of preparation, incubation, illumination, and verification, and thus laying the foundation for a better understanding of creative processes in students. However, such creative processes have not yet been investigated in larger groups of students or with younger students. The overarching question of our study is therefore: What characterizes the creative process of students in their work with MSTs? More specifically: *Which phases of Hadamard’s model can be identified in students’ individual approaches, and how are these phases characterized*? And: *what invariant parts emerge across approaches, and what general process model can be derived from them?*

THEORETICAL BACKGROUND

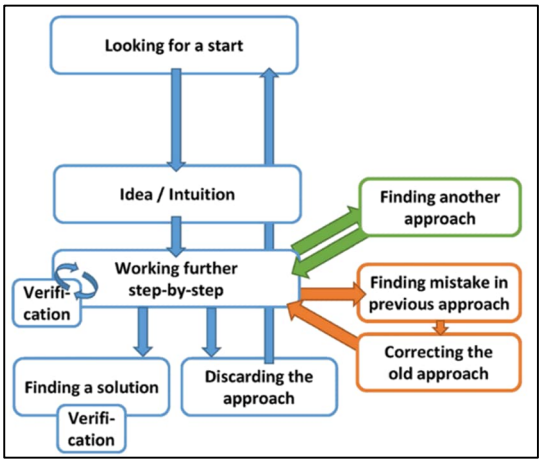
In mathematics education research, interest in creativity grows. There is a wide variety of ways in which mathematical creativity is perceived and conceptualized (Joklitschke

2026. In H. Krzywacki, M. Hähkiöniemi, E. Haataja, M.S. Hannula & P. Hästö (Eds.). *Proceedings of the 49th Conference of the International Group for the Psychology of Mathematics Education: Research Reports*, pp. 707-714. PME.

et al., 2021; Leikin & Sriraman, 2022). Based on the Torrance Test of Creative Thinking (Torrance, 1974), which was developed in the domain of psychology, MSTs were developed to quantitatively measure mathematical creativity in students (Leikin, 2009). In MSTs, students are asked to find as many solutions as possible to a mathematical problem (see Fig. 2 for an example). The *products* are then scored regarding fluency, i.e., the number of solutions, flexibility, i.e., the solution categories used, and originality, i.e., the rarity of solutions, resulting in a creativity score.

In addition to *product* orientation, research from mathematics education also addresses creative *processes*. In this regard, many researchers refer to Hadamard's (1945) stage model of the creative process, which he derived from interviews with successful mathematicians, drawing on ideas of Poincaré and Wallas. The first stage is *preparation*, in which the problem solver consciously engages with the task at hand. This is followed by a subconscious stage of *incubation*, in which the problem is not actively worked on. In the subsequent *illumination*, the third stage, the problem and the solution idea come back into consciousness and is then completed and reflected upon in the final *verification* stage. Although the model originates from work with the most renowned mathematicians and scientists of Hadamard's time, Hadamard writes that "between the work of a student who tries to solve a problem in geometry or algebra and a work of invention, one can say there is only a difference of degree" (Hadamard, 1945, p. 104). Thus, this stage model is not only meant to describe processes of those who make great discoveries but also applies to students. This is in line with Kaufman and Beghetto's (2009) Four C Model of different levels of creativity, who refer to great discoveries as Big-C and to creative achievements of students as little-c or mini-c. In this regard, we refer to the terminology of "phases" in the context of learners, as the term "stages" tends to imply a fixed sequential order. Using this adaptability, Liljedahl (2013) compared the processes of mathematicians and preservice teachers. In doing so, he emphasizes that it is the affective aspects like excitement that distinguish illumination from other mathematical experiences. This crucial link between affect and creativity is also evident in the exploratory study of Schindler and Lilienthal (2020). By recording eye movements during the MST and conducting stimulated recall interviews, the study reconstructed a student's individual creative processes (Fig. 1). A comparison with the stage model for creative processes shows that the model does not "fully capture the creative process" (p. 1580). No typical incubation phases could be identified in the process of the student

Figure 1: tentative model of a creative process (Schindler & Lilienthal, 2020, p. 1577). CC BY 4.0 (https://creativecommons.org/licenses/by/4.0/) — unchanged.

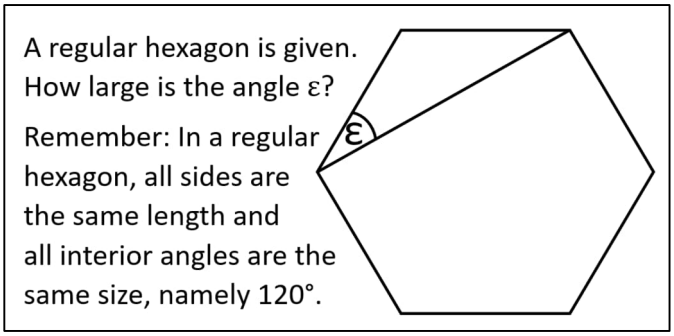


David. Instead, there were phases in which David temporarily returned to a previous approach or a new approach before bringing the current approach to a solution—these were called mini-incubation. The moments of illumination were similar to the descriptions given by mathematicians (Hadamard, 1945; Liljedahl, 2013), as they were characterized by excitement and certainty. In contrast to Hadamard's descriptions, verification phases did not only occur at the end, but also while David was following up on an approach to ensure that everything was correct. So, valuable insights have already been gained with regard to a better understanding of creative processes. However, these findings primarily relate to upper secondary and university students with affinity to mathematics. Building on Hadamard's phases of the creative process, the study at hand reconstructs the creative processes of average upper secondary students and subsequently examines the extent to which these reconstructions align with the model proposed by Schindler and Lilienthal (2020), thereby allowing for a form of triangulation.

METHODS

To systematically record students' creative mathematical processes, 15 students from upper secondary school worked for 20 minutes on an MST (Fig. 2). MSTs are typically used to assess creativity. The data generation corresponded to the study by Schindler and Lilienthal (2020). The students wore eye-tracking glasses (Tobii Pro Glasses 2, 100 Hz) to record where they looked at on the worksheet at what time and what was written down. Afterwards, the interviewer and student watched the eye-tracking video together in a stimulated recall interview: The video, in which the gaze was visualized as a moving dot, served as the stimulus. Both persons were able to pause the video to ask questions or provide further explanations. Due to the exploratory nature of their study, in which Schindler and Lilienthal (2020) used a purely inductive analysis through paraphrasing, this study opted for a structured qualitative content analysis (Kuckartz & Rädiker, 2023). This analysis involved developing the main categories deductively through the phases of Hadamard (1945). Subsequent affective descriptions and solutions found served as further categories. In an additional inductive phase, these main categories were refined to form subcategories. The individual approaches were then reconstructed on the basis of the coding. Bearing in mind the 20-minute limit of the MST processes, intensive preparation and longer-term incubation, as described by Hadamard, are not feasible here. A critical examination of this adaptation can be found in the discussion.

Figure 2: MST "hexagon"



In order to find out which phases from Hadamard's model also occur in students and how they are characterized (RQ1), all subcategories were operationalized by means of

so-called *category definitions*. According to Kuckartz and Rädiker (2023, p. 43), these include four to six aspects: "[1] Name of the category: Concise label as meaningful and precise as possible; [2] Content description: Description of the category, if applicable with theoretical background; [3] Category application: 'This category is coded if the following aspects are mentioned …'; [4] Examples of applications: Quotations with reference (document; paragraph or page); [5] Other applications (optional): 'The category is also coded when …' Quotations with reference (document; paragraph or page); [6] Differentiation from other categories (optional): 'The category is not coded if …; but in this case category z is used' Quotations with reference (document; paragraph or page)". The coded phases can then be represented in a flow chart, which provides a clear graphical visualization of the chronological sequence of the individual phases. As a reminder, MSTs require multiple solutions, so that several reconstructed approaches were created for each individual process.

For RQ2, we compared the flowcharts visualizing the approaches to identify recurring patterns and derive a general model of the creative process. We analyzed sequences of at least two consecutive phases occurring across multiple approaches and students. The sequence stability increases with both the number of consecutive phases and the number of processes in which it appears. Longer sequences recurring across more processes thus provide stronger evidence of invariant phase patterns.

RESULTS

The 15 students generated 83 solutions in total, with a number of one up to nine individual solutions per student.

Reconstruction of phases in a creative process

Based on Hadamard's (1945) phases, which served as deductive categories for coding, further subcategories were identified inductively and operationalized by means of a coding manual containing all *category definitions*. Overall, 1,208 meaning units (i.e., transcript segments assignable to a specific phase) were coded. The phases are presented below, along with examples of frequent subcategories.

Preparation: *Preparation* phases were by far the most frequently coded phases (527 times, 43.6 %). Explanations of the strategy used, reading task instructions, and searching for mathematical properties to solve the MST were particularly common. There were also characteristic phases in which the students reported *getting stuck* or not having any ideas (81 times). In general, this phase is characterized by students' strong engagement with the task. They search for clues and follow them in order to find the correct angle size. One additional core element of this phase is the subcategory *reference to a specific property*, in which students report on properties of the hexagon that could lead to a solution. In some cases, several properties are combined, such as identifying an isosceles triangle and the size of the interior angle sum in a triangle.

Incubation: The least frequently coded phases were those that could be attributed to incubation (19 times, 1.6 %). This occurred for only seven of the 15 processes. Here,

a distinction was made in the subcategories between whether the focus was a specific one or whether students reported having rather undefined thoughts, such as “yeah, I was just staring blankly at the epsilon” (Holger, turn 38). There was no pattern in which students presented a finished solution immediately after an incubation**.** In addition, follow-up questioning made it possible to assess their level of consciousness during this phase**,** as illustrated in the following excerpt (Benedict, turn 60-61):

Interviewer: I think you just took another close look at exactly this triangle (right-angled triangle). (8 sec) [B: yes] yes (..) were you already consciously aware of that at the time, or was it more...

Benedict: More subconscious.

Hadamard’s (1945) describes incubation as a period during which one is engages in completely different activities. This is not the case in the design of this study; however, the references to subconscious engagement with MST are worth mentioning.

Illumination: Phases of illumination were identified relatively rarely (74 times, 6.1 %; due to differently structured subcategories, this number does not reflect the number of completed solutions). One category definition of the subcategories revealed that illumination refers to those reported mathematical properties that led immediately and directly to a completed solution. Getting stuck or switching back to a preparation phase is not seen here, though. Some coded phases also clearly show the intuitive character that a phase of illumination can have: “I knew—well, for me, something kind of clicked that something else had to happen with the 120 degrees” (Conny, turn 38). However, immediate access to a solution does not mean that the found property does not still need to be worked out. Furthermore, our understanding of illumination differs from Liljedahl’s (2013) conceptualization in that it places no restrictions on affective aspects for this category. Nevertheless, there were also phases of illumination that were definitely linked to an affective phase, as Diana illustrates: “Whenever a new idea came up, it was like a little glimmer of light” (Diana, turn 93).

Verification: The conceptualization of the verification phase differs significantly from Hadamard’s (1945) model, who describes this phase as the final stage in the process. For all students in this study, verification played an important role throughout the entire process (385 times, 31.9 %). Even when searching for properties, students explain that they increasingly verify references to specific properties before continuing to work on the approach. This is particularly the case shortly before the students write something down. Thus, the category definitions also have a rather metacognitive character. Beyond that, verification does not always take place at the end of an approach. This is understandable given the task format, as a result of 30° was often sufficient confirmation. In addition, phases of *postponing, suspending, or discarding* plans proved to be important in many processes.

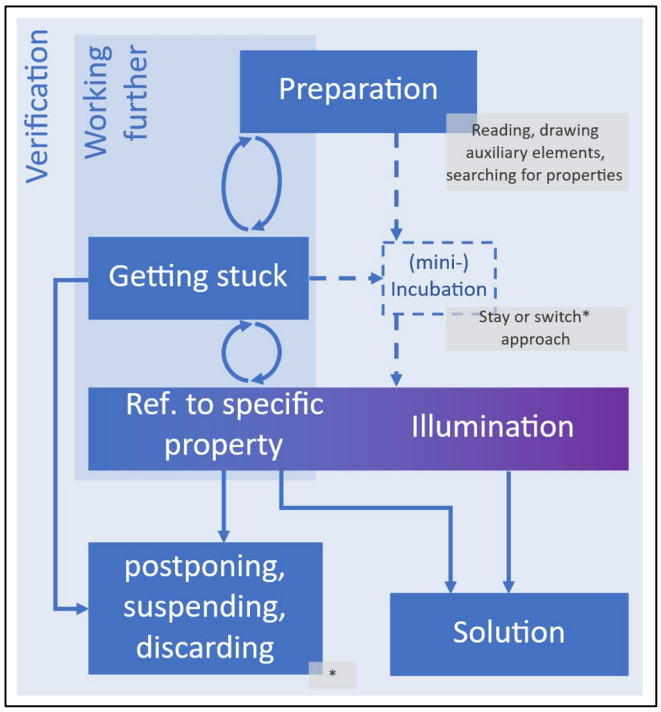
Invariant parts leading to a process model

All approaches to solve the MST were compared to identify invariant sequences of phases. Although diverse, many of these approaches exhibit a similar phased structure. When these are overlaid (Figure 3), they form a model describing the creative process we identified for the students. As with Hadamard (1945), the *preparation* phase here can be interpreted as a conscious examination of the task. A direct solution through *illumination* is also similar to Hadamard. At the same time, a second pathway emerged. Through *reference to specific property* and an interplay between this and the *preparation* and *getting stuck* phases, many approaches resulted in a *solution*. *Illumination* and *reference to a specific property* can be understood as a continuum, depending on whether the students arrive at a solution directly or have to go through many phases of hard work. If no solution is found, the approach is *postponed*, *suspended*, or *discarded*. In some cases, students came also back to an approach, and had a new idea on how to bring it to a solution—so we found that postponing and suspending could also be interpreted as a variant of *incubation*. Finally, *verification* is not merely a concluding phase (as described by Hadamard) but a form of continuous monitoring cutting across all other phases.

DISCUSSION

The analysis of the reconstructed approaches to a geometric MST shows that all phases of Hadamard's (1945) model can be identified. We used category definitions based on Kuckartz and Rädiker (2023) to operationalize these phases, which resulted in a general model for students' creative process. Hadamard's (1945) model is thus rediscovered in the presented student model. Hadamard's "degree of difference" is confirmed, situating the model within the scope of mini-c and little-c (Kaufman & Beghetto, 2009).

Figure 3: Model of students' creative process (with the hexagon MST)



In contrast to *preparation*, which also follows Hadamard's (1945) descriptions in the case of students, *incubation* plays only a minor role. The task format with a time limit is not designed for (intensive) incubation. Furthermore, it is unclear to what extent incubation phases can be reliably reconstructed in students. In contrast, *illumination* phases played a more central role, as students often arrived at a solution immediately from this point. However,

illuminations were conceptualized more broadly than in Hadamard's (1945) descriptions. Importantly, the category definition was not tied to affective descriptions but rather to the rapid "seeing" or "grasping" of a solution. While previous studies have consistently linked illuminations to affective experiences (Liljedahl, 2013), the present data show that affective descriptions do not necessarily coincide with illumination phases, even though such connections can be observed in some cases. This points to a more differentiated relationship between cognitive and affective aspects of creative processes. In the context of MSTs, affective moments are particularly difficult to reconstruct, both because the task format involves very short processes and because students do not always have a clear awareness of emerging affective experiences related to mathematical problem solving or choose not to report them for various subjective or situational reasons.

Across approaches, clear parallels emerge to Schindler and Lilienthal's (2020) tentative model. Due to our study's deductive-inductive analysis, the terminology of the phases of the model at hand are more reminiscent of Hadamard's (1945) model. For instance, we use *preparation* and *illumination* instead of Schindler and Lilienthal's "Looking for a start" and "Idea/intuition", even though the underlying interpretations are similar. What has been added to the current model is *getting stuck*, as this was an essential part of many processes, as was the cyclical change between the phases of *preparation*, *getting stuck*, and *reference to specific property*. The activity "Working further step-by-step" (Schindler & Lilienthal, 2020) is of enormous importance, but in our analyses, we did not include it as a single phase; rather, we added the activity of *working further* in the background. This raises the question of how broadly creative processes should be defined. While some approaches show clear indicators of creativity, such as affective "aha"-moments while *illumination* or productive *incubation* phases, others remain inconclusive. However, this should not suggest that these are not creative processes. We argue for an open perspective in which the task format of an MST generally enables creative processes and allows them to be reconstructed and analyzed within a creative process framework. This discussion is necessarily incomplete and subject to further debate.

In Schindler and Lilienthal's (2020, 2022) analyses, there were no classic *incubation* phases; instead, phases in which the student dealt with a different approach were interpreted as "(mini-)incubation". These phases are also found more frequently in this study and can be easily identified with the flowcharts. A current approach is postponed in favor of (a) correcting, (b) completing, and (c) continuing to work on a previous approach, or (d) searching for a completely new approach. Beyond this, the present study provides evidence for incubation phases *within* an approach, thereby extending previous research and suggesting that even in such short processes, forms of incubation can be reconstructed. In addition, the widespread occurrence of *verification* is visualized as a background activity and not assigned to individual phases. In particular, it is not consistently seen after finding a *solution*. This rather metacognitive interpretation is also shared by Schindler and Lilienthal (2020). Thus, through a form

of triangulation, our findings largely support the model of a creative process proposed by Schindler and Lilienthal (2020), while also extending it in selected aspects.

Methodological limitations, particularly the time constraints of the interviews, may have limited the level of detail in the reconstructed processes. The eye-tracking recordings proved highly effective as a stimulus for reflections on the process, as they prompted students to provide detailed comments even during periods of seeming inactivity. The findings have direct implications for learning environments. They point to the importance of emphasizing final verification as a form of critical thinking and of deliberately creating incubation spaces, either through engagement with alternative approaches or through activities unrelated to the task. Finally, the MST task format proves beneficial for both lower- and higher-performing students. Overall, the study demonstrates that mathematical creativity is not limited to highly gifted students, but can also be meaningfully reconstructed in students with average performance.

References

Hadamard, J. (1945). *The psychology of invention in the mathematical field*. Dover Pub.

Joklitschke, J., Baumanns, L., Rott, B., Schindler, M., & Liljedahl, P. (2022). Literature review on empirical findings on creativity in mathematics among secondary school students. In S. A. Chamberlin, P. Liljedahl, & M. Savić (Eds.), *Mathematical creativity: A developmental perspective* (1st ed. 2022, pp. 81–103). Springer.

Joklitschke, J., Rott, B., & Schindler, M. (2021). Notions of creativity in mathematics education research: A systematic literature review. *International Journal of Science and Mathematics Education*, *20*, 1161–1181.

Kaufman, J. C. & Beghetto, R. A. (2009). Beyond big and little: The Four C Model of creativity. *Review of General Psychology*, *13*(1), 1–12. https://doi.org/10.1037/a0013688

Kuckartz, U. & Rädiker, S. (2023). *Qualitative content analysis: Methods, practice and software* (2nd edition). Sage.

Leikin, R. (2009). Exploring mathematical creativity using multiple solution tasks. In R. Leikin, A. Berman, & B. Koichu (Eds.), *Creativity in mathematics and the education of gifted students* (pp. 129–145). Sense Publishers.

Leikin, R. & Sriraman, B. (2022). Empirical research on creativity in mathematics (education): From the wastelands of psychology to the current state of the art. *ZDM—Mathematics Education*, *54*(1), 1–17. https://doi.org/10.1007/s11858-022-01340-y

Liljedahl, P. (2013). Illumination: An affective experience? *ZDM*, *45*(2), 253–265.

OECD. (2024). *PISA 2022 results: Creative minds, creative schools* (Vol. 3). OECD Pub.

Schindler, M. & Lilienthal, A. J. (2020). Students' creative process in mathematics: Insights from eye-tracking stimulated recall interview on students' work on multiple solution tasks. *International Journal of Science and Mathematics Education*, 18, 1565–1586.

Schindler, M. & Lilienthal, A. J. (2022). Students' collaborative creative process and its phases in mathematics: An explorative study using dual eye tracking and stimulated recall interviews. *ZDM—Mathematics Education*, *54*(1), 163–178.

Torrance, E. P. (1974). *Torrance Tests of Creative Thinking*. Scholastic Testing Service.