UPPER SECONDARY STUDENTS' IMPASSES DURING JOINT PROBLEM-SOLVING IN A "THINKING CLASSROOM"

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Researchers have found that desirable difficulties contribute to student learning. Therefore, we carried out a study that examines impasses in a teaching approach that mathematics teachers increasingly implement, thinking classrooms. In this study, we use video data from an upper secondary classroom to examine the impasses a group of three students encounter when solving a problem-solving task on vertical whiteboards and how they overcome the impasses. Impasses and navigation out of them occurred in various ways. We also found that the students depended on external assistance to overcome most of the impasses, drawing help from neighbouring groups and from the teacher. When the students navigated out of impasses by collaboration, the study illustrates that students established a joint problem space (JPS).

INTRODUCTION

In recent years, the teaching approach *thinking classrooms*, developed by Liljedahl (2016), has become increasingly prevalent in mathematics education internationally, and in Norwegian schools. In short, a thinking classroom involves dividing the students into random groups of 2-4, before letting the groups work jointly with problem-solving tasks and use vertical whiteboards as their work surfaces (Liljedahl, 2016). In this context, a problem-solving task can be understood as a task that students do not initially know how to solve. However, they should be able to use their previous experiences and mathematical knowledge to develop a solution to the problem. Throughout the years, many problem-solving models have been developed, typically describing the process as stepwise, systematic, and planned work, and without addressing nuanced and non-linear aspects and occurrences of failure and detours (Rott et al., 2021). However, there are situations where the problem-solving process is halted. Such situations can be termed impasses (Ohlsson, 2011).

Previous research indicates that desirable difficulties contribute to students' learning by enhancing knowledge construction and memory retrieval (Jonsson et al., 2014). When VanLehn et al. (2003) examined impasses, they found that for learning to be successful, and even to occur, students must reach an impasse. However, it can be challenging for students to move beyond an impasse and for teachers to facilitate this process, especially when aiming to avoid disruption of students' thinking or reduction of the complexity of the activity. There is a lack of research that include issues related to impasses in a thinking classroom context. To contribute to the research field our study examines the occurrences of impasses in thinking classrooms and how students

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navigate out of them, with or without peer or teacher facilitation. In particular, this study aims to answer the following research question: *When a group engages in problem-solving in a thinking classroom, which strategies do they use to overcome impasses and how are these strategies related to the groups' joint problem space?*

THEORETICAL BACKGROUND

The current study draws on the concepts of joint problem-solving and impasses. A joint problem space (JPS) is a socially negotiated knowledge structure comprising a shared understanding of goals, the current problem state, and available problem-solving actions (Roschell and Teasley, 1994). The concept is based on a conception of collaboration during joint problem-solving as "a coordinated, synchronous activity that is the result of continued attempts to construct and maintain a shared conception of a problem" (Roschell and Teasley, 1994, p. 70). During groupwork each student needs to contribute with their own ideas and resources to construct a JPS. Through dialogue, negotiation, and shared actions such as drawings and gestures, each individual contribution results in a shared goal, shared knowledge, or problem-solving strategies. To maintain the JPS, students must continuously monitor the process, identify and repair misconceptions or divergence, and choose appropriate strategies moving forward in the problem-solving process (Roschell and Teasley, 1994). The teaching approach thinking classrooms aims for students to develop knowledge and understanding collaboratively through discussion (Liljedahl, 2016). Therefore, establishing JPS may be regarded as a key condition for effective group work in a thinking classroom.

To map out students' problem-solving processes we will use a model that comprises four sequential steps that must be undertaken to solve a problem-solving task: (1) understanding the problem, (2) devising a plan, (3) carrying out the plan, and (4) looking back (Pólya, 1945). During the problem-solving process, students may reach impasses (Ohlsson, 2011). These are situations wherein students are unable to generate new solution strategies, stop attempting to reach a solution, or repeat previous strategies (Ohlsson, 2011; Fedor et al., 2015). Students can also become fixated on pursuing non-productive strategies, often referred to as a wild goose chase (Schoenfeld, 1987), which can be understood as another type of impasse.

Encountering impasses has some resemblance to struggle. Warshauer (2014) identified four categories of struggle that occur during challenging mathematical task-solving. Students struggle with (1) initiating a task, (2) carrying out a procedure or process, (3) explaining or understanding what they have done or why they select particular strategies, and (4) expressing misconceptions or errors. These types of struggles might be underlying reasons for impasses during problem-solving.

METHODS

Video data is collected to observe student groups in upper secondary mathematics classrooms as they work jointly with problem-solving tasks in a thinking classroom

context. Video observation is a suitable method for studying complex social situations (Blikstad-Balas, 2017), such as group work on vertical whiteboards. This method allows us to investigate a phenomenon in a naturalistic setting (Blikstad-Balas, 2017) and the collected videos can be reviewed multiple times, enabling detailed analysis (Derry et al., 2010), which is advantageous when aiming to analyse problem-solving processes in a classroom environment.

Data collection

The sample in this study is a subsample of a larger overarching study and consists of one student group: three students (aged 15–16) from an upper secondary class in theoretical mathematics. In addition, one of the neighbouring groups and the mathematics teacher, are also included when the sample group interacts with peers and their teacher. Written consent letters were signed by the participating students, their guardians for those under 16, and the class teacher. The data collection took place during a single lesson.

The groups worked with a problem-solving task, provided orally by the teacher, challenging them to find the best deal for a school visit to a museum:

> There are two museums that the class can be sent to. Tickets for museum A cost 90 Norwegian kroner per person, but there must be at least three people. Museum B costs 100 Norwegian kroner and there must be at least one person. Which museum is it most economical to send the class to?" (translated from Norwegian)

Analytical approach

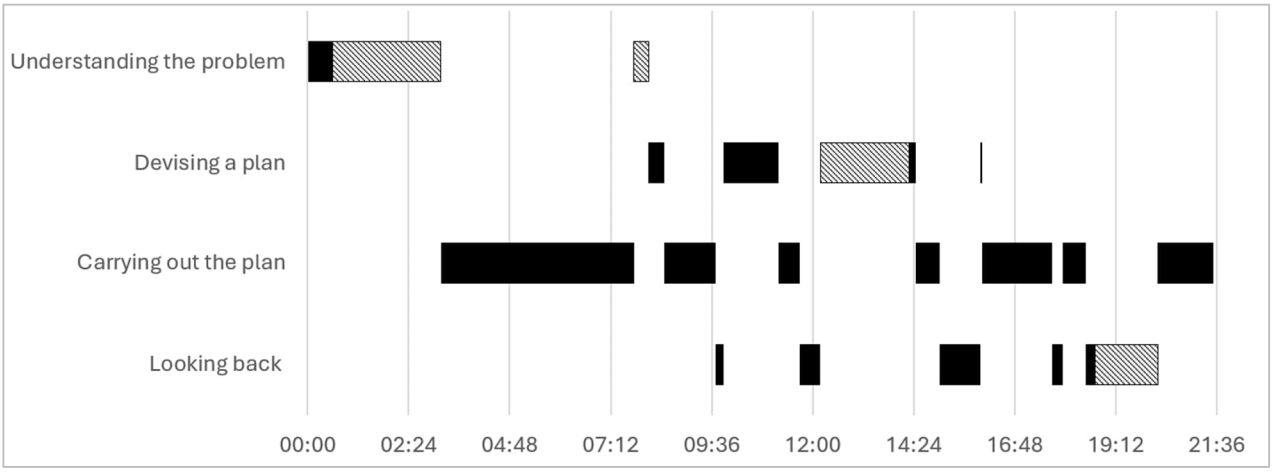
The video data were analysed in three stages, with a distinct analytical lens applied in each stage. First, the problem-solving stages were determined using the deductive codes *understanding the problem*, *devising a plan*, *carrying out the plan*, and *looking back* (cf. Pólya, 1945). Next, the deductive codes *unable*, *stopping*, *repeating*, and *wild-goose chase* (cf. Ohlsson, 2011; Schoenfeld, 1987) were applied to analyse moments of impasses. Lastly, the students' attempts to overcome impasses were analysed. We utilized inductive coding, resulting in the codes *asking teacher*, *asking neighbour group*, *looking at neighbouring vertical surface*, *collaboration*, and *teacher initiating guidance*.

Drawing on a timeline representation used for illustrating behavioural phases during mathematical problem-solving developed by Schoenfeld (1985), illustrations of the students' problem-solving processes was developed. The illustration consists of Pólya's four problem-solving stages and moments of impasses, with the time of identification and duration of each. In addition, another illustration was produced to depict the nature of each impasse, and the students attempts to overcome it. These sequences are subsequently described in detail, and excerpts of the dialogues are included to substantiate the interpretations.

RESULTS AND DISCUSSION

The group encountered a total of four impasses during their problem-solving. Figure 1 shows where in the problem-solving process these occurred and how long each lasted. The initial impasse occurred during the first step of the problem-solving process, *understand the problem*, and was the longest of the four impasses. The group did not understand how to mathematically interpret the task content in terms of equations. One of the students expressed "I didn't quite understand this". The group's first reaction was to look at the neighbouring group's vertical surface. A student in the neighbouring group articulated their reasoning aloud "That's 90 multiplied and then minus 3, no plus 3", and wrote the expression "$90x + 3$" on the vertical surface. One of the students in the studied group responded, "Huh? Why?". The neighbouring group attempted to explain, but the studied group disagreed, stating, "I don't think it will be that. It won't be that". The studied group stopped interacting with the neighbouring group and continued the discussion internally. They proposed their own ideas and collaboratively constructed the expression "$90x + 270$" for the costs of going to museum A. For the second time they looked at the neighbouring vertical surface and formulated the expression for the costs of going to museum B as the neighbouring group had written it, "$100x + 100$". However, they were still uncertain regarding how to interpret the task and asked the teacher for assistance. One of the students explained their understanding of the problem to the teacher, and the teacher confirmed their interpretation. This led to the group being able to continue the problem-solving process. Figure 1 shows that much of the first stage in the problem-process was devoted to attempts to overcome the impasse. The students spent a total of 2 minutes and 34 seconds on this impasse.

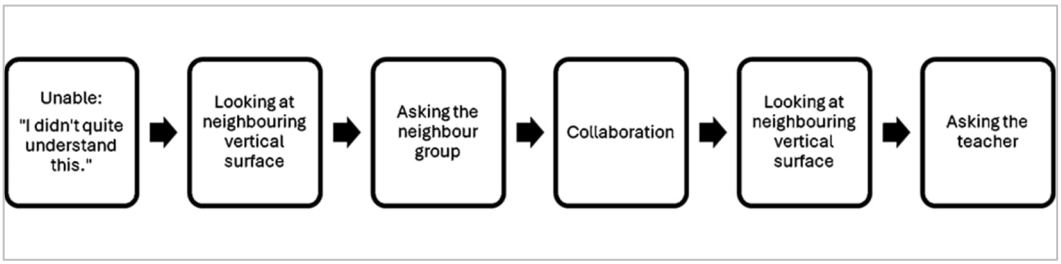
Figure 1: The group's problem-solving process. The shaded areas show where impasses occur.



The initial impasse can be categorized as the impasse type *unable* (Ohlsson, 2011), resembling one of the struggles identified by Warshauer (2015), namely *struggle with initiating a task*. The group used the strategies *asking the neighbour group* and *looking*

at neighbouring vertical surface. The establishment of JPS depends on students possessing ideas and resources (Roschell and Teasley, 1994). During this sequence, the students turned to the neighbouring group, which might indicate that they lacked sufficient ideas and resources within their own group and needed to obtain them from outside their group. The group did not agree with the neighbouring group's interpretation of the problem. Consequently, these strategies did not enable them to overcome the impasse. However, the group was able to generate their own ideas after interacting with the neighbouring group. It appears that the group needed something to base their discussion on before they could progress. The discussion resulted in shared knowledge among the student which may indicate that the group constructed a JPS. *Asking the teacher* was also a part of their navigation out of the impasse, although the teacher's contributions was limited to listening and confirming the group's interpretation. Thus, one may question whether asking the teacher was necessary for overcoming this impasse. Alternatively, one can argue that the teachers' confirmation functioned as an authority, prompting the students to consider their own approach as worthy of further pursuit. In total, the group used four strategies in this sequence, and figure 2 presents an overview of the group's path out of the initial impasse.

Figure 2: The group's process of overcoming their first impasse.

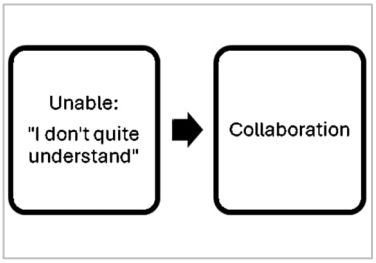


After overcoming the first impasse the group started to carry out a plan but soon realized that they still did not understand the task. They returned to the first stage of problem-solving *understanding the problem*, and a second impasse occurred. The group halted the calculation they had been doing and fell silent. The students said, "I don't quite understand", "I don't really get the problem" and "I just don't understand why?". Then one of the students noted that there are many people in a class, and another one specified that there must be more than three people, which would make museum A the best option. The third student characterized the task as a trick question, suggesting that the problem was simpler than they had assumed. The students' exchange of ideas led them to devising a plan, and the impasse was overcome.

In the beginning the sequence can be categorized as the impasse type *stopping* (Ohlsson, 2011). After being silent for a while, the students express their struggle with understanding the problem, and the impasse type transition to *unable* (Ohlsson, 2011). All the individual contributions result in a problem-solving strategy, which may indicate that the group constructed a JPS. Through dialogue, negotiation, and shared

actions each student's contribution resulted in a problem-solving strategy and the group being able to overcome the impasse through *collaboration* (see figure 3).

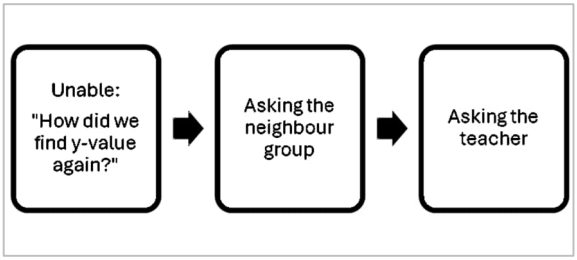
Figure 3: The group's process of overcoming their second impasse.



The third impasse occurred when the group tried to figure out when a given function is greater than another function. The group aimed to determine the y-value for the intersection but was unable to generate a solution strategy, indicating an impasse. Attempting to navigate out of the impasse, the group reached out to a neighbouring group but failed to understand their provided suggestions. Therefore, the group turned to their teacher in a new attempt to navigate out of the impasse. The teacher replied by decomposing the solution with several questions. This led to the group being able to determine the y-value.

The sequence can be categorized as the impasse type *unable*. Their impasse resembles *struggle with carrying out a procedure or process*, one of the categories of struggles identified by Warshauer (2015). The impasse can also be understood as a *wild goose chase* (Schoenfeld, 1987), as the solution strategy fails to provide information about when the function is greater and thus is non-productive. The teacher guided them out of the impasse type unable by posing questions decomposing the solution strategy. This indicates that the students needed assistance from the teacher in uncovering and structuring the necessary knowledge to get "back on track" in their problem-solving process. However, the group was still on a wild goose chase. A likely reason for this, is that the students were unaware of this impasse. In addition, the teacher did not notice that their solution strategy fails to provide information about when the function is greater or chose not to address it to let them discover it on their own. To summarize, the group used two strategies when attempting to overcome this impasse (see figure 4). They became able to determine the y-value, but were still on a wild-goose chase, so the impasse was only partly overcome.

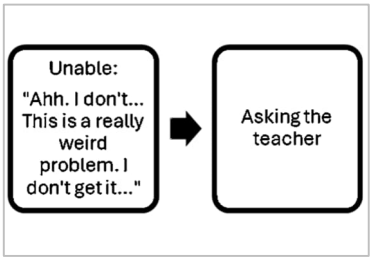
Figure 4: The group's process of partly overcoming their third impasse.



The group decided to use the x-value for the intersection in the equations for both museums to find the y-value. After carrying out their plan, the group got two different answers, 1530 NOK for museum A and 1800 NOK for museum B, which led them to encounter another impasse. One student became visibly frustrated and said, “Ahh. I don’t... This is a really weird problem. I don’t get it”. The group asked the teacher for assistance. They explained why they were unable to proceed, and the teacher responded, “Are you confident about that calculation? “. The students replied that they were unsure and then began to redo the calculation. The teacher left them to continue the work. They discovered the error in the previous calculation and thereby overcame the impasse.

The impasse can be categorized as the impasse type *unable*, resembling Warshauer’s (2015) *struggle with expressing misconceptions or errors*. Attempting to progress, the group used the strategy *asking the teacher* (see figure 5). After the teacher asked the group if they were confident about their calculation, the students became aware of a possible reason for why they had obtained two different answers. This indicates that, in some cases, students need guidance to discover their own errors, which is an important part of maintaining a JPS (Roschell and Teasley, 1994). In this sequence, the teacher might assist them in maintaining their JPS by reducing the complexity of the students’ impasse. It is likely that the students believed their difficulties stemmed from their lack of understanding the task, when in fact the issue was with the calculation.

Figure 5: The group's process of overcoming their fourth impasse.



CONCLUDING REMARKS

To summarize, this study has shown that impasses and navigation out of them during collaborative problem solving can occur in various ways. The three students encountered impasses in their problem-solving process because they initially were unable to understand the task and to carry out a plan, a phenomenon frequently observed in previous research (Schoenfeld, 1985; Warshauer, 2014). Our observations confirmed that the students did not know how to solve the task beforehand, indicating that the task could be considered as a mathematical problem for the group. The students depended on external assistance to overcome the impasses, drawing help from neighbouring groups and from the teacher. Of all the strategies for overcoming impasses, the only one we did not observe was *teacher initiating guidance*. It is noteworthy that in the two sequences in which students sought assistance from a neighbouring group, they subsequently asked the teacher for help, indicating that even in student-centred activities, the teacher is regarded as an authority. When

collaboration was a part of the navigation out of an impasse, the study illustrates that the students established a joint problem space (JPS), and that this JPS can emerge from interaction with a neighbouring group.

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