EPISTEMOLOGICAL BELIEFS AND PEDAGOGICAL APPROACHES TO GEOMETRY: A PHENOMENOLOGICAL STUDY OF PRE-SERVICE PRIMARY TEACHERS

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This study examines preservice primary teachers' epistemological beliefs about geometry and their implications for pedagogical orientations. Using a phenomenological qualitative design, data were collected through a structured reflective form focusing on definitions of geometry, interpretations of Poincaré's statement, conceptual meanings of basic geometric objects, and beliefs about the source of knowledge. Thematic analysis revealed two dominant epistemological orientations: a school-oriented/instrumental view and a life-oriented/ontological view of geometry. Based on these findings, the study proposes the Dual Epistemological Filter Model, offering a theoretical lens for understanding epistemologically grounded differences in preservice teachers' approaches to geometry teaching.

INTRODUCTION

Geometry is widely recognized as a central domain in mathematics education, supporting the development of spatial reasoning, mathematical argumentation, and problem solving from the early grades onward (Battista, 2007; Sinclair & Bruce, 2015). Beyond the recognition or drawing of shapes, geometry involves relational thinking, generalization, and the construction of mathematical meaning. Nevertheless, research consistently shows that both students and pre-service teachers (PST) often experience geometry as a rule-driven, drawing-oriented, and examination-focused subject, which limits its conceptual depth and meaning-making potential (Boakes, 2009). Such challenges in geometry teaching cannot be explained solely by instructional methods or materials. How teachers and pre-service teachers conceptualize geometry—what kind of knowledge they believe it represents and how geometric knowledge is justified—plays a critical role in shaping pedagogical decisions. From this perspective, teachers' epistemological beliefs about mathematical knowledge are particularly significant. Epistemological beliefs refer to individuals' assumptions about the source, certainty, and justification of knowledge, and they strongly influence teaching and learning processes (Hofer & Pintrich, 1997; Schommer-Aikins, 2004). Whether pre-service teachers view geometric knowledge as transmitted by external authority or as constructed through reasoning and experience has important implications for how geometry is taught and learned. Despite this importance, the epistemological dimension of geometry has received relatively limited attention in qualitative research, particularly in studies focusing on pre-service primary teachers. Existing studies have

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often emphasized attitudes, self-efficacy, or instructional practices, while teachers' epistemological orientations shaped by prior geometry learning experiences remain underexplored. Yet geometry plays a critical role in teacher education by supporting transitions from concrete visual representations to abstract concepts and by fostering reasoning-based mathematical understanding.

This study presents a qualitative investigation of pre-service primary teachers' epistemological beliefs about geometry and their pedagogical orientations. It examines how pre-service teachers define geometry, how they make sense of fundamental geometric concepts (such as point, line, and plane), and how they justify the source of their geometric knowledge. To make participants' epistemological assumptions visible, Henri Poincaré's statement "*Geometry is the art of good reasoning from poor drawings*" was used as an epistemological prompt. This statement was selected for its potential to problematize taken-for-granted assumptions about the role of visual representations and to foreground reasoning as a core component of geometric knowledge. The central contribution of the study is the proposal of a Dual Epistemological Filter Model that explains how pre-service teachers' conceptions of geometry are shaped through two contrasting epistemological orientations: a school-oriented/instrumental filter, in which geometry is viewed as a rule-based, authority-driven subject, and a life-oriented/ontological filter, in which geometry is understood as a meaning-making and reasoning-centred activity connected to lived experience. These orientations are reflected consistently across participants' definitions of geometry, interpretations of Poincaré's statement, conceptualizations of fundamental geometric ideas, and beliefs about the source of knowledge.

Accordingly, the study is guided by the following main research question: How are pre-service primary teachers' epistemological beliefs about geometry and their pedagogical orientations structured?

METHOD

This study was designed as a qualitative inquiry aimed at exploring pre-service elementary teachers' epistemological beliefs about geometry and the ways these beliefs relate to their pedagogical orientations. Given the focus on participants lived experiences, perceptions, and meaning-making processes regarding geometry, the study adopted an exploratory qualitative design with phenomenological sensitivity. Phenomenological approaches seek to capture how individuals experience a given phenomenon and how they construct meaning from those experiences, thereby providing insight into the essence of the phenomenon under investigation (Moustakas, 1994).

In line with this perspective, the study prioritizes depth of experience and contextual relevance over representativeness or generalizability. Particular attention was given to foregrounding participants' personal learning histories and intuitive understandings of geometry rather than responses shaped by formally structured instructional practices.

The participants consisted of 40 third-year pre-service elementary teachers enrolled in a teacher education program at a public university in Turkey. All participants were undergraduate students actively taking mathematics education courses as part of their program. Consistent with the phenomenological orientation of the study, participants with prior structured teaching experience in geometry were intentionally excluded in order to better capture epistemological beliefs grounded in lived learning experiences.

The research data were collected through a structured reflective form consisting of open-ended questions. The form was designed around four core dimensions aimed at making preservice primary teachers' individual experiences with geometry, epistemological beliefs, and ways of thinking explicit. Definition of Geometry: Participants were asked the question "What is geometry?" to express their personal definitions of geometry. This dimension aimed to reveal whether preservice teachers conceptualized geometry primarily as a curricular subject or as a mode of thinking. Epistemological Reflection: Henri Poincaré's statement, "Geometry is the art of correct reasoning on incorrect figures," was presented to participants, who were asked to reflect on the relationship between visual representations and reasoning. This statement was used as an epistemological prompt to elicit participants' beliefs about the nature of mathematical knowledge and how geometric knowledge is constructed through representations and reasoning. Conceptual Meaning-Making: Participants were asked to define and interpret basic geometric concepts such as point, line, and plane. This dimension enabled an examination of whether their conceptualizations were predominantly rote/formal or relational/conceptual in nature. Awareness of the Source of Knowledge: Through the question "Where do you know this from?", participants were asked to articulate the sources on which they based their geometric knowledge (e.g., school, teachers, textbooks, everyday life, personal experiences). This dimension aimed to identify whether knowledge was grounded primarily in external authority or experiential understanding. Taken together, these four dimensions were designed to reveal not only what preservice teachers know about geometry, but also how they know it, on which experiences their knowledge is based, and within which epistemological frameworks it is constructed. The development of the reflective form was informed by questions and activities drawn from the geometry section of Mathematics for Elementary Teachers, which conceptualizes geometry not merely as drawing shapes or applying procedures, but as involving definition, visualization, reasoning, conceptual connections, and justification. This perspective provided a theoretical and pedagogical reference point for structuring the reflective questions used in the study.

Data were analysed using an inductive thematic analysis approach. The analysis began with line-by-line reading of participants' responses, followed by the identification of meaning units and the generation of initial codes grounded in the data. Similar codes were then grouped into broader themes, and relationships among themes were examined comparatively. The resulting patterns clustered around two contrasting epistemological orientations: a school-oriented/instrumental view of geometry and a

life-oriented/ontological view. These themes were interpreted in dialogue with relevant theoretical perspectives in the literature, leading to the development of the Dual Epistemological Filter Model, which serves as an integrative framework for explaining participants' approaches to geometry. Ethical principles were observed throughout the study, with voluntary participation ensured and participant identities anonymized through coding (for example PST1, PST2 etc.).

RESULTS

This section presents the findings derived from the thematic analysis of pre-service elementary teachers' written reflections on geometry. To provide an integrated and analytically transparent account, the results are first summarized in Table 1, which displays the dominant epistemological orientations identified across themes, along with illustrative participant excerpts. The table is followed by a synthesized narrative that highlights cross-theme patterns and epistemological consistencies.

Table 1: Overview of Themes and Epistemological Orientations in Pre-service Teachers' Geometry Conceptions

Theme	Epistemological Orientations	Illustrative PSTs' statements
Defining Geometry	School-oriented / Instrumental	"Geometry is a subject where we draw shapes and apply formulas taught at school." (PST2)
	Life-oriented / Meaning-based	"Geometry is a way of understanding order in nature and daily life." (PST8)
Nature of Geometric Knowledge (Poincaré Reflection)	Reasoning-centred	"Even if the drawing is poor, what matters is the reasoning behind it." (PST6)
	Visual-centred	"If the drawing is incorrect, it becomes difficult to understand the concept." (PST3)
Conceptualization of Basic Concepts	Formal–definitional	"A point is the trace left by a pencil." (PST11)
	Metaphorical–abstract	"A point can be thought of as the beginning of everything." (PST15)
Source of Knowledge	Authority-based	"We know these concepts from teachers and textbooks." (PST1)
	Experience-based	"We learn geometry from observing shapes in everyday life." (PST9)

Across all four themes, the findings reveal two coherent and recurring epistemological orientations that cut across participants' definitions, reflections, conceptualizations, and accounts of knowledge sources. Some pre-service teachers consistently positioned geometry as a school-bound, procedural, and authority-driven subject, emphasizing correct representations, formal definitions, and externally validated knowledge. Others, in contrast, articulated a life-oriented and ontological view of geometry, foregrounding reasoning, abstraction, personal meaning-making, and experiential grounding. Importantly, these orientations did not appear in isolation within single themes. Rather, participants who defined geometry instrumentally also tended to rely on visual accuracy and authoritative sources, whereas those who framed geometry as a way of thinking more often emphasized reasoning over drawing, metaphorical abstraction of concepts, and experiential sources of knowledge. This cross-theme consistency suggests that pre-service teachers' responses are shaped by underlying epistemological filters rather than by isolated opinions about geometry.

Taken together, the results indicate that pre-service teachers do not hold a uniform view of geometry; instead, their understandings are structured around distinct epistemological orientations that influence how they define geometry, interpret representations, conceptualize foundational concepts, and justify what counts as geometric knowledge. These patterned orientations form the empirical basis for the Dual Epistemological Filter Model, which is elaborated in the following section.

DISCUSSION AND CONCLUSION

This result aligns with previous research emphasizing that teachers' beliefs about the nature of mathematical knowledge play a central role in shaping instructional decisions and pedagogical orientations (Hofer & Pintrich, 1997; Schommer-Aikins, 2004).

Epistemological Tensions in Interpreting Geometry

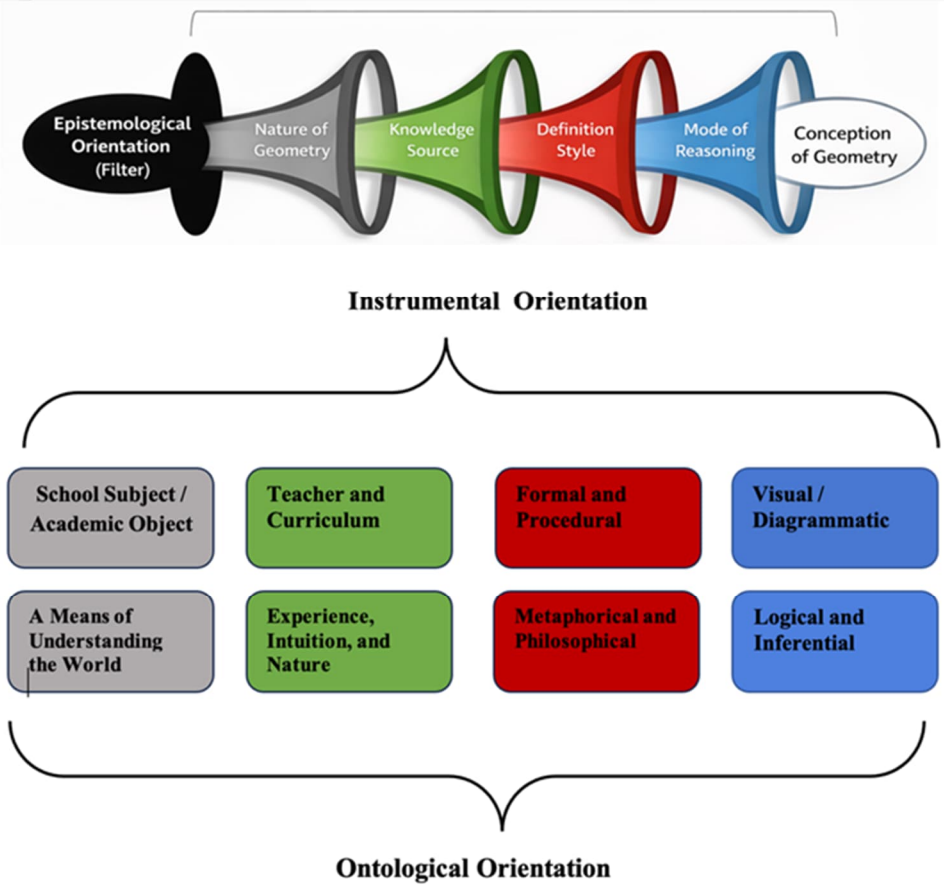
The first dimension of the findings concerns the meanings that preservice teachers attribute to geometry. The distinction between those who define geometry as a school-based, technical, and exam-oriented subject and those who associate it with life, nature, and thinking reflects not merely a definitional difference but an epistemological positioning regarding the nature of knowledge. This distinction aligns with studies emphasizing that mathematics learning is not only a cognitive but also an epistemological process (Schoenfeld, 1985; Ernest, 1989). Similarly, responses to Poincaré's statement made visible a tension regarding whether geometric knowledge is grounded primarily in visual representations or in reasoning. The literature likewise highlights that geometric thinking develops not solely through visual representations but through the construction of logical relationships underlying those representations (Duval, 1995; Sinclair & Bruce, 2015). In this sense, participants' drawing-centred approaches and their reasoning-oriented approaches can be interpreted as reflections of different epistemological orientations. This divergence becomes even more pronounced in the ways basic geometric concepts are conceptualized. Interpreting

concepts such as point, line, and plane through textbook-based and formal definitions corresponds to an instrumental understanding, whereas associating them with metaphorical and experiential meanings points to a relational understanding (Skemp, 1976). Likewise, responses concerning the source of geometric knowledge indicate differing epistemic orientations regarding whether knowledge is grounded in external authority or in experience and awareness. This finding is consistent with studies showing that preservice teachers' prior learning experiences play a significant role in shaping their epistemological beliefs (Lortie, 1975; Raymond, 1997).

The Dual Epistemological Filter Model

Taken together, the findings suggest that pre-service teachers' interpretations of geometry are systematically shaped by underlying epistemological filters rather than by isolated beliefs or individual preferences. In response, this study proposes the Dual Epistemological Filter Model as a theoretical synthesis of the patterns identified through thematic analysis. The model was not imposed a priori but emerged inductively from the data as a coherent explanatory framework. Figure 1 illustrates the Dual Epistemological Filter Model, which synthesizes the patterned relationships identified across participants' definitions of geometry, interpretations of geometric knowledge, conceptions of basic geometric concepts, and beliefs about the sources of this knowledge.

Figure 1: A layered epistemological–ontological model of geometry conceptions



According to the model, some pre-service teachers interpret geometry through a school-oriented/instrumental filter, viewing it as a rule-governed, externally validated body of knowledge. This orientation aligns with epistemological beliefs that conceptualize mathematical knowledge as fixed, transmissible, and authority-driven (Ernest, 1989; Schommer-Aikins, 2004). In contrast, others adopt a life-oriented/ontological filter, conceptualizing geometry as a meaning-making activity embedded in reasoning, experience, and interpretation. This latter orientation resonates with higher levels of geometric thinking described by Van Hiele (1986) and with Skemp's (1976) notion of relational understanding.

Importantly, the model suggests that similar instructional experiences may lead to qualitatively different understandings of geometry depending on the epistemological filter through which they are interpreted. Making these filters visible within teacher education contexts may therefore be a critical step toward supporting more reflective and conceptually grounded pedagogical practices. The layered structure of the proposed Dual Epistemological Filter Model is depicted in Figure 1.

Implications for Teacher Education

The purpose of this study was to examine pre-service elementary teachers' epistemological beliefs about geometry and the potential pedagogical implications of these beliefs. The findings indicate that pre-service teachers' ways of defining geometry, conceptualizing the nature of geometric knowledge, making sense of fundamental geometric concepts, and justifying the sources of this knowledge are not independent or fragmented. Rather, these dimensions form coherent and patterned orientations that reflect underlying epistemological positions. These findings indicate that discussions of geometry teaching in teacher education cannot be limited to methods, materials, or activities alone. Transforming preservice teachers' pedagogical approaches requires, first and foremost, making their epistemological assumptions about the nature of geometric knowledge visible and open to critical reflection. This emphasis is consistent with research highlighting the importance of epistemological awareness in teacher education (Ball et al., 2008; Furinghetti & Pehkonen, 2002). Using Poincaré's statement as an epistemological prompt enabled preservice teachers to question the relationship between visual representations and reasoning, thereby rendering their implicit assumptions about the nature of geometric knowledge explicit. Similarly, engaging with fundamental geometric concepts at metaphorical and experiential levels may support preservice teachers in viewing geometry not merely as content to be transmitted, but as a domain of thinking to be jointly constructed with students.

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